Stabilizing a Class of Switched Nonholonomic Mechanical Systems

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Abstract: Structurally reconfigurable or variant robot systems can provide better mobility and environmental adaptability. In this paper it is shown that a class of nonholonomic constraints robot systems can be changed to a class of special linear time varying (LTV) systems, and by applying switching control strategy the control singular problems of nonholonomic systems caused by the local coordinates can be overcome, and provides a flexible approach of optimal motion planning for mobile robotic systems. For variant robot systems with switched discontinuous dynamics, it is shown that the switching control approaches can be used to stabilize a class of switched heterogeneous nonholonomic systems. Some numerical simulation results also demonstrate the effectiveness of the control strategy proposed in this paper.

Keywords: Nonholonomic constraints; Robots; Stabilization; Switching control; Switched systems.

1. INTRODUCTION

Reconfigurable or variant robot systems have a wide application prospect in modern industry, extra-terrestrial exploration, and field working [1-3]. However, variant robot systems are commonly a class of hybrid dynamic systems [4], of which the motion control is generally a difficult subject that has not been thoroughly studied so far. Switched systems are a special class of hybrid systems and have been investigated more than twenty years [5, 6]. Due to the complexity of the control problems of switched systems, the switched linear or nonlinear systems are currently still an important and rather active research direction in control fields [7-9].

For investigating the feasibility of developing variant mobile robot systems, we study the stabilization issues of a class of switched nonholonomic systems, which are a class of special hybrid nonlinear dynamic systems [7-9]. For the switched systems, to date the main research results are presented for switched linear time invariant (LTI) systems [7, 8] or some switched nonlinear systems with special properties [9]. For the switched nonholonomic systems, only a few research results can be found in literatures. In this research direction, switched nonholonomic systems are primarily caused by introducing some kinds of discontinuous feedback based on the Brockett's theorem [10], which shows that nonholonomic systems can-not be stabilized by continuous differentiable, time invariant, pure state feedback control law. As shown by Bloch in [11], owing to the discontinuous feedback property sliding mode controller can be applied to stabilize some nonholonomic systems, although the method is difficult

to be generalized to stabilize general nonholonomic systems. In reference [12] the time invariant sliding mode control for nonholonomic systems was also extended to time varying sliding mode control that relied on finding a set of sufficiently smooth periodic odd functions. Astolfi also showed that the first order nonholonomic systems could be mapped to disconti-nuous nonholonomic systems by discontinuous coordi-nate transformation [13], and then the discontinuous systems can be used to design an exponentially stable controller for stabilizing the original systems. In [14] it was also shown that an underactuated autonomous surface vessel could be stabilized by a switching controller on the basis of the discontinuous coordinate transformation, which is similar to that used in [13]. With the aid of fractional power feedbacks, in the literature [15] switching controllers were also used to stabilize several kinds of underactuated nonholonomic systems in finite time. In [16] a logic-based switching controller was presented to stabilize the nonholonomic integrator. A similar strategy has also been used to stabilize a nonholonomic system with robots with a differential-drive mechanism [17], Pendubot systems [18], or mobile robotic systems [19]. Unfortunately, all of these presented approaches for stabilizing the nonholonomic systems or underactuated systems com-monly relate to the specific controlled plants, and the controlled plants are usually in single dynamic mode. From the point of view of switching control systems, these switched nonholonomic subsystems are *homo-geneous* [20].

For the purpose of searching for a feasible control scheme for reconfigurable or variant mobile robot systems, stabilization of switched nonholonomic mech-anical systems with *hetero- generous* [21] subsystems is investigated in this paper. By combing the globally

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exponential coordinate transformations and switching control approaches, we show that a class of variant mobile robot systems can be globally stabilized under certain conditions. Since a class of nonholonomic systems could be changed to a kind of special LTV system by time varying coordinate transformations (TVCT) [22, 23], and stabilizing the nonholonomic systems can be changed to stabilize the special LTV systems, then on the basis of switching control theory, we show that not only the global stabilization issues of nonholonomic systems with single model can be resolved conveniently, but also globally stabilizing a class of variant mobile robot systems with multi-models is possible under rather relaxed conditions.

This paper is organized as follows. In section Ⅱ, the theoretical basis of stabilizing a class of special switched LTV systems are presented. The main results of the paper are presented in section Ⅲ, where two propositions based on the TVCT methods and two theorems for stabilizing the switched nonholonomic systems are presented. To the best knowledge of the authors, it is the first time to discuss the global stabilization issues of switched nonholonomic mechanical systems with multi locomotion modes. We show that a class of multimode nonholonomic systems can be globally stabilized by isomorphic controllers, even though the different modes of the nonholonomic system are heterogeneous. This is helpful for simplifying the controllers of complex plants, such as the variant robot systems. In section Ⅳ, a variant mobile robot system, which is the combination of a wheeled mobile robot and a hovercraft robot, is introduced and the numerical simulation results are given in detail for the purpose of demonstrating the effectiveness of the proposed switching control strategy for stabilizing the switched nonholonomic systems. The last section includes the conclusions and some discussions.

2. STABILITY OF A CLASS OF SWITCHED LTV SYSTEMS

In this article, the following continuous time LTV systems are considered

$$\dot{x} = \left(A_1 + A_2(t)\right)_\sigma x + B_\sigma u \qquad (1)$$

where $x \in R^n$ is the continuous state, $u \in R^m$ is the control input, $\sigma : [0, \infty) \to \aleph$ is a piecewise constant switching signal, $\aleph := \{1, 2, \ldots, s\}$ is a index set, the matrix pairs $\left(A_{1\sigma}, B_\sigma\right)$ are controllable, and the time varying matrix $A_{2\sigma}(t)$ satisfies the relationships

$$\lim_{t \to \infty} A_{2\sigma}(t) = 0, \text{ and } \int_0^\infty \left\|A_{2\sigma}(t)\right\| dt < \infty . \qquad (2)$$

Due to the following lemma, the issues of stabilizing a class of switched LTV systems can be further discussed.

Lemma 1. [24] The LTV system

$$\dot{x} = \left(A_1 + A_2(t)\right)x , \ x(0) = x_0 \in R^n \qquad (3)$$

where A_1 is a *Hurwitz* matrix, and $A_2(t)$ is a time-varying continuous matrix. If $A_2(t)$ satisfies the conditions (2), i.e., $\lim_{t \to \infty} A_2(t) = 0$ and $\int_0^\infty \left\|A_2(t)\right\| dt < \infty$, then the LTV system (3) is exponentially stable.

Remark 2. Lemma 1 is included in some literatures, such as [24], but its proof is not very clear. For the purpose of helping to understand the main results of the paper, Lemma 1 is concisely analyzed as follows.

The solution of the LTV system (3) can be written as

$$x(t) = x_0 + \int_0^t \left(A_1 + A_2(s)\right)x(s)ds$$

It can be shown that

$$\|x(t)\| \leq \left\|x_0 + \int_0^t A_1 x(s)ds\right\| + \int_0^t \left\|A_2(s)x(s)\right\|ds . \qquad (4)$$

The first term of the right hand side of the inequality (4) satisfies

$$\left\|x_0 + \int_0^t A_1 x(s)ds\right\| \leq e^{-\alpha t}\|x_0\|$$

where α is a positive constant that can always be found since A_1 is a Hurwitz matrix. According to Cauchy inequality, the second term of the right hand side of (4) satisfies

$$\int_0^t \left\|A_2(s)x(s)\right\|ds \leq \left(\int_0^t \left\|A_2(s)\right\|ds\right)\left(\int_0^t \|x(s)\|ds\right),$$

and due to the given condition $\int_0^\infty \left\|A_2(t)\right\|dt < \infty$, there exists a sufficiently large constant M such that $\int_0^t \left\|A_2(s)\right\|ds < M$, $t \in [0, T]$. Accordingly, for the inequality (4), we have $\|x(t)\| \leq e^{-\alpha t}\|x_0\| + M\int_0^t \|x(s)\|ds$ for $t \in [0, T]$.

According to Gronwall's inequality (see [25]) it follows that

$$\|x(t)\| \le e^{(-\alpha + M)t}\|x_0\|, \ t \in [0, T].$$

On the other hand, by the given condition $\lim_{t \to \infty} A_2(t) = 0$, we can conclude that there exists a sufficiently large time T, so that M is sufficiently small and then $-\alpha + M < 0$. Therefore the LTV system (3) is exponentially stable. $\diamond$

On the basis of Lemma 1, the issues of stabilizing the switched LTV systems (1) can be simplified to that of stabilizing the linear time invariant (LTI) systems

$$\dot{x} = A_\sigma x + B_\sigma u, \ \sigma \in \aleph \tag{5}$$

where $A_\sigma \in R^{n \times n}$ and $B_\sigma \in R^{n \times m}$ are constant matrices. Since all subsystems of (1) are assumed to be controllable, all the pairs of $[A_{1\sigma} \ \ B_\sigma]$ ($\sigma \in \aleph$) are controllable. Suppose the feedback $u = -K_\sigma x$ ($\sigma \in \aleph$) stabilize the subsystem (5), where $K_\sigma, \sigma \in \aleph$ are the gain matrices of each mode respectively, then the closed-loop subsystems

$$\dot{x} = \overline{A}_\sigma x, \ \sigma \in \aleph \tag{6}$$

are exponentially stable, and $\overline{A}_\sigma = A_\sigma - B_\sigma K_\sigma, \ \sigma \in \aleph$ are Hurwitz matrices. It is well known that a switched system with all stable modes may be unstable under inappropriate switching modes, while a switched system with some unstable modes may be stabilized by proper switching modes. For the issues of stabilizing the switched nonholonomic systems, both of the two aspects are involved. The fact is that, on one hand the nonholonomic systems are a class of special nonlinear systems of which the linear approximation systems are general uncontrollable [11,12]. On the other hand, owing to the Brockett's theorem [10] that demonstrates a nonholonomic constraint system could not be stabilized by any time invariant and smooth pure state feedback, discontinuous controllers are often employed to stabilize the nonholonomic systems.

The nonholonomic systems generally appear in mechanical systems, and it is different from a theoretically switched system or a switched electronic/circuit system. Consequently the switched nonholonomic systems are generally a class of slow switching systems due to the non-ignorable system inertia. As an important foundation of investigating the stabilization of switched nonholonomic systems, the slowing switching theorem is presented as follows since it will be used in the sequel.

Lemma 3 [7]. Assume that all subsystems in a switched linear system are exponentially stable. There exists a scalar $\tau_a^* > 0$ such that the switched system is exponentially stable if the average dwell time is larger than τ_a^*.

Remark 4. The *average dwell time* τ_a for a switching signal σ is defined to be $\tau_a \le \dfrac{t - \tau}{N_\sigma(t, \tau) - N_0}$, where $t > \tau > 0$ and $N_\sigma(t, \tau)$ denotes the number of mode switches of a given switching signal σ over the interval (τ, t), the scalar $N_0 \ge 0$ is the *chatter bound* that is the maximal number of allowable switching over the interval (τ, t). In other words, in order to exponentially stabilize a switched linear system that has stable modes, the number of mode switches over the interval (τ, t) should satisfy $N_\sigma(t, \tau) \le N_0 + \dfrac{t - \tau}{\tau_a}$. Therefore, if the average dwell time τ_a is sufficiently large, then the switched linear system (6) is exponentially stable with the assumption that all the subsystems are exponentially stable.

3. STABILIZING A CLASS OF SWITCHED NONHOLONOMIC SYSTEMS

Nilpotent or nilpotentizable nonholonomic systems are a class of important mechanic systems in robotics field [26]. An essential property of the nilpotent or nilpotentizable nonholonomic systems is that the special nonlinear systems can be possibly changed to a chained form by nonlinear coordinate and feedback transformations [27]. For the first-order nonholonomic systems with two inputs, it is shown that the nonlinear systems could be globally transformed into the following normal form.

$$\dot{x}_1 = u_1, \qquad \dot{x}_i = u_1 x_{i+1}, \qquad \dot{x}_n = u_2 \tag{7}$$

Where $i = 2, \cdots\cdots, n - 1$, x are the state variables of the system, u_i, $i = 1, 2$ are the inputs. By the method of TVCT presented in [22, 23], we can present the following proposition, which shows that the chained form systems (7) can be further transformed into a LTV system.

Proposition 5: By applying the following TVCT

$$y_i = x_i e^{\alpha t}, i = 2, 3, \ldots, n - 1$$
$$y_n = x_n \tag{8}$$

where t is time variable, and selecting an input

$$u_1 = c_0 e^{-\alpha t} + c_1 e^{-\beta t} \tag{9}$$

where c_0 and c_1 are two constants defined by the initial state $(\int_0^t x_1(s)ds, x_1(0))$ of the subsystem $\dot{x}_1 = u_1$ at $t = 0$, and $\beta > \alpha > 0$ are two arbitrary constants. The chained form nonlinear system (7), excluding the linear subsystem $\dot{x}_1 = u_1$, can be transformed into the following LTV system

$$\dot{y} = (A_1 + A_2(t))y + Bu_2 \tag{10}$$

where $y = [y_2 \quad y_3 \quad \dots \quad y_n]^T$, $B = [0 \quad \cdots \quad 0 \quad 1]^T$,

$$A_1 = \begin{bmatrix} \alpha & 0 & \dots & 0 & 0 \\ 0 & \alpha & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & \alpha & 0 \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}, \text{ and}$$

$$A_2(t) = \begin{bmatrix} 0 & c_0 e^{-\alpha t} + c_1 e^{-\beta t} & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & c_0 e^{-\alpha t} + c_1 e^{-\beta t} \\ 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Proof: It is clear that the linear subsystem $\dot{x}_1 = u_1$ of (7) is exponentially stable by applying the input (9). By the TVCT given by (8) for $i = 2,3,\dots,n$, it follows that

$$\begin{aligned} \dot{y}_i &= \dot{x}_i e^{\alpha t} + \alpha x_i e^{\alpha t} \\ &= u_1 x_{i+1} e^{\alpha t} + \alpha y_i \\ &= \left(c_0 e^{-\alpha t} + c_1 e^{-\beta t}\right) x_{i+1} e^{\alpha t} + \alpha y_i \end{aligned}$$

for $2 \le i \le n-1$. Note that $x_{i+1} = y_{i+1} e^{-\alpha t}$ due to the TVCT defined by (8), then it follows that

$$\dot{y}_i = \left(c_0 e^{-\alpha t} + c_1 e^{-\beta t}\right) y_{i+1} + \alpha y_i.$$

Note that the constant matrix A_1 in (10) is given by the coefficients of the time-invariant terms of $\dot{y}_i$ for $i = 2,3,\dots,n$, while the time varying matrix $A_2(t)$ is given by the coefficients of the time-varying terms of $\dot{y}_i$ for $i = 2,3,\dots,n$. Thus the nonlinear subsystem x_i $(i = 2,3,\dots,n)$ of (7) can be transformed into the LTV system (10), and the time varying matrix $A_2(t)$ satisfies the convergence conditions $\lim_{t \to \infty} A_2(t) = 0$ and

$\int_0^\infty \|A_2(t)\| dt < \infty$. This completes the proof. $\diamondsuit$

Remark 6. For the first-order nonholonomic system given by the kinematics equations (7), the responses of the input u_1 are determined by the initial state $(x_0(0), x_1(0))$ and the control parameters α and β, where $x_0 = \int_0^t x_1(s)ds$ is a virtual value and can be arbitrarily chosen. If the model of the system is given by dynamics equations, then the input u_1 is determined by the actual initial state $(x_1(0), \dot{x}_1(0))$ and the parameters α and β.

In more recent years, it was also shown that some underactuated mechanical systems could be transformed into chained forms expressed by second-order differential equations

$$\ddot{x}_1 = u_1, \qquad \ddot{x}_i = u_1 x_{i+1}, \qquad \ddot{x}_n = u_2 \tag{11}$$

The relevant systems include some underactuated manipulators in weightless field [22], underactuated rigid body [28] and underactuated hovercraft system [29] etc [30]. By applying the following coordinate transformations

$$z_{2i-1} = x_i, \qquad z_{2i} = \dot{x}_i, \ i = 1,2,3,\dots,n \tag{12}$$

the system (11) can be rewritten as

$$\dot{z}_1 = z_2, \ \dot{z}_2 = u_1 \tag{13a}$$

and

$$\begin{aligned} \dot{z}_3 &= z_4 \\ \dot{z}_4 &= u_1 z_5 \\ &\vdots \\ \dot{z}_{2n-3} &= z_{2n-2} \\ \dot{z}_{2n-2} &= u_1 z_{2n-1} \\ \dot{z}_{2n-1} &= z_{2n} \\ \dot{z}_{2n} &= u_2 \end{aligned} \tag{13b}$$

For the special nonlinear system (13), the following proposition can be presented.

Proposition 7: By applying the following TVCT

$$\begin{aligned} y_i &= z_i e^{\alpha t}, i = 3,4,\dots,2n-2 \\ y_{2n-1} &= z_{2n-1} \\ y_{2n} &= z_{2n} \end{aligned} \tag{14}$$

and using the input

$$u_1 = c_0 e^{-\alpha t} + c_1 e^{-\beta t} \tag{15}$$

where c_0 and c_1 are two constants defined by the initial state $(z_1(0), z_2(0))$ of the subsystem (13a) at $t = 0$, and $\beta > \alpha > 0$ are two arbitrary constants, then the nonlinear subsystem (13b) can be transformed into the following LTV systems

$$\dot{y} = \left(A_1 + A_2(t)\right)y + Bu_2 \tag{16}$$

Where $y = \begin{bmatrix} y_3 & y_4 & \cdots & y_n \end{bmatrix}^T$, $B = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^T$,

$$A_1 = \begin{bmatrix} \alpha & 1 & 0 & \cdots & 0 & 0 \\ 0 & \alpha & 0 & \cdots & 0 & 0 \\ 0 & 0 & \alpha & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \text{ and}$$

$$A_2(t) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & c_0 e^{-\alpha t} + c_1 e^{-\beta t} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & c_0 e^{-\alpha t} + c_1 e^{-\beta t} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Proof: By proceeding along a similar line as the proof of Proposition 5, it can be shown that

$$\begin{aligned} \dot{y}_i &= \dot{z}_i e^{\alpha t} + \alpha z_i e^{\alpha t} \\ &= z_{i+1} e^{\alpha t} + \alpha y_i \\ &= \alpha y_i + y_{i+1} \end{aligned}$$

for $i = 3, 5, \ldots, 2n-3$, and

$$\begin{aligned} \dot{y}_i &= \dot{z}_i e^{\alpha t} + \alpha z_i e^{\alpha t} \\ &= \left(u_1 z_{i+1}\right) e^{\alpha t} + \alpha y_i \\ &= \left(c_0 e^{-\alpha t} + c_1 e^{-\beta t}\right) z_{i+1} e^{\alpha t} + \alpha y_i \\ &= \left(c_0 e^{-\alpha t} + c_1 e^{-\beta t}\right) y_{i+1} + \alpha y_i \end{aligned}$$

for $i = 4, 6, \ldots, 2n-2$. Then the nonlinear subsystem (13b) can be transformed into the LTV system (16), and the time varying matrix $A_2(t)$ satisfies the conditions $\lim_{t \to \infty} A_2(t) = 0$ and $\int_0^\infty \|A_2(t)\| dt < \infty$. This completes the proof. $\diamond$

It is worth noting that, even though Propositions 5 and 7 demonstrate that, some classes of nonlinear

constraints systems or underactuated systems, which are generally nonlinear in nature, can be changed to special LTV systems by the presented TVCT (8) or (14), the obtained LTV systems are not always controllable because of the introduced parameters (α, β) and (c_0, c_1) in (10) and (16). For instance, the LTV system (10) and (16) are not controllable if $\alpha = 0$. This reflects that it is not always possible for a nonholonomic constraint system to move between two arbitrarily given points in the state space, while the value of the parameters (c_0, c_1) are determined by the two given points and the control parameters (α, β). A feasible approach of removing the singular points of nonholonomic constraints systems is to use switching control. By adding an intermediate point between the two given points, control of a smooth LTV system (10) or (16) changes to control of a switched LTV system. Thereby for stabilizing a nonholonomic system by switching control, a theorem can be presented as follows.

Theorem 8. If a nonholonomic constraints system can be changed to a LTV system that is given by

$$\dot{y} = \left(A_1 + A_2(t)\right)y + Bu \tag{17}$$

where $y \in R^n$ is the state variables, $u \in R^m$ is the inputs, A_1 and B are time invariant matrices, the time varying matrix $A_2(t)$ satisfies the convergence conditions

$$\lim_{t \to \infty} A_2(t) = 0 \text{ and } \int_0^\infty \|A_2(t)\| dt < \infty \tag{18}$$

and for the origin $y = 0$ and an initial point y_0, there exists a series of intermediate points y_i ($i = 1, 2, \ldots, p-1$) between the origin and initial point y_0, so that all curve segments L_i connecting two adjacent points y_i and y_{i+1} of the piecewise path $L = \bigcup_{i=0}^{p-1} L_i$ are controllable, then the nonholonomic system can be exponentially stabilized to origin from initial point y_0 by slowing switching control in accordance with the switching sequence $\sigma = 0 \to 1 \to 2 \to \ldots \to p-1$.

Proof: Since all adjacent points are controllable, for every pair y_i and y_{i+1} there exists a set of feasible parameters $(\alpha, c_0)_\sigma$ so that the pair $(A_1(\alpha, c_0), B)_\sigma$ is controllable for $\sigma \in \aleph = \{0, 1, 2, \ldots, p-1\}$. Suppose $u_\sigma = K_\sigma y$ is a feasible feedback for mode σ, then the smooth LTV system (17) is changed to a switched LTV system

$$\dot{y} = \left[\left(A_{1\sigma} - BK_\sigma\right) + A_{2\sigma}(t)\right]y, \quad \sigma \in \aleph = \{0, 1, 2, \ldots, p-1\} \tag{19}$$

where $A_{1\sigma} - BK_\sigma$ is a Hurwitz matrix, and the sequence of switching signal σ is given by

$$\sigma = 0 \rightarrow 1 \rightarrow 2 \rightarrow \ldots \rightarrow p-1 .$$

Without loss of generality, suppose τ^* is a permitted minimize dwell time of all modes, then by applying Lemma 1 and Lemma 3, it can be shown that along the curve segment L_i there exists a sufficient large constant $\tau > 0$ and a sufficient small positive constant $\varepsilon_i > 0$, so that $\tau > \tau^*$ and $\|y_i(t)\| \leq e^{-\varepsilon_i t} \|y_i(0)\|$, $t \in [0, \tau)$, $y_{i+1}(0) = y_i(\tau)$ for all $i \in \{0,1,2,\ldots,p-1\}$. Then along the overall path $L = \bigcup_{i=0}^{p-1} L_i$ it follows that

$$\|y(t)\| \leq e^{-\varepsilon_{p-1}t} \|y_{p-1}(0)\|$$
$$\leq e^{-\varepsilon_{p-2}\tau} e^{-\varepsilon_{p-1}t} \|y_{p-2}(0)\|$$
$$\ldots\ldots$$
$$\leq e^{-\sum_{i=0}^{p-1}\varepsilon_i\tau - \varepsilon_{p-1}t} \|y_0\|$$

for $t \in [0, \tau]$. Because of $\varepsilon_i > 0$ for all $i \in \{0,1,2,\ldots,p-1\}$, and the average dwell time $\tau > \tau^*$, then $\lim_{\tau \to \infty} \|y(t)\|_{t \in [0,\tau]} = 0$. This completes the proof. $\diamond$

It is important to point out that a set of coordinates usually selected in Cartesian space is local coordinates for a nonholonomic system since the configuration space of a nonholonomic system is commonly not diffeomorphic to the Cartesian space. Therefore the selected local coordinates usually lead to control singularity for large range or global motion control tasks. At a control singular point the control inputs of a controlled system are infinite. Nevertheless, Theorem 8 reveals that many nonholonomic constraint systems can be stabilized by switching control approaches even if the given initial point y_0 with respect to the origin is not controllable, or in other words, y_0 is a control singular point with respect to the origin in local coordinate system. By properly selecting an intermediate point y^*, the system can be stabilized to origin from any given initial point y_0 if the initial point is controllable with respect to the intermediate point y^*, while the intermediate point is controllable with respect to the origin. Therefore, even though the switching control strategy changes the control issues of a smooth system to the control issues of a discontinuous system, it is obvious that the switching control strategy

commonly provides a larger stabilization range, or even global stabilization capability for controlling a non-holonomic system if the nonholonomic system can be changed to a chained form by nonlinear coordinate and input transformations. As it will be demonstrated in the next section, this property of the switching control approach for nonholonomic constraints systems is appealing for solving the issues of obstacle avoidance of robot system in complex environments, which is still a tricky problem that has not been thoroughly investigated for general nonholonomic systems.

Another feature of Theorem 8 is that it does not depend on a specific nonholonomic system. For a re-configurable or variant robot system with nonholonomic constraints, a claim can be presented as follows based on Theorem 8.

Theorem 9. Suppose the dynamics of a variant robot system with nonholonomic constraints in all locomotion modes can be changed to the LTV systems as given by (1), and the time varying matrix $A_{2\sigma}(t)$ satisfies the conditions (2) for all $\sigma \in \aleph = \{1,2,\ldots,s\}$, which denotes the modes of the variant robot system, then for a given initial point x_0, if there exists a series of intermediate points x_i $(i = 1,2\ldots,p)$, and every curve segment L_i connecting the two adjacent points x_i and x_{i+1} is controllable with regard to at least one robot mode σ, then the switched nonholonomic system can be stabilized from the initial point x_0 to origin by slow switching control.

Proof: Since the feasible path from x_0 to origin is connected by a curve segment L_i, which is the curve segment between the two adjacent points x_i and x_{i+1} for $i = 1,2,\ldots,p$, and there exists a feasible control corresponding to a mode $\sigma \in \aleph = \{1,2,\ldots,s\}$ so that $\overline{A}_\sigma = A_\sigma - B_\sigma K_\sigma$ is Hurwitz. Then there exists a positive constant $\varepsilon_i(\sigma) > 0$ and a sufficiently large dwell time $\tau > 0$ so that the state of the controlled system along the curve segment L_i satisfies $\|x_i(t)\| \leq e^{-\varepsilon_i(\sigma)t} \|x_i(0)\|$, $t \in [0, \tau)$, $x_{i+1}(0) = x_i(\tau)$ for $i = 1,2,\ldots,p$ and $\sigma \in \aleph = \{1,2,\ldots,s\}$. For the overall path $L = \bigcup_{i=1}^{p} L_i$, it follows that

$$\|x(t)\| \leq e^{-\varepsilon_p(\sigma)t} \|x_p(0)\|$$
$$\leq e^{-\varepsilon_{p-1}(\sigma)\tau} e^{-\varepsilon_p(\sigma)t} \|x_{p-1}(0)\|$$
$$\ldots\ldots \qquad \leq e^{-\sum_{i=1}^{p-1}\varepsilon_i(\sigma)\tau - \varepsilon_p(\sigma)t} \|x_0\|$$

for $t \in [0, \tau]$. This completes the proof. $\diamond$

Remark 10. In Theorem 8, the switching sequence $\sigma = 0 \to 1 \to 2 \to \ldots \to p-1$ just depends on the state of single LTV system, where the switching characteristic is caused by applying switching control, while the open loop dynamics of the robot system is invariant. Therefore the subsystems of the switched LTV systems are generally homogeneous. However, in Theorem 9, the switching sequence $\sigma = 0 \to 1 \to 2 \to \ldots \to p-1$ not only depends on the state of single LTV system, but also depends on the open loop dynamics of the robot mechanisms. Accordingly, the subsystems of the switched LTV systems permit to be heterogeneous. Theorem 9 reveals that a class of switched nonholonomic system can be stabilized under rather relaxed conditions, thus it is feasible to design a variant mobile robot system.

4. NUMERICAL SIMULATIONS OF STABILIZING THE SWITCHED NONHOLONOMIC SYSTEMS

To substantially improve the mobility of locomotion robotic systems, structurally variable robot systems or reconfigurable robot systems are a promising research direction. In this section, the dynamics and control for a dual-mode mobile robot are investigated. As illustrated in Fig. (**1**), the variant robot system is a combination of a wheeled mobile robot and a hovercraft robot. In both locomotion modes, the robot system is driven by two forces in horizontal plane. In land movement pattern, the robot system is a two wheeled mobile system with an auxiliary omni-directional wheel, while in water surface or swamp movement pattern, the system is a hovercraft system. Since suspension control can be decoupled from the propulsion control for a floating base robot system [31], suspension control in hovercraft mode will not be considered in this paper.
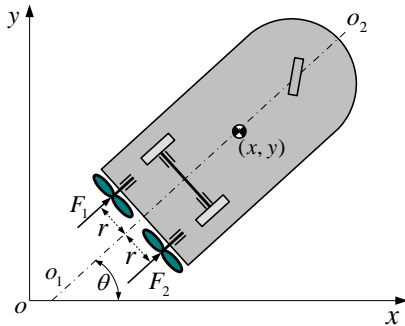
Figure 1: Diagram of a dual model mobile robot system.

In this section, the system parameters of the variant robot are given as follows. The mass of the robot is $m = 3.0 \mathrm{kg}$, the moment of inertia with respect to the center of mass is $J = 1/3 \mathrm{kg} \cdot \mathrm{m}^2$, the distance between the two thrust forces is $D = 2r = 0.6 \mathrm{m}$, and the acting lines of the two forces are symmetric about the principal axis $o_1 o_2$ of the robot.

A. Mode of a Wheeled Robot Actuated by Two Forces

In land movement pattern, the dynamics of a two wheeled mobile robot can be written as [27]

$$m\ddot{x} = \lambda \sin\theta + (F_1 + F_2)\cos\theta$$
$$m\ddot{y} = -\lambda \cos\theta + (F_1 + F_2)\sin\theta$$
$$J\ddot{\theta} = (F_2 - F_1)r \tag{20}$$

and the motions of the system are restricted by the first order nonholonomic constraint $\dot{x}\sin\theta - \dot{y}\cos\theta = 0$, where (x, y, θ) is the local coordinates of the mobile robot as shown in Fig. (**1**), λ in (20) is a scalar that denotes the constraint force caused by the nonholonomic constraint, F_1 and F_2 are two thrust forces. By the following coordinate changes

$$x_1 = x\cos\theta + y\sin\theta$$
$$x_2 = \theta \tag{21}$$
$$x_3 = -x\sin\theta + y\cos\theta$$

and the input changes

$$u_1 = v_1 - x_3 v_2 - x_1 \dot{x}_2^2, \qquad u_2 = v_2$$

where

$$v_1 = \frac{F_1 + F_2}{m}, \qquad v_2 = \frac{(F_2 - F_1)r}{J}$$

the dynamics (20) can be transformed to

$$\ddot{x}_1 = u_1, \qquad \ddot{x}_2 = u_2, \qquad \dot{x}_3 = -x_1 \dot{x}_2 \tag{22}$$

By applying a new coordinate transformation

$$z_1 = x_1, z_2 = \dot{x}_1, z_3 = x_3, \; z_4 = x_2, z_5 = \dot{x}_2 \tag{23}$$

the system (22) can be rewritten as

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = u_1$$
$$\dot{z}_3 = -z_1 z_5 \tag{24}$$
$$\dot{z}_4 = z_5$$
$$\dot{z}_5 = u_2$$

If the input $u_1(t)$ in (24) is chosen to be

$$u_1(t) = c_1 \alpha^2 e^{-\alpha t} + c_2 \beta^2 e^{-\beta t} \qquad (25)$$

where

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -\alpha & -\beta \end{bmatrix}^{-1} \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} \qquad (26)$$

and the two parameters α and β satisfy $\beta > \alpha > 0$. By further applying a TVCT

$$y_3 = z_3 e^{\alpha t}, \qquad y_4 = z_4, \qquad y_5 = z_5 \qquad (27)$$

the subsystem (z_3, z_4, z_5) of (24) can be changed to a form as the LTV system given by (17), and can be written as

$$\dot{y} = (A_1 + A_2(t))y + Bu_2 \qquad (28)$$

where

$$A_1 = \begin{bmatrix} \alpha & 0 & -c_1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, A_2(t) = \begin{bmatrix} 0 & 0 & -c_2 e^{(\alpha-\beta)t} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$ Note that the system (28) is different from any

of the systems (10) and (13) since here the dynamics system of a wheeled robot is considered. However, it is obvious that the time-varying matrix $A_2(t)$ also satisfies the convergence condition (18). From the example given above it is also observed that the time-varying coordinate transformations presented by (8) and (14) can be extended to a wider class of nonholonomic systems.

Now we show the switching control can be used to stabilizing the wheeled mobile robot system with obstacle avoidance effects. Suppose the initial state of the wheeled robot is given by $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0 = (14,14, \pi/2, 0, 0, 0)$ and the control parameters of u_1 in (25) are selected as $(\alpha, \beta) = (1, 3)$, then by applying (26) we have $(c_1, c_2) = (-16.3915, 2.3915)$. The control gains of u_2 in (28) is calculated by LQR optimal control method with optimization criterion $S = \int_0^\infty [y^T Q y + u_2^T R u_2] dt$, where the weight matrices Q and R are chosen as $Q = E \in R^{3 \times 3}$ an identity matrix, and $R = 1$.

The responses of the position and the speed of mobile robot are illustrated in Figs. (**2** and **3**) demonstrates the corresponding animations of the movements

of the controlled robot. If there are two obstacles in the motion space of the robot as shown in Fig. (**3**), the generated trajectory will be infeasible and the trajectories could not be changed in a large range by only adjusting the control parameters of a non-switched controller.
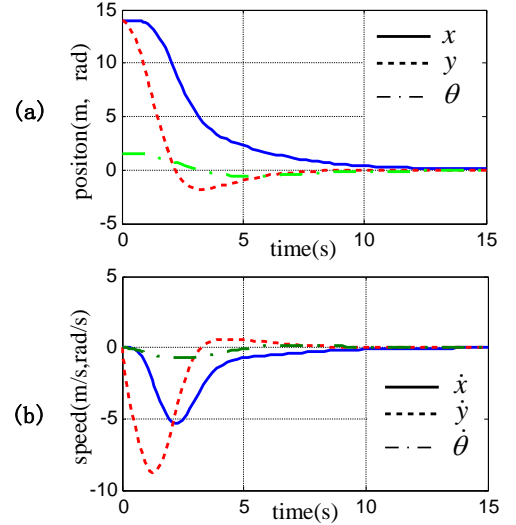
Figure 2: (a) The position trajectory of a two wheeled mobile robot controlled by non-switched controller; (b) The speed trajectory of a two wheeled mobile robot controlled by non-switched controller.
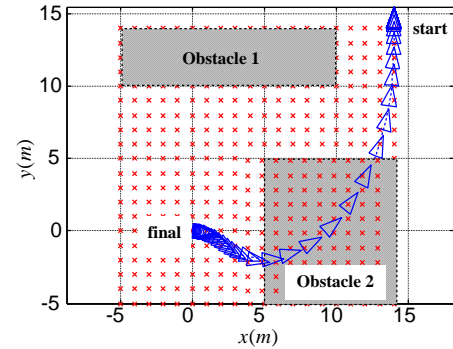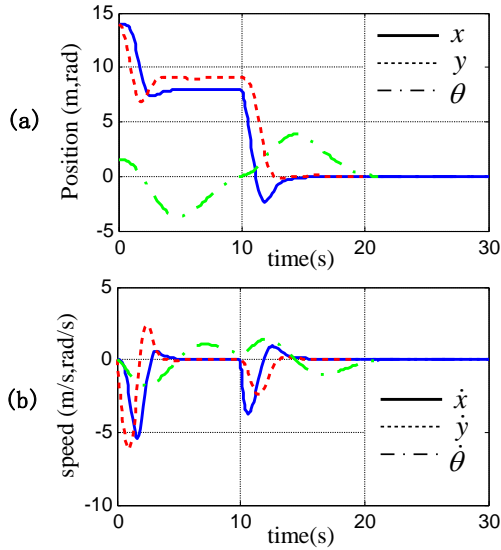

Figure 3: Animations of a two wheeled mobile robot controlled by non-switched controller.

However, by applying switching control strategy, the mobile robot can be stabilized from the initial point to origin as shown by Fig. (**4**), and the animations illustrated in Fig. (**5**) visually show the robot can avoid the two obstacles and finally arrive at the origin. In this numerical simulation, the initial state and the control parameters of u_1 are not changed, i.e. $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0 = (14,14, \pi/2, 0, 0, 0)$ and $(\alpha, \beta) = (1, 3)$. Whereas, the smooth LTV system (28) is changed to a switched LTV system (1) since the relative states are changed. By selecting an intermediate point $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_1 = (8, 9, 0, 0, 0, 0)$, and a minimum dwell time $\tau^* = 10s$, then

along the curve segment $L_1(P_0 \to P_1)$ the structure parameters (c_1, c_2) in system (28) are calculated to be $(c_1, c_2) = (-10,5)$ according to (26), while along the curve segment $L_2(P_1 \to o)$ the structure parameters in (28) are calculated to be $(c_1, c_2) = (-15.52, 7.76)$.

Figure 4: (a) The position trajectory of a two wheeled mobile robot controlled by a switched controller; (b) The speed trajectory of a two wheeled mobile robot controlled by a switched controller.
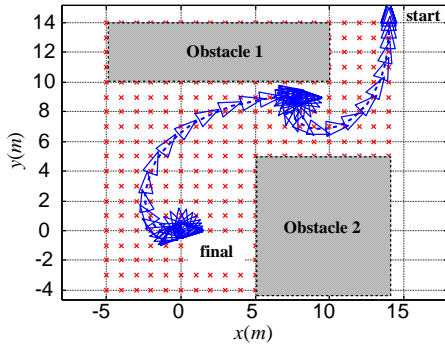
Figure 5: Animations of a two wheeled mobile robot controlled by a switched controller.

B. Mode of Hovercraft System Driven by Two Forces

The dynamics of a hovercraft system can be written as [27]

$$m\ddot{x} = (F_1 + F_2)\cos\theta$$
$$m\ddot{y} = (F_1 + F_2)\sin\theta \qquad (29)$$
$$J\ddot{\theta} = (F_2 - F_1)r$$

It is different from the wheeled mobile robot mode, and in hovercraft mode the dynamics of the robot system are not restricted by any external constrains. However, the motions of the hovercraft system are restricted by the second order nonholonomic constraint $\ddot{x}\sin\theta - \ddot{y}\cos\theta = 0$, which is caused by the internal dynamics of the underactuated system.

Using the following coordinate transformations

$$x_1 = x, \qquad x_2 = \tan\theta, \qquad x_3 = y \qquad (30)$$

and the input changes

$$u_1 = \frac{1}{m}(F_1 + F_2)\cos\theta$$
$$u_2 = \frac{r}{J}(F_2 - F_1)\sec^2\theta + 2\dot{\theta}^2\tan\theta\sec^2\theta \qquad (31)$$

system (29) can be transformed into the following chained form that is given by the second-order differential equations

$$\ddot{x}_1 = u_1, \qquad \ddot{x}_2 = u_2, \qquad \ddot{x}_3 = x_2 u_1 \qquad (32)$$

Furthermore, by using the coordinate transformations

$$z_1 = x_1, \; z_2 = \dot{x}_1, \; z_3 = x_3, \; z_4 = \dot{x}_3, \; z_5 = x_2, \; z_6 = \dot{x}_2 \qquad (33)$$

the state space equations of the system (32) can be presented as

$$\begin{aligned}
&&\dot{z}_3 &= z_4 \\
\dot{z}_1 &= z_2 & \dot{z}_4 &= u_1 z_5 \\
\dot{z}_2 &= u_1 & \dot{z}_5 &= z_6 \\
&& \dot{z}_6 &= u_2
\end{aligned} \qquad (34)$$

Note that the system (34) is a special form of (13). However, here the input $u_1(t)$ is chosen to be

$$u_1(t) = c_1 \alpha^2 e^{-\alpha t} + c_2 \beta^2 e^{-\beta t}, \qquad (35)$$

in order to solve the coefficients (c_1, c_2) more easily by the initial state of the subsystem (z_1, z_2) of the system (34), and the coefficients (c_1, c_2) are calculated by

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -\alpha & -\beta \end{bmatrix}^{-1} \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix}, \qquad (36)$$

and the two parameters α and β satisfy $\beta > \alpha > 0$. By further using the following TVCT

$$y_3 = z_3 e^{\alpha t}$$
$$y_4 = z_4 e^{\alpha t}$$
$$y_5 = z_5 \quad\quad\quad\quad\quad\quad\quad (37)$$
$$y_6 = z_6$$

the subsystem (z_3, z_4, z_5, z_6) of (34) can be changed to a form as the LTV system (17), which can be specified as

$$\dot{y} = (A_1 + A_2(t))y + B u_2 \quad\quad\quad (38)$$

where

$$A_1 = \begin{bmatrix} \alpha & 1 & 0 & 0 \\ 0 & \alpha & c_1\alpha^2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & c_2\beta^2 e^{(\alpha-\beta)t} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^\mathrm{T}.$$

It is obvious that the time varying matrix $A_2(t)$ in (38) satisfies the convergence condition (18). As demonstrated in the numerical simulations by a hovercraft system, switching control strategy can be used to overcome control singularity of the nonholonomic systems. With the assumptions that the parameters m, J and r are same as the wheeled mobile robot, and herein the controller parameters in $u_1(t)$ are selected to be $(\alpha, \beta) = (2,3)$, while the initial state of robotic system is given by $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0 = (0,14,0,0,0,0)$. Even though this is a parallel parking problem where the initial state of the system is controllable, a non-switching controller will result in infeasible large control inputs. However, as shown in Fig. (**6**), the control inputs can be reduced to a reasonable level if switching control is applied. In Figs. (**6** and **7**), the switching state is casually selected to be $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0 = (-10,0,0,0,0,0)$. Fig. (**8**) shows the animations of the hovercraft system that is stabilized to origin from a control singular configuration $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0 = (0,10,\pi/2,0,0,0)$. Refer to (30), the hovercraft system is in a control singular configuration if $\theta = \pm k\pi/2$, $(k \in \mathrm{N}^*)$ because of the local coordinates. By casually selecting an intermediate point $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0 = (-10,7,\pi/4,0,0,0)$ and using switching control approach, the control singular configuration is removed.
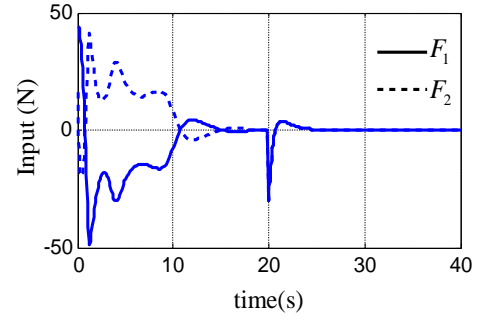
Figure 6: The input trajectory of parallel parking control a hovercraft system by a switching controller.
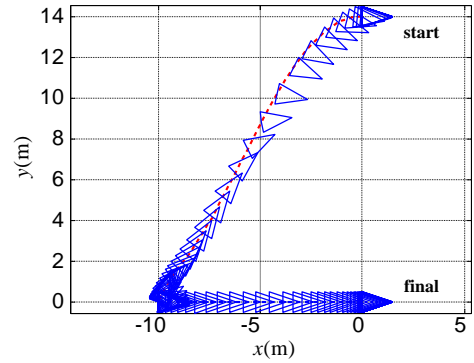
Figure 7: Animations of parallel parking control a hovercraft system by a switching controller.
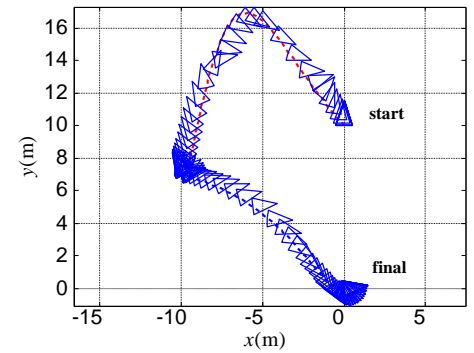

Figure 8: Animations of stabilizing a hovercraft system from a control singular configuration by a switching controller.

## C. Switching Control of a Dual Mode Locomotion Robot System

In this subsection, we show that a dual-mode variant robot system as illustrated in Fig. (**1**) can be stabilized to origin by switching control approaches. The robot is a combination of a two wheeled mobile robot and a hovercraft, both of which are actuated by two forces. Refer to the dynamics of the variant robot

(20) and (29), it is obvious that the dynamics of the variant robot in various mode are different. Even if the different dynamics can be changed to a class of similar LTV system as shown by (28) and (38), the switched nonholonomic system is heterogeneous from a point of view of control engineering, since the state space dimensions of the two modes are different. As claimed by Theorem 9, the switched system could be globally stabilized if there is a feasible piecewise path from the initial point to origin, and there at least exists one mode with respect to a curve segment that is controllable with respect to the current mode.

Suppose that the given initial state is $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})_0$ $= (20, 20, 0, 0, 0, 0)$, the input u_1 is selected as (25) or (35), of which the control parameters are selected to be $(\alpha, \beta) = (2, 3)$, and applying the LQR optimal control method to LTV subsystems (28) and (38), as shown by Figs. (**9** to **13**), the dual-mode variant robot system can be stabilized to origin by switching control. The closed-loop trajectories of the position, the speed and the inputs of the variant robot system are plotted in Figs. (**9-11**) respectively. The corresponding switching sequence is illustrated in Fig. (**12**), and the animations of stabilizing process of the variant robot system are plotted in Fig. (**13**).
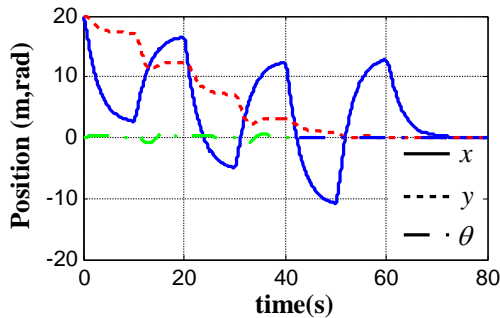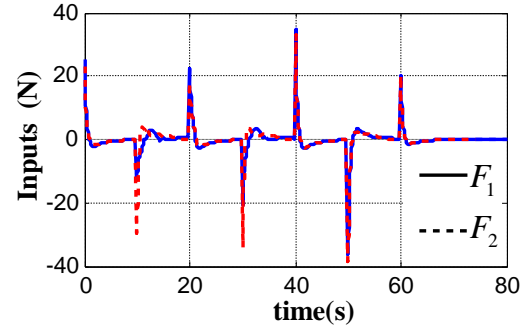

Figure 9: Position trajectory of the dual mode variant robot system stabilized by a switching controller.
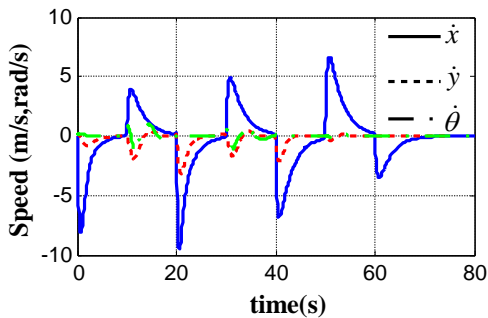

Figure 10: Speed trajectory of the dual mode variant robot system stabilized by a switching controller.

It is worth pointing out that the control parameters α and β in (25) for wheeled mode and in (35) for hovercraft mode can be selected under the single condition $\beta > \alpha > 0$ since the nonholonomic systems considered in this paper can be changed to the chained normal forms. In practice, a larger value of $\beta - \alpha$ is helpful for accelerating the decay of the time-varying terms in $A_2(t)$ of the equations (28) and (38). Therefore, it is helpful for obtaining a locomotion trajectory with smaller fluctuation, and thereby permits accelerate the switching frequency of the controllers when it is necessary. A larger $\alpha > 0$ is also advantageous to improve the dynamic responses speed of the closed-loop system. However, large control parameters α and β will result in large control inputs. As a class of slow switching systems due to the large inertia of the variant robot systems, the contradiction of selecting control parameters α and β is not prominent.
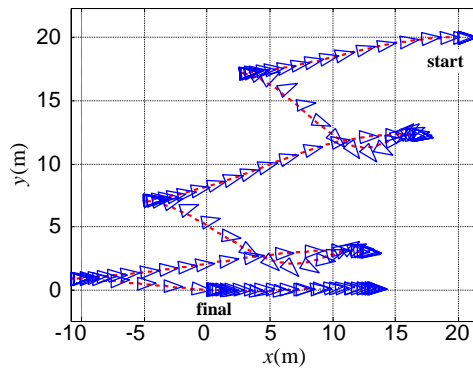

Figure 11: Inputs trajectory of the dual mode variant robot system stabilized by a switching controller.
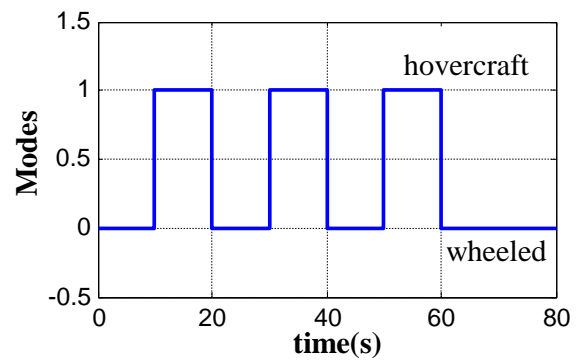
Figure 12: The switching sequence of the dual mode variant robot system in switching control.

CONCLUSIONS

In order to essentially increase the mobility of a robotic system, mechanically reconfigurable or variant robot system is a promising research direction. In this

Figure 13: Animations of the dual mode variant robot system stabilized by a switching controller.

paper we show that the dynamics of a class of nonholonomic constraint robot systems can be changed to a class of special LTV systems based on the TVCT method, and by applying switching control strategy the motion control issues of nonholonomic systems can be resolved with better flexibility. On one hand, by changing a nonholonomic system to LTV system, we show that switching control approaches provide the capability of overcoming the control singular problems for nonholonomic systems, and provide a dexterous way for optimal motion planning of mobile robot system such as obstacle avoidance. On the other hand, we show that the switching control approaches can be used to stabilize a switched nonholonomic system even if the various modes of the switched nonholonomic system are heterogeneous. Thus the combination of TVCT and switching control of LTV systems provides a feasible approach to globally stabilize a class of variant mobile robot systems.

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