An Optimum Design for Laminated Steel Leaf Springs

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Abstract: This study presents an optimum design methodology for laminated semi-elliptic steel leaf springs used in vehicle suspension systems, with the primary objective of minimizing spring weight while satisfying structural, geometric, and dynamic requirements. Analytical expressions for bending stress, mid-span deflection, stiffness, and natural frequency are derived based on classical beam theory and expressed in non-dimensional form to enable a generalized and systematic optimization framework. The design variables considered include strip thickness, strip width, and number of leaves, while constraints are imposed on allowable bending stress, maximum deflection, bounded geometry, and natural frequency to ensure structural safety and acceptable ride comfort. The resulting nonlinear optimization problem is solved using MATLAB for a carbon steel (AISI 1020) leaf spring subjected to a specified vertical load. The optimized design consists of four steel strips with a width of approximately 75 mm and a thickness of about 7.5 mm, yielding a total spring weight of approximately 19 kg. The results demonstrate that significant weight reduction can be achieved without compromising stress limits, deflection requirements, or dynamic performance. The proposed methodology provides a reliable and efficient tool for the optimal design of steel laminated leaf springs and can be extended to other materials, loading conditions, and vehicle applications. The results obtained from this study have been compared with available numerical results obtained by FEA and a good agreement is aobtained.

Keywords: Leaf Spring, Optimal design, Steel laminated spring, Weight minimization, Natural frequency, Structural constraints.

1. INTRODUCTION

Leaf springs are among the most widely used elastic elements in the suspension systems of road vehicles, particularly in trucks, buses, and trailers, due to their simplicity, robustness, and ability to carry heavy loads while providing vertical compliance. In addition to supporting vehicle weight, laminated leaf springs play a vital role in maintaining ride comfort, controlling axle movement, and isolating the chassis from road-induced vibrations. Consequently, their design must simultaneously satisfy requirements related to strength, stiffness, fatigue life, and dynamic performance.

The performance of a laminated leaf spring is governed by several interrelated parameters. Material properties such as Young's modulus, density, and allowable stress directly influence stiffness, weight, and fatigue resistance. Geometric parameters, including the number of leaves, strip thickness, strip width, and eye-to-eye length, significantly affect stress distribution, deflection behavior, and load-carrying capacity. Furthermore, dynamic characteristics, particularly the natural frequency of the spring, are of great importance, since resonance with road excitation frequencies can lead to excessive vibration, noise, and reduced ride comfort.

In recent decades, numerous studies have investigated the analysis and optimization of leaf springs using analytical, numerical, and experimental approaches [1, 2]. Finite element methods have been widely employed to evaluate stress, deflection, and fatigue behavior under static and dynamic loading conditions. Optimization techniques have been applied to minimize spring weight while satisfying strength and deflection constraints, often considering parameters such as strip geometry and number of leaves. A considerable portion of the published literature has focused on composite leaf springs, motivated by their potential for significant weight reduction compared to conventional steel springs.

Shaikh *et al*. [3] studied composite leaf springs with uniform thickness and compared their performance with steel leaf springs, reporting reduced weight but lower stiffness. Xue *et al*. [4] performed a dynamic simulation of the composite leaf spring on Unmanned Aerial Vehicles. Venkatesan and Helmen [5] optimized the design of lightweight automobile leaf springs and analyzed their performance

Reddy *et al*. [6] investigated the effects of leaf width, length, and thickness on the optimal design of automobile leaf springs using finite element analysis, showing that stress levels decrease with increasing leaf width and thickness. Fakkir *et al*. [7] presented an optimization study emphasizing weight reduction in suspension systems through the use of composite materials, achieving weight savings of approximately 10–20% compared to traditional steel springs.

Despite the extensive research on composite leaf springs, steel laminated leaf springs remain widely used in commercial vehicles due to their cost-effectiveness, manufacturing simplicity, durability,

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and well-established design standards. However, many existing studies on steel leaf springs rely on conventional design approaches or purely numerical simulations, with limited integration of analytical formulations, dynamic constraints, and systematic optimization techniques.

Therefore, the present study proposes an optimum design methodology for laminated semi-elliptic steel leaf springs with the primary objective of minimizing spring weight while satisfying strength, deflection, geometric, and natural frequency constraints. Analytical expressions for bending stress, deflection, stiffness, and natural frequency are derived and expressed in non-dimensional form to enable a generalized and efficient optimization framework. The resulting nonlinear optimization problem is solved using MATLAB for a carbon steel (AISI 1020) leaf spring subjected to a specified load. The proposed approach provides clear insight into the influence of key design parameters and offers an effective tool for achieving lightweight and dynamically acceptable steel leaf spring designs.

The work performed in this paper consider both the dynamic effect (natural frequency) and the design analysis (stress and deflection) of leaf springs.

2. DESIGNS AND METHODOLOGY

Laminated leaf springs are widely used in vehicle suspension systems, particularly in trucks and trailers, due to their ability to support heavy loads while providing adequate flexibility and vibration isolation. A typical laminated semi-elliptic leaf spring consists of several arc-shaped steel strips stacked together and connected to the vehicle chassis and axle as illustrated in Figure **1** [8].

The design of a leaf spring is governed by multiple interacting parameters, including material properties, geometric dimensions, and loading conditions. Material properties such as Young's modulus, density, and allowable stress significantly influence stiffness, weight, and fatigue resistance. Geometric parameters, including the number of leaves, strip width, strip thickness, and eye-to-eye length, control stress distribution, deflection behavior, and load-carrying capacity. In addition, fatigue strength and dynamic performance are essential requirements to ensure safe and reliable operation under repeated loading.

For the purpose of analytical modeling and optimization, the following assumptions are adopted in the present study:

- Linear elastic material behavior.
- Small deflections.
- Uniform cross-section along the spring length.
- Symmetric loading applied at the mid-span of the spring.

2.1. Spring Stresses and Deflection

A laminated leaf spring can be modeled as a beam subjected to a transverse load acting at its midpoint. The bending moment varies along the spring length and reaches its maximum value at the center. For a leaf spring with a total width B and thickness t, the maximum bending stress is expressed as [9], Figure **2**:

$$\sigma_{max} = \frac{My}{I} = \frac{3\,F\,L}{2\,B\,t^2}$$

and the corresponding spring stiffness is defined as:

$$\delta = \frac{3\,F\,L^3}{8\,E\,B\,t^3}$$

In practical applications, the total width B is divided into n identical strips, each having a width b, such that $B = nb$. Accordingly, the maximum bending stress and deflection in a laminated spring composed of n leaves becomes:

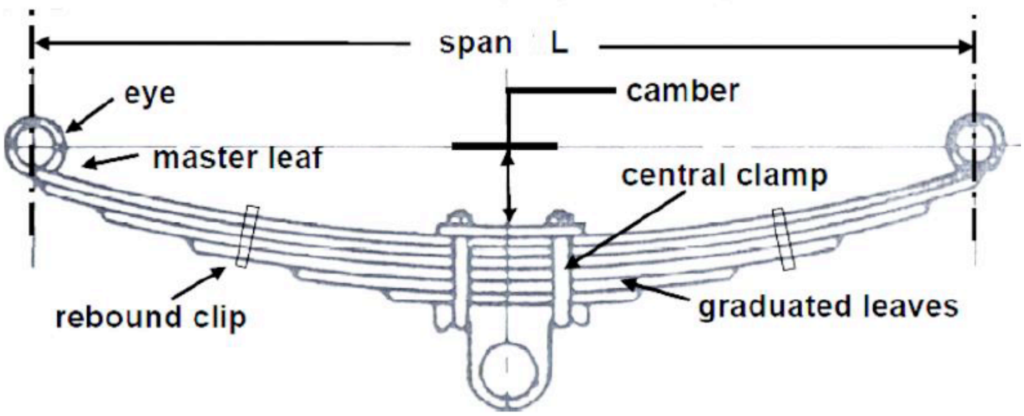

Figure 1: Semi elliptic leaf spring characteristics [8].

$$\sigma_b = \frac{3\,F\,L}{2\,n\,b\,t^2}$$

$$\delta = \frac{3\,F\,L^3}{8\,E\,B\,t^3}$$

The spring stiffness is:

$$K = \frac{8\,E\,n\,b\,t^3}{3\,L^3}$$

Where;

F= applied load on spring

L= length of leaf spring between eye to eye

n= number of total leaves

b= width of each leave

t= thickness of each leaf

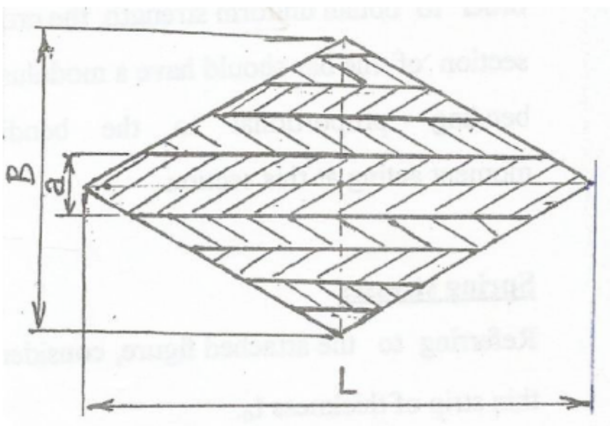

Figure 2: Cross-section of a laminated a leaf spring.

Hence the spring stiffness, $K = \frac{F}{\delta} = \frac{8\,E\,B\,t^3}{3\,L^3}$

2.2. Natural Frequency of Leaf Springs

Ride comfort and vibration control are closely related to the natural frequency of the suspension system. Very low natural frequencies may cause motion sickness, while high frequencies lead to harsh ride conditions. Therefore, the natural frequency of the leaf spring is considered a key design constraint.

The leaf spring is modeled as a single-degree-of-freedom spring–mass system, and the fundamental natural frequency is expressed as:

The natural frequency (f_n) of a leaf spring could be calculated using the formula for a simple spring-mass system as: $f_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$

By substituting the expressions for stiffness K and mass M of the leaf spring, the natural frequency can be written as:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{8\,E\,t^2}{3\,\rho\,L^4}}$$

2.3. Dimensionless Formulation

To facilitate the optimization process and generalize the results, the stress, deflection, and natural frequency equations are expressed in dimensionless forms as follows:

$$\sigma_b = \frac{3\,F}{2\,n\,\beta\,(L\,\tau)^2}$$

$$\delta = \frac{3\,F}{8\,E\,n\,\beta\,L\,\tau^3}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{8\,E\,\tau^2}{3\,\rho\,L^2}}$$

These dimensionless expressions provide a compact and efficient framework for formulating and solving the optimization problem.

3. OPTIMUM DESIGN OF THE LEAF SPRING

The optimum design of the laminated leaf spring is formulated as a constrained nonlinear optimization problem in which multiple design variables are considered simultaneously to determine the optimal spring configuration. The primary objective of the optimization is to minimize the total weight of the leaf spring while satisfying structural, geometric, and dynamic performance requirements. The optimization problem is solved numerically using MATLAB.

The design variables selected for the optimization process include the strip thickness, strip width, and number of leaves. The constraints imposed on the optimization problem are based on practical design considerations and recommendations reported in the literature, as described below.

Design Parameters for Optimum Design

3.1. Natural Frequency Constraint

The natural frequency of a leaf spring is a critical parameter affecting ride comfort and vibration behavior. There is no unique allowable value for the natural frequency, as it depends on vehicle type, loading conditions, and suspension configuration. However, it is generally recommended that the fundamental natural frequency of the suspension system be selected to avoid resonance with road excitation frequencies.

Previous studies reported by Khudhair *et al.* [10] mentioned that, for heavy vehicles, a natural frequency between 1 to 10 HZ for the rear suspension which carries more load is suitable. Where, frequencies under 1 HZ can cause notion sicknes and those over 10 Hz can cause harsh ride. Based on these considerations, the natural frequency is treated as a design constraint to ensure acceptable vibration isolation and ride comfort.

3.2. Deflection Constraint

The allowable deflection of a leaf spring depends on the vehicle application and loading conditions. Typical allowable deflection values range from approximately 40 mm for trailer suspension systems to about 170 mm for high-deflection composite leaf springs [10]. In the present study, an upper limit on the spring deflection is imposed to prevent excessive deformation, maintain structural integrity, and ensure safe and comfortable vehicle operation.

3.3. Number of Leaves Constraint

The number of leaves significantly influences the stiffness, load-carrying capacity, and durability of the leaf spring. There is no fixed limitation on the number of leaves for all vehicle types, as it depends on the required load capacity and design requirements. In practical applications, heavy-duty trucks may employ a main leaf pack consisting of up to ten leaves, with additional leaves added for overload conditions. In the present optimization framework, the number of leaves is treated as a bounded discrete design variable.

3.4. Geometric Constraints on Strip Thickness and Width

Geometric constraints are imposed on the strip thickness and width to ensure manufacturability and compliance with practical design limits. The thickness of leaf spring strips varies widely depending on the application, with typical values for steel leaf springs ranging up to approximately 9.5 mm. Similarly, the strip width commonly used in automotive applications is around 65 mm, although standard widths may range from 32 mm to 125 mm depending on vehicle load and design requirements. Accordingly, appropriate upper and lower bounds are specified for the strip thickness and width in the optimization problem.

4. PROBLEM FORMULATION

Objective Function and Constraints

The objective of this optimization work is minimizing the spring weight. The spring material and geometrical constraints are shown in Table **1**. The design of that spring is based on factor of safety equals 3 and natural frequency not more than 10 Hz. The load acting on the spring is taken as 1000 N.

Table 1: Spring Specification

Specification	Value
Material	Carbon steel AISI 1020
Yielding strength	300 MPa
Ultimate strength	420 MPa
Young’s modules	200 GPa
Density	7800 kg/m^3
Eye-eye lebgth	1100 mm
Maximum deflection	170 mm
Maximum strip thickness	10 mm
Maximum strip width	75 mm

Problem Formulation

Minimize spring weight:

$$W = n\,\beta\,\tau\,\rho\,L^3$$

Subject to:

Non-equality constraints:

$$\frac{3\,F}{2\,n\,\beta\,(L\,\tau)^2} \le 100 \times 10^6 \qquad \text{Constraint (1)}$$

$$\delta = \frac{3\,F}{8\,E\,n\,\beta\,L\,\tau^3} \le 0.150 \qquad \text{Constraint (2)}$$

Bounded constraints:

$$1 \le \frac{1}{2\pi}\sqrt{\frac{8\,E\,\tau^2}{3\,\rho\,L^2}} \le 8 \qquad \text{Constraint (3)}$$

$$1 < n < 6 \qquad \text{Constrain (4)}$$

$$0.03 \le \beta \le 0.08 \qquad \text{Constrain (5)}$$

$$\le \tau \le 0.009 \qquad \text{Constrain (6)}$$

5. OPTMIZATION RESULTS

Since the above optimization problem has multiple nonlinear constraints, it is classified as a nonlinear constrained optimization problem. Such problems are inherently more complex than linear optimization problems, as the optimal solution is not necessarily located at the vertices of the feasible region. Therefore, specialized numerical optimization techniques are required to obtain a feasible and optimal solution that satisfies all imposed constraints simultaneously.

Various solution strategies can be employed to address nonlinear optimization problems, including gradient-based methods and evolutionary algorithms. In the present study, the optimization problem is solved numerically using MATLAB, where the optimal solution is obtained by satisfying all constraint conditions related to stress, deflection, geometry, and natural frequency.

The optimized design parameters obtained from the MATLAB optimization procedure are summarized as follows:

β = 0.069. *i.e.* b = 76 mm

τ = 0.0066. *i.e.* t = 7.3 mm

N = 4 strips

Optimized Weight = 17.2 kg

The corresponding optimized spring weight is approximately 17.2 kg. Considering practical manufacturing dimensions and rounding of geometric parameters, the final optimized configuration for the steel laminated leaf spring consists of four strips with a width of about 75 mm and a thickness of approximately 7.5 mm, resulting in a total spring weight close to 19 kg.

Influence of Design Parameters on the Spring Weight

The following Tables **2** & **3** show the optimum design parameters obtained from optimization work performed on various leaf springs made from different loading forces and lengths. The obtained optimum parameters are mainly related to spring geometry as: strip numbers, strip thickness and strip width for different material properties as: Young's modules and density. The optimization objective in all work is minimizing the spring weight. The results are represented graphically in Figures **3** - **11**.

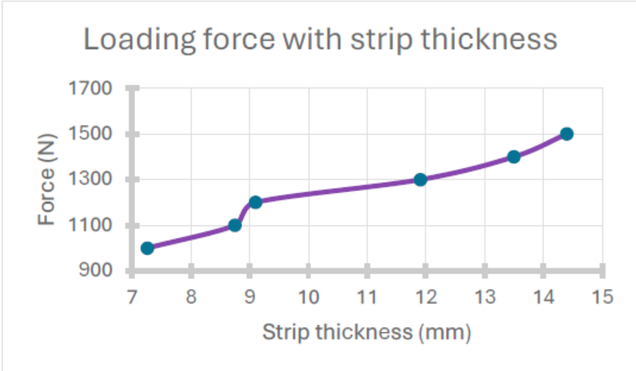

Figure 3: Variation of load with strip thickness.

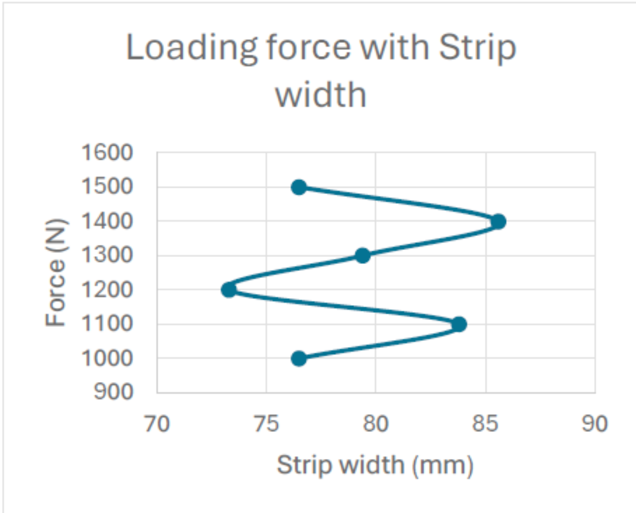

Figure 4: Variation of spring weight with strip thickness.

Table 2: Optimum Weight for Springs of Same Eye to Eye Length

	F (N)	n	*t* (mm)	*b* (mm)	*W (kg)*
L= 1100 mm	1000	4	7.26	76.5	19.2
	1100	4	7.26	83.8	21.1
	1200	5	7.26	73.3	23.1
	1300	5	7.26	79.4	25.0
	1400	5	7.26	85.6	27.0
	1500	6	7.26	76.5	28.9

Table 3: Optimum Weight for Springs under Same Loading

	L (mm)	n	*t* (mm)	*b* (mm)	*W (kg)*
F= 1000 N	1100	4	7.26	76.5	19.2
	1200	3	8.75	78.4	19.2
	1300	2	9.10	92.7	19.2
	1400	2	11.91	72.8	19.2
	1500	2	13.50	61.7	19.5
	1600	2	14.40	57.9	20.8

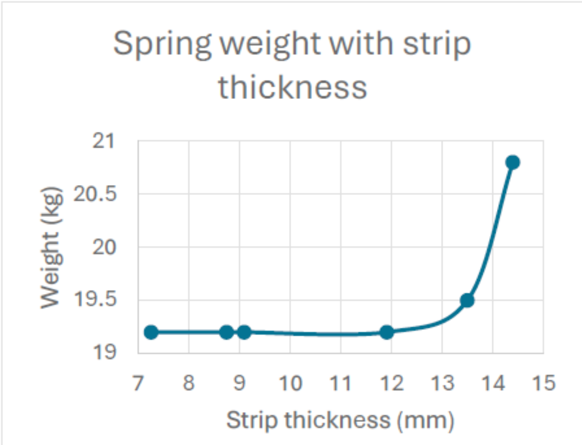

Figure 5: Variation of spring weight with strip thickness.

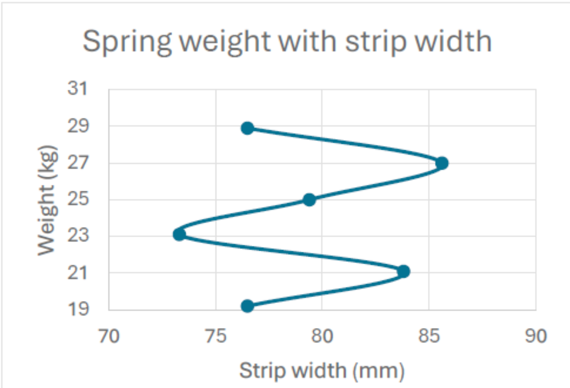

Figure 6: Variation of spring weight with strip width.

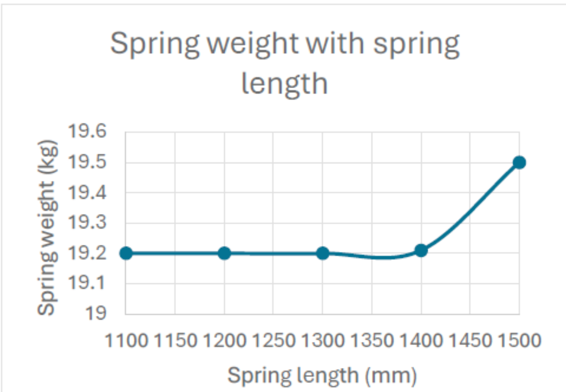

Figure 7: Variation of spring weight with spring length.

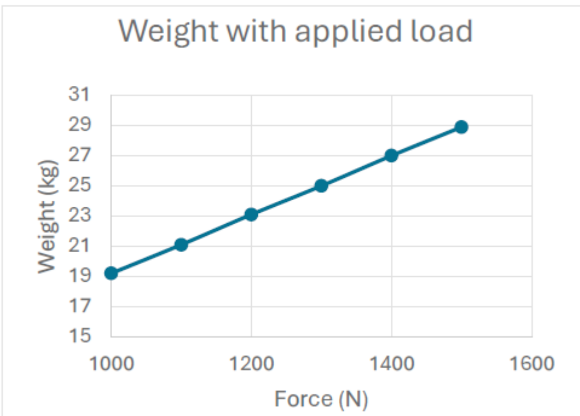

Figure 8: Variation of spring weight with loading force.

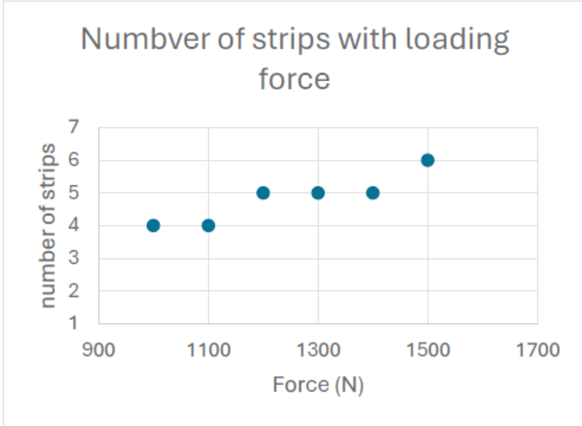

Figure 9: Variation of strips number with loading force.

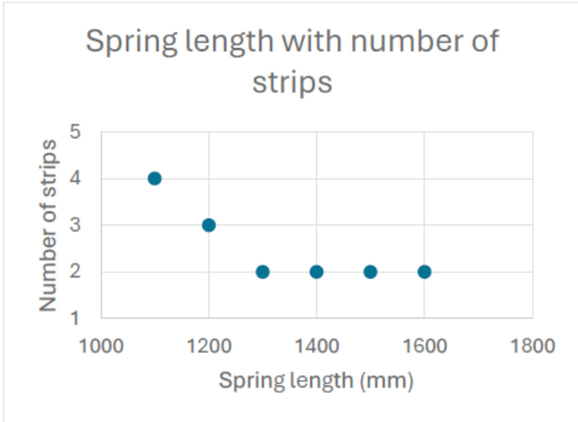

Figure 10: Variation of number of strips with spring length.

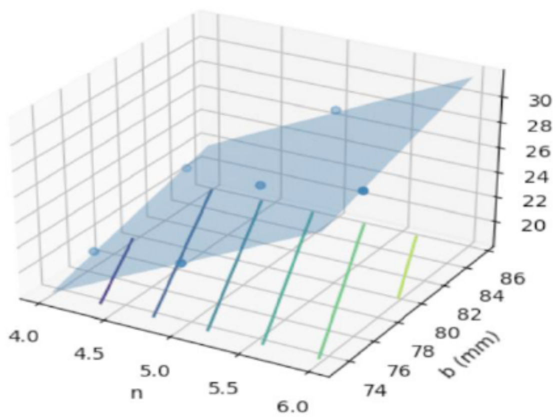

Figure 11: Variation of spring weight with spring width for different number of strips.

6. DISCUSSION OF RESULTS

A clear positive relationship between strip thickness and loading force, Figure **3**. As thickness increases, the force rises steadily, indicating a greater load-carrying capacity with thicker strips. There is a noticeable change in slope around strip thickness of 9 mm, suggesting a transition in stiffness or contact behavior, after which the force continues to increase more smoothly. The trend of variation of the load with strip thickness is nonlinear, with the rate of load increase becoming slightly higher at larger thicknesses, which enhances the structural rigidity as the strip gets thicker. Conversely, regarding the strips thickness, increasing the width does not lead to a simple, steady increase in force. This suggests that factors such as load distribution, edge effects, or interaction with other geometric parameters influence the response. Generally, the strip width affects the loading force in a more complex manner than the strip thickness." A non-monotonic relationship between strip width and loading force is indicated as shown in Figure **4**. The relationship between the spring weight and the strips thickness is illustrated in Figure **5**. For the initial range of strip thicknesses, from approximately 7.5 mm up to 12 mm, the spring weight remains remarkably constant and low around 19.2 kg. This suggests that within this range, increasing the strip thickness has negligible impact on the final weight of the spring. All strip thicknesses between 7.5 mm and 12 mm, showing a fixed weight of 19.2 kg. Critical threshold and sharp increase after 12 mm. Once, the strip thickness exceeds 12 mm, the spring weight starts to increase significantly showing a distinct, non-linear, and increasing trend. The chart in Figure 6 illustrates that the strip width increases from 77 mm to 86 mm, the weight first increases from 19 kg to 25 kg, then decreases to 21 kg and finally increases again to 27 kg. The minimum weight recorded is 19 kg at a strip width of approximately 77 mm. The maximum weight recorded is 27 kg at a 86 mm strip width. This may refer to the weight capacity of a spring might be affected by an interplay between material strength which might increase with width and buckling/instability which might increase at certain widths.

Figure **7** illustrates a very specific and relatively stable relationship between the spring length and its weight. The most striking feature is the extremely small variation in weight across a significant change in length. The most significant change occurs between 1400 mm and 1500 mm. The weight increases sharply from just over 19.2 kg at 1400 mm to the maximum recorded weight of 19.5 kg at 1500 mm. The entire range of weights is very narrow, spanning only 0.3 kg from the minimum of 19.18 kg to the maximum of 19.5 kg, while the length changes by 400 mm from 1100 mm to 1500 mm.

Figure **8** shows that there is a strong, direct linear relationship between the applied load and the spring weight. As load increases from 1000 N to 1500 N, weight increases from 19 kg to 29 kg. The spring has a consistent increase of 2 kg in its weight for every 100 N of applied load. The spring weight is nearly constant across most of the length range, with a sharp increase at the maximum length.

The optimum number of strips for a specified load is illustrated in Figure 9. For a low loading force of 1000 N and 1100 N, the system uses 4 strips. As the force increases to 1200 N and up to 1400 N, the number of strips increases to 5 strips. This indicates a design requirement to add structural support (strips) once the load exceeds 1100 N. At the highest measured force of 1500 N, the number of strips increases again to the maximum of 6 strips. This shows a second threshold at crossed loads between 1400 N and 1500 N, which requiring the maximum support structure.

Figure **10** illustrates an inverse relationship between the number of strips and the spring length. As the spring length increases from about 1100 to 1300 mm, the required number of strips drops sharply from 4 to 2. Beyond roughly 1300 mm, the curve flattens and the number of strips remains constant at around 2 despite further increases in length up to 1600 mm.

From data shown in 3D-chart given in Figure **11**, it is shown that a smooth trend for increasing the spring weight with both the number of strips and the strip width. As the number of strips increases from 4 to 6 and the strip width increases from roughly 75 to 85 mm, the spring weight increases steadily and align closely with the fitted surface, indicating a strong, predictable dependence and minimal scatter. This trend of weight variation may be considered as a stable design region with no abrupt nonlinearities, making optimization process lies within this domain reliable and well-behaved.

7. CONCLUSIONS

An optimum design approach for laminated leaf springs has been successfully developed and applied by integrating analytical modeling with nonlinear optimization techniques. Based on the results of the present study, the following conclusions can be withdrawn:

- An effective mathematical formulation for leaf spring optimization was established using analytical expressions for bending stress, deflection, stiffness, and natural frequency, allowing multiple design constraints to be handled simultaneously.
- The optimization results confirm that significant weight reduction can be achieved without compromising structural integrity or ride comfort, provided that appropriate bounds are imposed on stress, deflection, and natural frequency.
- Strip thickness has a strong and nonlinear influence on load-carrying capacity and spring weight, whereas strip width exhibits a more

complex, non-monotonic effect due to interactions with other geometric and loading parameters.

- The number of strips increases discretely with increasing load, indicating the existence of load thresholds beyond which additional leaves are required to maintain safe stress and deflection levels.
- The optimized steel leaf spring configuration for the selected design conditions consists of four strips with a width of approximately 75 mm and a thickness of about 7.5 mm, resulting in a total weight close to 19 kg.
- The natural frequency constraint plays a critical role in ensuring acceptable ride quality, and the optimized design satisfies the recommended frequency limits for vehicle suspension systems.
- The proposed methodology is flexible and can be extended to other materials, including composite leaf springs, as well as to different loading conditions and vehicle applications.

Overall, the study demonstrates that combining analytical modeling with numerical optimization provides a robust and efficient tool for the optimum design of laminated leaf springs with improved performance and reduced weight. These results will be verified practically by fabricating samples of these optimized springs and tested experimentally.

NOMENLATURE

M= bending moment (N-mm)

L= eye to eye spring length (mm)

B= total width of strips (mm)

b= strip width (mm)

t= strip thickness (mm)

τ= non-dimensional thickness ratio (=t/L)

β= non-dimensional width ratio (=b/L)

F= applied force (N)

δ= spring deflection at mid-point of spring (mm)

K= spring stiffness (N/mm)

n= number of strips

f_n= natural frequency (HZ)

ρ= density (kg/m^3)

E= Young's modules (GPa)

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