Multi-Objective Design Optimization of Thermal Engineering Systems using Rao Algorithms

Ravipudi Venkata Rao*

Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat, India

Abstract: Thermal engineering systems include devices or processes where heat transfer plays a major role, such as heat exchangers, refrigeration units, combustion engines, solar collectors, cooling systems in electronics, etc. These systems often have multiple performance objectives like minimizing energy consumption, maximizing heat transfer, reducing costs, improving efficiency and reliability, etc. Design optimization aims to determine the best configuration and parameters for these systems within physical, operational, and economical constraints. The number of objectives considered for optimization may be one or more than one. This study employs three optimization algorithms, known as Rao algorithms, to handle both constrained and unconstrained thermal system optimization problems under single- and multi-objective settings. Their multi-objective extensions-MO-Rao algorithms-are effectively utilized to solve selected two- and three-objective thermal engineering case studies. Additionally, the BHARAT decision-making method is applied to identify the best compromise solution from the set of Pareto-optimal non-dominated solutions. Because the optimization is performed on fast-evaluating RSM models, the obtained Pareto fronts can be readily integrated into intelligent energy management frameworks, where real-time operating conditions can be mapped to pre-optimized design or control configurations. Researchers and practitioners across scientific and engineering domains may find these algorithms advantageous in solving real-world, constrained, and non-convex single-, multi-objective optimization problems.

Keywords: Thermal engineering systems, Multi-objective optimization, Rao algorithms, BHARAT method.

1. INTRODUCTION

Thermal engineering systems are at the heart of many engineering applications — from power generation and heating, ventilation, and air conditioning (HVAC) to electronic cooling, renewable energy systems, and industrial processes. These systems must handle heat effectively, efficiently, and sustainably. However, designing such systems poses several challenges due to competing requirements, operational constraints, and the need for high performance. This makes design optimization not just useful but essential. For example, optimal designs of thermal systems lead to improved energy efficiency (improved heat transfer efficiency, improved thermal performance, minimum energy input), reduced operational cost (energy savings translate into reduced operating and life cycle costs, lower fuel usage, and low material waste, contributing to economic efficiency), compact and lightweight designs, environmental sustainability (reduced emissions, enhanced use of renewable energy, and lower carbon footprint), and improved product performance and reliability (improved safety and reliability of electronic devices, engines, and mechanical systems).

Design optimization of thermal engineering systems aims to determine the best configuration and parameters for these systems within physical, operational, and economical constraints. Optimization may involve a single objective or multiple objectives. In practice, most thermal engineering systems include several conflicting goals—such as maximizing heat transfer, minimizing pressure drop, improving efficiency, and reducing cost—making multi-objective optimization essential.

Multi-objective optimization techniques enable the exploration of trade-offs among conflicting objectives and the generation of Pareto-optimal solutions, thereby supporting informed decision-making. Numerous studies have applied population-based advanced optimization algorithms to improve the design and performance of various thermal systems [1-5]. Their adaptive nature and capability to navigate high-dimensional, nonlinear, multimodal landscapes make them especially valuable when conventional deterministic or gradient-based methods fail to deliver viable solutions. These heuristic methods progressively improve a set of candidate solutions, balancing exploration and exploitation of the search space to converge toward globally optimal results.

The widespread adoption of metaheuristics is driven by their advantages, including flexibility, gradient-free search, strong global exploration, multi-objective compatibility, and ease of adaptation. Nevertheless, their limitations include the absence of guarantees for global optimality, potential convergence issues in intricate or large-scale problems, and the need for

*Address correspondence to this author at the Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat, India;
E-mail: rvr@med.svnit.ac.in

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careful configuration of algorithmic parameters. Many such techniques, barring a few notable exceptions like Jaya and Rao algorithms [6], are heavily reliant on algorithm-specific tuning parameters. Improper calibration may lead to suboptimal convergence or excessive computational overhead.

In recent years, numerous metaphor-based metaheuristics have emerged, drawing inspiration from nature, physics, astronomy, and cultural behaviors. However, many of these approaches offer only superficial links between the metaphor and the mathematical model, resulting in limited scientific justification. Critics have increasingly voiced concerns about this trend. Rao [6] advocated for a paradigm shift, encouraging the design of simple, robust algorithms free from metaphorical distractions. Instead of crafting yet another metaphor-laden heuristic, he proposed that researchers develop efficient strategies with clear logic and effective performance. Sörensen [7] emphasized the dangers of pursuing novelty without rigor, cautioning that the field might suffer long-term credibility damage and a decline in scientific integrity due to metaphorical excess. Similarly, Campelo and Aranha [8] underscored that emulating the real-world phenomena without mathematical soundness contributes little to scientific advancement. Aranha *et al*. [9] further criticized the prevalence of algorithms grounded in implausible processes, exposing the lax methodological standards behind many such publications.

A recurring issue in metaphor-based optimization algorithms is the disconnection among three elements: the metaphor, the mathematical formulation, and the actual implementation [10, 11]. Recent literature echoes this sentiment. Sarhani *et al*. [12] categorized algorithm initialization strategies, while Rajwar *et al*. [13] reviewed over 500 metaheuristics, questioning whether superficial modifications of existing search strategies truly constitute innovation. Velasco *et al*. [14] analyzed 111 articles proposing “new” or “hybrid” techniques, highlighting the common omission of the No Free Lunch theorem in their foundations. Benaissa *et al*. [15] critiqued the inflation of expectations surrounding metaphor-driven methods, warning against catchy names that misrepresent algorithmic novelty. Given these observations, this work is driven by the conviction that effective optimization algorithms need not rely on metaphorical constructs.

Recently, Rao and Davim [16, 17] introduced two advanced optimization algorithms—Best-Worst-Random (BWR) and Best-Mean-Random (BMR)—and successfully demonstrated their applicability in optimizing various manufacturing processes. Rao *et al*. [18] proposed BWR, BMR, and BMWR for the optimization of heat transfer systems. These algorithms proved that the optimization methods can be devised without any metaphorical inspiration. Rao algorithms developed by Rao [19] are also simple, metaphor-free, and algorithm-specific parameter-independent algorithms. The objective of the present work is to extend the application of Rao algorithms for the single- and multi-objective optimization problems of different thermal engineering systems. The convergence performance and solution accuracy of the proposed algorithms will also be evaluated.

The proposed algorithms are described in the next section.

2. MATERIALS AND METHODS

2.1. Rao Algorithms for Single-Objective Optimization

Let the objective function be denoted as *f(x)*, where the aim is either minimization or maximization. Consider an optimization problem with m decision variables *Xv*, *v = 1,2,…,m* and a population of solutions indexed by *k = 1,2,…,c*, evolving through iterations indexed by i. Further, let *X{v,b,i}*: best value of variable *v* in iteration *i*; *X{v,w,i}*: worst value of variable *v* in iteration *i*; and *X{v,k,i}*and *X{v,l,i}*: randomly selected individuals, and *r1* and *r2* are uniformly distributed random numbers in [0,1].

Update Rules [19]:

Rao1: $X'_{v,k,i} = X_{v,k,i} + r_1 (X_{v,b,i} - X_{v,w,i})$ (1)

Rao2: $X'_{v,k,i} = X_{v,k,i} + r_1 (X_{v,b,i} - X_{v,w,i}) + r_2 (|X_{v,k,i} \text{ or } X_{v,l,i}| - |X_{v,l,i} \text{ or } X_{v,k,i}|)$ (2)

Rao3: $X'_{v,k,i} = X_{j,k,i} + r_1 (X_{v,b,i} - |X_{v,w,i}|) + r_2 (|X_{v,k,i} \text{ or } X_{v,l,i}| - (X_{v,l,i} \text{ or } X_{v,k,i}))$ (3)

In Eqs. (2) and (3), the term $X_{v,k,i}$ or $X_{v,l,i}$ indicates that the candidate solution *k* is compared with any randomly picked candidate solution *l* and the information is exchanged based on their fitness values. If the fitness value of *k*th solution is better than the fitness value of *l* th solution then the term “$X_{v,k,i}$ or $X_{v,l,i}$” becomes $X_{v,k,i}$. On the other hand, if the fitness value of *l*th solution is better than the fitness value of *k*th solution then the term “$_{v,k,i}$ or $_{v,l,i}$” becomes $X_{v,l,i}$.

Table **1** shows the pseudocode of the Rao algorithms.

The description and code of the Rao algorithms is also available at https://sites.google.com/view/raoalgorithms/.

Table 1: Pseudocode of Rao Algorithms

```
Define the objective function f(x) to be minimized or maximized.
Set the number of decision variables m, population size c, maximum number of iterations MaxIter, and variable bounds [Xmin, Xmax].
Select the Rao variant: Rao1, Rao2, or Rao3.

Randomly initialize the population X(v,k,0), v = 1,…,m; k = 1,…,c, within the specified bounds.
Evaluate the fitness f(Xk) of all candidate solutions.

For iteration i = 1 to MaxIter do
  Identify the best solution X(v,b,i) and the worst solution X(v,w,i) in the current population.
For each candidate solution k = 1 to c do
  Randomly select another candidate solution l such that l ≠ k.
    Generate two random numbers r1 and r2 uniformly in [0,1].

    If f(Xk) is better than f(Xl) then
      X^+(v,i) = X(v,k,i) and X^-(v,i) = X(v,l,i);
    Else
      X^+(v,i) = X(v,l,i) and X^-(v,i) = X(v,k,i);
    End if

    For each decision variable v = 1 to m do
      If Rao1 is selected then
        X′(v,k,i) = X(v,k,i) + r1 [ X(v,b,i) − X(v,w,i) ];
      Else if Rao2 is selected then
        X′(v,k,i) = X(v,k,i)
         + r1 [ X(v,b,i) − X(v,w,i) ]
         + r2 [ |X^+(v,i)| − |X^-(v,i)| ];
      Else if Rao3 is selected then
        X′(v,k,i) = X(v,k,i)
         + r1 [ X(v,b,i) − |X(v,w,i)| ]
         + r2 [ |X^+(v,i)| − |X^-(v,i)| ];
      End if
    End for
    Repair X′k if any variable violates its bounds.
    If f(X′k) is better than or equal to f(Xk) then
      Replace Xk with X′k.
    End if
  End for
End for

Output the best solution X_best and its corresponding fitness value f(X_best).
```

2.3. Multi-Objective Optimization Rao Algorithms

The procedural steps of multi-objective Rao algorithms (MO-Rao) are briefly given in Table **2**.

The MO-Rao algorithms produce a Pareto front of non-dominated solutions, all of which are considered acceptable. When a decision-maker needs to prioritize objectives, a multi-attribute decision-making (MADM) method can be applied. Recently, Rao [20] introduced the BHARAT method, which provides a simple and logical way to assign objective weights and rank the solutions.

The following subsection outlines the BHARAT method, which is used to identify the most suitable compromise solution from the set of non-dominated Pareto-optimal solutions.

2.4. Selecting the Best Compromise Solution from the Pareto Front using the BHARAT Decision-Making Method

BHARAT (Best Holistic Adaptable Ranking of Attributes Technique), developed by Rao [20] in 2024, can be used alongside the R-method proposed by Rao and Lakshmi [21] in 2021.The method is suitable for both individual and group decision-making contexts,

Table 2: Procedural Steps of MO-Rao Algorithms

Step No.	Step Title	Description
1	Start	Begin the algorithm.
2	Generate Initial Population	Create an initial set of candidate solutions within variable boundaries.
3	Elite Seeding	Insert high-quality or historically strong solutions into the population.
4	Fast Non-dominated Sorting	Classify solutions into Pareto fronts and compute crowding distances.
5	Constraint Check and Repair	Correct constraint-violating solutions where possible.
6	Apply penalties	Penalize any remaining constraint violations to guide the search.
7	Objective Evaluation	Compute all objective function values for each solution.
8	MO-Rao1/MO-Rao2/MO-Rao3 Update	Generate improved solutions using the respective update mechanism.
9	Check termination	Verify whether the stopping condition (*e.g.*, maximum iterations) has been reached.
10	Continue search?	If not completed, return to the constraint-handling step.
11	Output Pareto Front	Provide the final set of non-dominated (Pareto-optimal) solutions.

emphasizing simplicity, transparency, and consistency rather than computational intensity. The steps involved in applying the BHARAT method to multi- or many-objective optimization problems are outlined below.

Step 1: Define the decision-making problem

- Determine the set of Pareto-optimal alternatives.
- Indicate which objectives are beneficial (to be maximized) and which are non-beneficial (to be minimized).

Step 2: Assign weights to the objectives

- Rank the objectives based on importance.
- Use R-method (based on reciprocals of cumulative reciprocal ranks) to convert ranks into weights (w_i) [20].

Step 3: Normalize the objective values corresponding to each Pareto-optimal alternative.

For *beneficial* objectives:

$$x_{ji}^{\text{normalized}} = x_{ji}/x_i^{\text{best}} \tag{4}$$

- For non-beneficial attributes:

$$x_{ji}^{\text{normalized}} = x_i^{\text{best}}/x_{ji} \tag{5}$$

Step 5: Compute total score

- For each alternative, compute:

$$\text{Total Score}j = \sum_{i=1}^{m} w_i \cdot x_{ji}^{\text{normalized}} \tag{6}$$

Step 6: Rank the alternative Pareto-optimal solutions

Higher total score ⇒ better alternative.

Now, the Rao algorithms are attempted on the single- and multi-objective optimization of selected thermal engineering systems.

3. APPLICATION OF RAO ALGORITHMS FOR SINGLE-OBJECTIVE OPTIMIZATION OF THERMAL ENGINEERING SYSTEMS

An example is presented to demonstrate and validate the Rao algorithms for single-objective optimization of a thermal engineering system.

Nekahi *et al*. [22] used a multi-nozzle microchannel heat sink with six different fin forms in order to identify the ideal fin design. Figure **1** shows the six different fin forms. The design that offers the greatest heat transfer and the least pressure drop was chosen as the contender for more optimization after all six fins were first numerically simulated. Then, 27 tests were planned to investigate how the geometric characteristics of the ideal fin affected the Nusselt number (*Nu*) and the pressure drop (ΔP). The data collected was then used to train the response surface models (RSM) and artificial neural network models.

Three geometric characteristics of the selected fin, including length (L_f), horizontal pitch (W_{bf}), and vertical pitch (H_{bf}), were optimized within predetermined limits using these models. In the final stage of the optimization process, the neural network model and a genetic algorithm (GA) performed optimization with the aim of finding the ideal balance between Nusselt number and pressure drop with maximum efficiency.

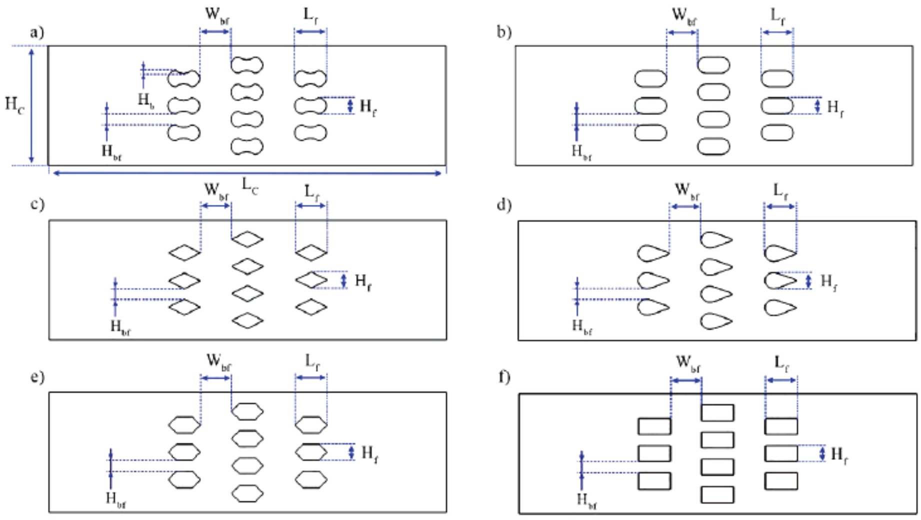

Figure 1: Different fin forms of the microchannel heat sink and their geometry [22].

The design variables are: length (L_f), horizontal pitch (W_{bf}), and vertical pitch (H_{bf}). The bounds of the variables are: 15μm ≤ W_{bf} ≤ 145μm, 15μm ≤ H_{bf} ≤ 41μm, 50μm ≤ L_f ≤ 110μm. The objective function *Nu* is expressed as given by Eq. (7).

Maximize $Nu = 17.7245 + 0.0286 \times W_{bf} + 0.2066 \times H_{bf} + 0.0469 \times L_f - 1.4500 \times 10^{-4} \times W_{bf} \times H_{bf} - 5.9831 \times 10^{-6} \times W_{bf} \times L_f - 4.0197 \times 10^{-4} \times H_{bf} \times L_f - 7.6095 \times 10^{-5} \times W_{bf}^2 - 3.6800 \times 10^{-3} \times H_{bf}^2 + 1.2305 \times 10^{-4} \times L_f^2$ (7)

Now the Rao algorithms are used to find the maximum value of *Nu*. A population size of 20 and iterations of 50 are considered for carrying out the optimization. The maximum value of *Nu* obtained by these three algorithms is the same, and it is *Nu* = 28.18156, corresponding to W_{bf} of 145μm, H_{bf} of 19.20629μm, and L_f of 110μm. Figure **2** shows the convergence graph for the Rao algorithms. The maximum value of *Nu* = 28.18156 is achieved within the first few iterations of Rao algorithms. This shows that the Rao algorithms converge much faster and reaches the global best quickly.

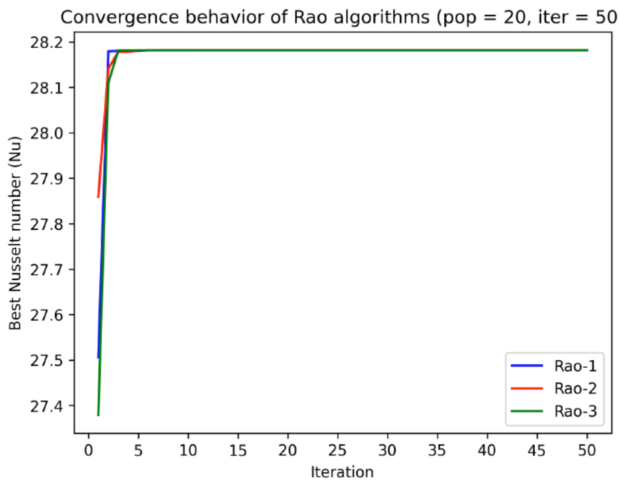

Figure 2: Convergence behavior of Rao algorithms for *Nu* (microchannel heat sink).

Nekahi *et al*. [22] presented another objective function of minimising pressure drop (ΔP) as a single objective for optimization. The objective function is given by Eq. (8).

$$\Delta P = 0.3833 + 9.8615 \times 10^{-4} \times W_{bf} + 0.0106 \times H_{bf} + 2.1589 \times 10^{-3} \times L_f - 5.4390 \times 10^{-5} \times W_{bf} \times H_{bf} - 2.7906 \times 10^{-6} \times W_{bf} \times L_f + 1.1603 \times 10^{-5} \times H_{bf} \times L_f + 1.9099 \times 10^{-6} \times W_{bf}^2 - 4.7824 \times 10^{-5} \times H_{bf}^2 + 3.1364 \times 10^{-6} \times L_f^2 \quad (8)$$

Now the Rao algorithms are used to find the minimum value of ΔP. The minimum value of ΔP obtained by these two algorithms is the same, and it is ΔP = 0.6569191, corresponding to W_{bf} of 15μm, H_{bf} of 15μm, and L_f of 50μm. Figure **3** shows the convergence graph.

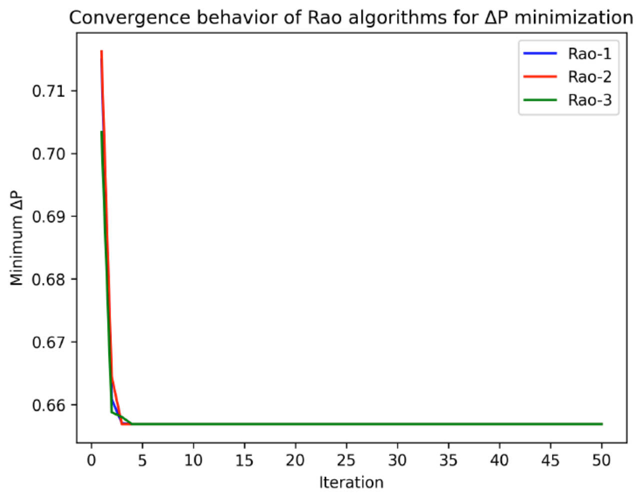

Figure 3: Convergence behavior of Rao algorithms for ΔP (microchannel heat sink).

The minimum value of ΔP = 0.6569191 is achieved after first few iterations for the Rao algorithms. The three algorithms rapidly found the global minimum at the very start—showing that the minimum ΔP occurs right at the lower bound of the design space.

In this example of optimization of a finned microchannel heat sink, the two objectives (*i.e.*, *Nu* and ΔP) are optimized independently, considering them as individual single-objective functions. The values of optimum variables are W_{bf} =145μm, H_{bf} = 19.20629μm, and L_f =110μm to give a maximum *Nu* value of 28.18156. However, the same values of variables may not give the minimum value of ΔP. The values of optimum variables are W_{bf} =15μm, H_{bf} = 15μm, and L_f =50μm to give a minimum ΔP value of 0.6569191. These are the optimal values when the two objectives are considered independently. However, in practical applications, both Nu and ΔP must be optimized simultaneously to determine variable values that satisfactorily meet both objectives. This places the problem within the domain of multi-objective optimization, which is addressed in the next section.

4. APPLICATION OF MO-RAO ALGORITHMS FOR MULTI-OBJECTIVE OPTIMIZATION OF THERMAL ENGINEERING SYSTEMS

To demonstrate the practical applicability of the proposed MO-Rao algorithms, two thermal engineering case studies are examined.

4.1. Multi-Objective Optimization of a Finned-Microchannel Heat Sink with 2-Objectives

In section 3, an example of optimization of a finned microchannel heat sink with two objectives (*i.e.*, *Nu* and ΔP) are optimized independently, considering them as individual single-objective functions. The values of optimum variables are W_{bf} =145μm, H_{bf} = 19.20629μm, and L_f =110μm to give a maximum *Nu* value of 28.18156. However, the same values of variables may not give the minimum value of ΔP. Similarly, the values of optimum variables W_{bf} =15μm, H_{bf} = 15μm, and L_f =50μm to give a minimum ΔP value of 0.6569191 may not give the maximum value of *Nu*. Hence, the MO-Rao algorithms are applied to find out the optimum values of design variables that give the maximum *Nu* and minimum ΔP simultaneously. The MO-Rao algorithms are applied with population size of 50 and number of iterations of 100. Table **3** gives the composite front containing a set of 136 non-dominated and non-repeated optimal solutions given by MO-Rao algorithms.

Nekahi *et al*. [22] derived the efficiency term η defined by Eq. (9).

$$\eta = (Nu/21.089)/ ((\Delta P/0.5091)^{1/3}) \tag{9}$$

The values in the denominator corresponded to a microchannel heat sink without fins and is considered as reference with η =1.0. The MO-Rao algorithms are run with the condition that the solution produced should give a η value greater than 1.0. The solutions are shown in Table **3**.

The Pareto front of the composite front is formed by combining the solutions of MO-Rao algorithms and it contains only the unique non-dominated solutions. Figure **4** shows the composite Pareto front. In fact, one can use only one of the MO-Rao algorithms. However, running the 3 algorithms and then combining them to form a composite front containing the non-repeated and non-dominated solutions will provide a much better solution path.

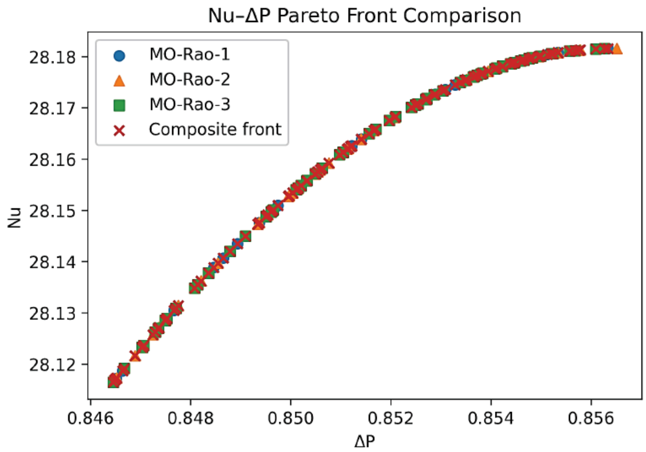

Figure 4: Pareto front obtained by the composite front for the finned-microchannel heat sink design.

From Table **3**, a list of top-performing 10 designs ranked by η (given by the composite front) is shown in Table **4**.

It can be understood that all the above designs are good. The MO-Rao algorithms have given η *of* 1.125482 with *Nu*=28.09718 and ΔP=0.844516. Thus, compared to the reference microchannel heat sink without fins, this corresponds to 12.5482% increase in the thermal efficiency.

It may be noted that the above optimization results by MO-Rao are based on the RSM models developed for *Nu* and ΔP by Nekahi *et al*. [22]. Those authors applied a multi-objective genetic algorithm (GA) to the RSM models to get the optimum values of the variables and the objectives. Hence, for a fair comparison, the present results are compared with the results of the GA used by Nekahi *et al*. [22]. Table **5** shows the comparison of the results. The values of the objective functions given by Nekahi *et al*. [22] for maximum *Nu* were incorrectly computed by them for W_{bf} =145μm, H_{bf} = 19.199μm, and L_f = 110μm. Hence, the values of *Nu*,

Table 3: Non-dominated Optimal Solutions Provided by MO-Rao Algorithms for the Finned-Microchannel Heat Sink

Solution No.	Design variables			Nu	ΔP	η	Algorithm
	W_{bf}	H_{bf}	L_f				
1.	140.2529	15	110	28.09718	0.844516	1.125482	MO-Rao-2
2.	139.8469	15	110	28.09537	0.844354	1.125481	MO-Rao-1
3.	137.8754	15	110	28.08623	0.843578	1.125461	MO-Rao-2
4.	143.4279	15	110	28.11045	0.845802	1.125442	MO-Rao-3
5.	145	15.35531	110	28.12699	0.847355	1.125416	MO-Rao-3
6.	145	15.269	110	28.12451	0.847137	1.125414	MO-Rao-2
7.	135.8946	15	110	28.07646	0.842813	1.125409	MO-Rao-3
8.	135.5271	15.25615	110	28.08262	0.843456	1.12537	MO-Rao-3
9.	141.4154	16.23193	110	28.13541	0.848297	1.125336	MO-Rao-2
10.	130.7372	15	110	28.04821	0.840891	1.125132	MO-Rao-2
11.	145	16.98994	110	28.16348	0.851348	1.125112	MO-Rao-3
12.	130.5162	15.13863	110	28.05142	0.841274	1.125091	MO-Rao-3
13.	127.876	15	110	28.03079	0.839869	1.12489	MO-Rao-1
14.	124.9818	15	110	28.0119	0.838867	1.124579	MO-Rao-2
15.	120.0385	15	110	27.9767	0.837229	1.123898	MO-Rao-3
16.	117.1174	15.18696	110	27.96056	0.837065	1.123323	MO-Rao-1
17.	115.0653	15	110	27.93753	0.835676	1.123019	MO-Rao-1
18.	107.2143	15	110	27.86803	0.833416	1.121237	MO-Rao-1
19.	106.2404	15	110	27.85875	0.833152	1.120982	MO-Rao-1
20.	111.0698	15.5608	108.3098	27.80931	0.832361	1.119347	MO-Rao-1
21.	98.29417	15	110	27.77768	0.831133	1.118624	MO-Rao-3
22.	97.58046	15	110	27.76992	0.830964	1.118388	MO-Rao-1
23.	136.6411	15.34507	104.8199	27.74719	0.830511	1.117675	MO-Rao-2
24.	117.6588	15.87528	105.4477	27.68424	0.827727	1.116388	MO-Rao-1
25.	121.3041	15.37023	104.5879	27.63893	0.824596	1.11597	MO-Rao-3
26.	117.6455	15	104.8392	27.6146	0.822647	1.115868	MO-Rao-2
27.	130.6902	15	102.9443	27.5802	0.822274	1.114646	MO-Rao-3
28.	127.124	15	101.4727	27.46208	0.817081	1.112219	MO-Rao-2
29.	116.3246	15	102.3133	27.43817	0.815504	1.111966	MO-Rao-1
30.	104.2223	15	103.4225	27.40161	0.814781	1.110812	MO-Rao-1
31.	108.8489	15	101.2481	27.30377	0.810309	1.108879	MO-Rao-3
32.	114.4301	15	99.44694	27.23619	0.807302	1.107506	MO-Rao-2
33.	137.5224	15.88112	95.04415	27.13892	0.80709	1.103647	MO-Rao-1
34.	118.8502	15	95.43751	27.01473	0.798322	1.102604	MO-Rao-1
35.	141.0503	15.80484	93.28175	27.04169	0.803796	1.101194	MO-Rao-3
36.	118.0956	15	92.76564	26.83964	0.791124	1.098771	MO-Rao-3
37.	106.8857	15.2836	94.44576	26.85968	0.792962	1.09874	MO-Rao-1
38.	113.8195	15	93.24111	26.83447	0.790844	1.098688	MO-Rao-3
39.	121.6465	15	91.0811	26.76099	0.788105	1.096947	MO-Rao-3
40.	121.5102	15	90.95694	26.75222	0.787735	1.09676	MO-Rao-3
41.	117.1895	15	91.32818	26.74212	0.787083	1.096648	MO-Rao-3
42.	122.2693	15	90.12444	26.70582	0.785894	1.095712	MO-Rao-3
43.	117.4376	15.06903	89.92596	26.65946	0.783828	1.09477	MO-Rao-3

44.	107.8434	15	90.14101	26.58603	0.780769	1.093179	MO-Rao-1
45.	121.6666	15	87.78042	26.556	0.779682	1.092451	MO-Rao-3
46.	113.7585	15	88.19693	26.51861	0.777786	1.091798	MO-Rao-3
47.	123.3858	15	86.69793	26.50178	0.777613	1.091186	MO-Rao-2
48.	123.6003	15	85.27158	26.41589	0.774094	1.089295	MO-Rao-1
49.	127.4171	15	84.63745	26.4032	0.774074	1.088782	MO-Rao-3
50.	112.5899	16.21352	86.10454	26.429	0.776834	1.088553	MO-Rao-2
51.	103.9546	15.00557	86.64051	26.33127	0.770445	1.087518	MO-Rao-3
52.	127.234	15	83.47822	26.33153	0.771093	1.087224	MO-Rao-3
53.	108.3147	15.43438	84.987	26.29154	0.769469	1.086335	MO-Rao-3
54.	108.3774	15.67238	84.47674	26.27116	0.769179	1.08563	MO-Rao-1
55.	123.2967	15.55449	81.13907	26.18747	0.765466	1.083918	MO-Rao-2
56.	120.5316	15	81.27142	26.15079	0.762795	1.083662	MO-Rao-2
57.	119.1007	15	81.3878	26.14677	0.762513	1.083629	MO-Rao-2
58.	104.7591	15.00656	82.94773	26.11371	0.761212	1.082875	MO-Rao-2
59.	103.3616	15	83.11297	26.10933	0.761137	1.082729	MO-Rao-3
60.	121.6847	15	80.28556	26.10027	0.760798	1.082514	MO-Rao-1
61.	109.8742	15.16105	81.11584	26.05942	0.758976	1.081684	MO-Rao-3
62.	123.6635	15	77.92752	25.97413	0.755762	1.07967	MO-Rao-2
63.	103.1235	15.52555	80.75292	25.98893	0.757367	1.079522	MO-Rao-3
64.	90.37257	15	83.75632	26.00319	0.758807	1.07943	MO-Rao-3
65.	123.5235	15	77.69732	25.95949	0.755132	1.079361	MO-Rao-3
66.	113.455	15	78.09043	25.90301	0.752028	1.078493	MO-Rao-3
67.	106.8402	15.77456	78.83202	25.9221	0.754712	1.078007	MO-Rao-3
68.	107.0224	15.57748	78.75862	25.91081	0.753757	1.077992	MO-Rao-3
69.	107.6609	15.14839	78.25195	25.86691	0.750889	1.077534	MO-Rao-3
70.	107.5157	15	77.78835	25.83076	0.749043	1.076912	MO-Rao-2
71.	114.1239	15.25677	74.63725	25.71762	0.744655	1.074296	MO-Rao-2
72.	118.977	15.2283	73.9012	25.71315	0.744645	1.074114	MO-Rao-2
73.	127.5073	15.76113	73.03681	25.74811	0.747754	1.074082	MO-Rao-2
74.	120.9018	16.39839	73.49113	25.75457	0.748362	1.074061	MO-Rao-3
75.	131.2742	15	72.99694	25.73484	0.747078	1.073853	MO-Rao-2
76.	112.4281	15	73.83344	25.6433	0.74101	1.072945	MO-Rao-2
77.	108.5049	15	73.78529	25.60451	0.739375	1.072111	MO-Rao-3
78.	116.6608	16.56694	72.29908	25.65992	0.744543	1.07194	MO-Rao-2
79.	88.6409	15	78.3711	25.65722	0.744355	1.071917	MO-Rao-3
80.	121.4042	15.14765	71.83666	25.60812	0.740304	1.071814	MO-Rao-1
81.	109.6055	15	73.21843	25.58179	0.738383	1.071639	MO-Rao-2
82.	120.5397	15.85518	69.99135	25.52858	0.737816	1.069685	MO-Rao-2
83.	120.8515	15.50192	69.87757	25.50833	0.73648	1.069482	MO-Rao-2
84.	102.1437	15.02908	72.86529	25.48906	0.734864	1.069457	MO-Rao-1
85.	107.8801	15	69.42044	25.34607	0.728309	1.066638	MO-Rao-2
86.	104.3974	15.81244	69.32624	25.34713	0.7302	1.065761	MO-Rao-1
87.	100.9814	15.00165	69.51151	25.28159	0.725953	1.065075	MO-Rao-2
88.	91.62728	15.87702	71.78441	25.35106	0.732816	1.064657	MO-Rao-1
89.	111.6827	15.5069	66.66293	25.25043	0.725011	1.064222	MO-Rao-1
90.	107.9082	15.48386	66.94509	25.23032	0.724191	1.063776	MO-Rao-2

91.	96.25049	15	69.06516	25.20386	0.72316	1.063165	MO-Rao-2
92.	115.7848	15	64.89916	25.16075	0.720509	1.062646	MO-Rao-1
93.	104.4759	15	66.52382	25.14751	0.71986	1.062406	MO-Rao-2
94.	104.6831	15	66.32054	25.1381	0.71944	1.062215	MO-Rao-2
95.	98.4473	15	66.18952	25.06476	0.716789	1.060421	MO-Rao-1
96.	113.5821	15	62.934	25.03195	0.714818	1.060005	MO-Rao-1
97.	103.849	16.00695	64.49289	25.07851	0.718883	1.059971	MO-Rao-2
98.	117.8043	15.28953	61.55593	25.00656	0.714363	1.059155	MO-Rao-2
99.	101.4895	15	63.34243	24.93765	0.710914	1.057941	MO-Rao-2
100.	119.3831	16.23849	59.50998	24.95357	0.713297	1.057436	MO-Rao-3
101.	107.7123	15.16937	61.41831	24.90309	0.709376	1.057238	MO-Rao-2
102.	103.4399	15	62.06163	24.88672	0.708543	1.056957	MO-Rao-1
103.	119.7291	15.74299	59.02641	24.9063	0.710673	1.056731	MO-Rao-2
104.	112.458	15.43226	59.83902	24.87429	0.708507	1.056447	MO-Rao-1
105.	112.8164	15	59.51114	24.83604	0.706269	1.055935	MO-Rao-1
106.	118.2319	15	58.19111	24.80987	0.70555	1.055181	MO-Rao-1
107.	101.3469	15	60.61023	24.78446	0.704196	1.054775	MO-Rao-1
108.	99.00037	15	60.32181	24.74317	0.702593	1.053818	MO-Rao-1
109.	102.8383	15.05458	59.24249	24.72805	0.7017	1.053621	MO-Rao-3
110.	105.853	15	58.56412	24.71851	0.701051	1.05354	MO-Rao-1
111.	100.8864	15	59.37337	24.71148	0.701015	1.053258	MO-Rao-1
112.	116.258	15	56.61358	24.70795	0.700914	1.053158	MO-Rao-2
113.	110.6862	15	57.29568	24.69581	0.700055	1.053071	MO-Rao-1
114.	117.9215	15	55.46946	24.65997	0.698984	1.05208	MO-Rao-2
115.	120.7337	15	55.18183	24.66682	0.699635	1.052045	MO-Rao-3
116.	97.41769	15	59.07559	24.65705	0.698961	1.051966	MO-Rao-1
117.	116.0812	15.22918	55.05224	24.63521	0.697947	1.051543	MO-Rao-2
118.	112.3655	15.45825	54.82767	24.6036	0.696617	1.050862	MO-Rao-3
119.	105.8078	15.96062	55.56961	24.60834	0.69771	1.050515	MO-Rao-3
120.	101.8619	15	56.195	24.54869	0.693734	1.049967	MO-Rao-1
121.	109.4767	15	54.34482	24.52532	0.692518	1.049582	MO-Rao-3
122.	95.37765	15.20419	57.4293	24.5558	0.695141	1.049562	MO-Rao-1
123.	124.7907	15.04324	52.5163	24.55707	0.695542	1.049414	MO-Rao-2
124.	112.0954	15.1766	52.51236	24.46204	0.690007	1.048141	MO-Rao-2
125.	104.6865	15	52.40911	24.37504	0.68587	1.046509	MO-Rao-2
126.	113.8139	15.45138	50	24.36058	0.685865	1.045891	MO-Rao-2
127.	105.9612	16.35191	51.08891	24.39275	0.688609	1.045879	MO-Rao-3
128.	89.0584	15	55.80467	24.37955	0.687942	1.045651	MO-Rao-1
129.	101.1615	15	51.79847	24.30579	0.682948	1.045022	MO-Rao-3
130.	96.95623	15	52.73067	24.30892	0.683477	1.044887	MO-Rao-2
131.	109.3165	15	50	24.29329	0.682215	1.044859	MO-Rao-3
132.	96.89992	15.93317	51.86136	24.31607	0.685531	1.044149	MO-Rao-3
133.	98.83496	15.34145	50	24.20613	0.679212	1.042642	MO-Rao-3
134.	97.08214	15	50	24.16581	0.677016	1.04203	MO-Rao-1
135.	93.11187	15	50	24.11954	0.675451	1.040837	MO-Rao-2
136.	93.00769	15.07213	50	24.12279	0.675749	1.040825	MO-Rao-2

Table 4: Ranked list of Top-Performing 10 Designs by *η* for the Finned-Microchannel Heat Sink Design given by the Composite Front

Design	W_{bf}	H_{bf}	L_f	*Nu*	*ΔP*	*η*	Algorithm
1	140.2529	15	110	28.09718	0.844516	1.125482	MO-Rao-2
1	139.8469	15	110	28.09537	0.844354	1.125481	MO-Rao-1
1	137.8754	15	110	28.08623	0.843578	1.125461	MO-Rao-2
2	143.4279	15	110	28.11045	0.845802	1.125442	MO-Rao-3
3	145	15.35531	110	28.12699	0.847355	1.125416	MO-Rao-3
4	145	15.269	110	28.12451	0.847137	1.125414	MO-Rao-2
5	135.8946	15	110	28.07646	0.842813	1.125409	MO-Rao-3
6	135.5271	15.25615	110	28.08262	0.843456	1.12537	MO-Rao-3
7	141.4154	16.23193	110	28.13541	0.848297	1.125336	MO-Rao-2
8	130.7372	15	110	28.04821	0.840891	1.125132	MO-Rao-2

Table 5: Comparison of Results Based on the Efficiency (*η)* for the Finned-Microchannel Heat Sink Design Problem

Optimization Method	W_{bf}	H_{bf}	L_f	*Nu*	*ΔP*	*η*	Remark
GA [22] for max. *η*	142.011	15.221	110	28.11147	0.845821	1.125475	The maximum value of *η* of of composite front is slightly better than that given by GA [22]
MO-Rao1/MO-Rao2/MO-Rao3 (Composite front) for max. *η*	**140.2529**	**15**	**110**	**28.09718**	**0.844516**	**1.125482**	
GA [22] for max. *Nu*	145	19.199	110	(28.2229*) 28.18156**	(0.8562*) 0.85634**	(1.1253*) **1.12364**	The corrected results match with the design variables of solution D5 given by [22].
Rao1/Rao2/Rao3 for max. *Nu*	145	19.20629	110	28.18156	0.85635	**1.12364**	
GA [22] for min. *ΔP*	15	15	50	22.72141	**0.656919**	0.989638	These results match and correspond to solution D6 of [22].
Rao1/Rao2/Rao3 for min. *ΔP*	15	15	50	22.72141	**0.656919**	0.989638	

*: Incorrect value given by [22]; **: Corrected value now.

ΔP, and *η* are corrected now by substituting the values of the design variables obtained by them in the Eqs. (7)-(9) and the corrected values are shown in Table **5**.

The maximum value of *η* of MO-Rao algorithms' composite front is slightly better than that given by multi-objective GA [22]. For the case of individual maximization of *Nu,* the corrected values of *Nu, ΔP,* and *η* for the obtained values of design variables by GA [22] are shown in the table, and these are same as those given by the Rao algorithms. These results correspond to the design variables of solution D5 of GA [22]. The GA [22] and the Rao algorithms have given the ideal minimum value of *ΔP,* however, as the *η* is less than 1.0 in that case, these results are not considered by Nekahi *et al*. [22] using GA, and the Rao algorithms, and hence are not given in Table **5**. The Rao algorithms used only a few function evaluations of 5000 (*i.e*., 50 x 100) compared to those used by multi-objective GA of Nekahi *et al*. [22]. This proves the usefulness of these algorithms for the optimal design of finned-microchannel heat sinks.

4.2. Multi-Objective Optimization of a Battery Thermal Management System (BTMS) with 3-Objectives

Dong *et al*. [23] used a multi-objective optimization approach that combined RSM and the NSGA-II to optimize a novel bionic lotus leaf (NBLL) channel and improve battery thermal management system (BTMS)'s performance. The design parameters included mass flow rate (*M*), channel spacing (*S*), width (*W*), and angle (*α*) are optimized in conjunction with heat production rates derived from lithium-ion battery (LIB) testing trials carried out under varied discharge rates. Figure shows the schematic of BTMS with NBLL channel.

A maximum temperature constraint *Tmax* of 30.895^{0}C was followed while optimizing the objective functions, which included the maximum temperature differential (Δ*Tmax*), pressure drop (Δ*P*), and heat transfer coefficient (*h*). The design points were chosen using Optimal Latin Hypercube Sampling (OLHS), then RSM created the mathematical models for the three objective functions as given below.

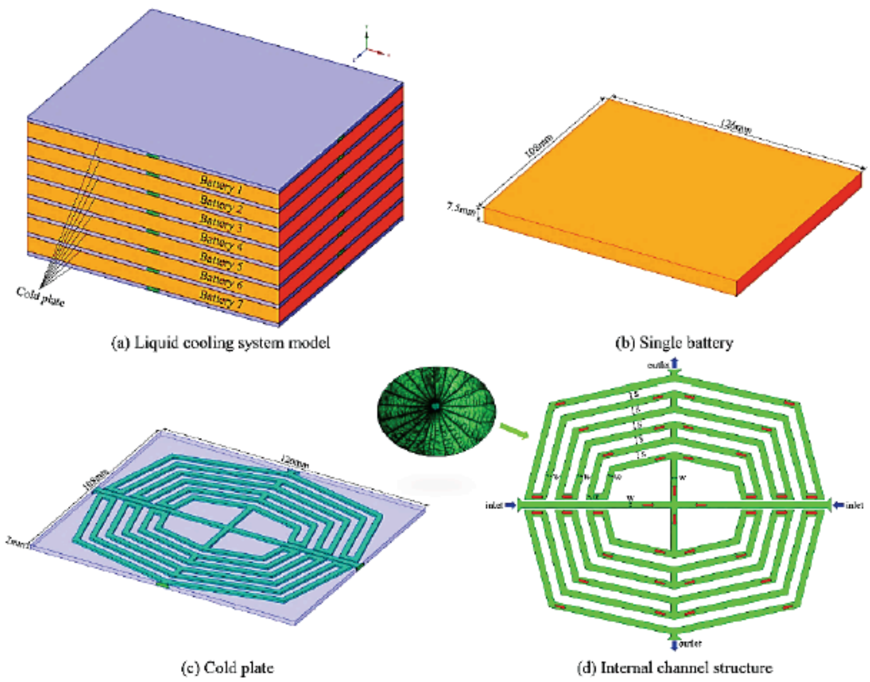

Figure 7: A BTMS configuration with NBLL channel used for energy-efficient heat removal and thermal safety control.

$$\text{Minimize } \Delta Tmax\ (M, S, W, \alpha) = 3.51 - 1.536M + 0.062S + 0.174W - 0.0180\alpha + 0.3677M^2 - 0.01724S^2 - 0.0278W^2 + 0.000069\alpha^2 + 0.00597MS + 0.0049MW - 0.00250M\alpha + 0.00365SW + 0.000716S\alpha - 0.00087W\alpha \quad (10)$$

$$\text{Maximize } h\ (M, S, W, \alpha) = 4940 - 4048M - 70S - 1367W - 14\alpha + 760M^2 + 76S^2 + 267W^2 - 0.11\alpha^2 - 71MS + 303MW + 46.8M\alpha - 65.0SW - 3.91S\alpha + 0.3W\alpha \quad (11)$$

$$\text{Minimize } \Delta P\ (M, S, W, \alpha) = 27083 + 5034M - 817S - 6319W - 460\alpha + 923M^2 + 69S^2 + 2017W^2 + 3.00\alpha^2 - 48MS - 2805MW + 26.4M\alpha + 31SW + 1.6S\alpha - 21.3W\alpha \quad (12)$$

Subject to the constraint of:

$$Tmax\ (M, S, W, \alpha) = 47.3 - 9.970M + 0.322S + 0.545W - 0.113\alpha + 2.343M^2 - 0.0772S^2 - 0.106W^2 + 0.00042\alpha^2 + 0.0514MS + 0.0932MW - 0.00697M\alpha + 0.0136SW + 0.00219S\alpha - 0.00327W\alpha - 30.895 \leq 0. \quad (13)$$

The bounds of the variables are: $0.8 \leq M \leq 2$ (g/s), $3 \leq S \leq 5$ (mm), $1.5 \leq W \leq 3$ (mm), $75 \leq \alpha \leq 90$ (^{o}). (14)

ΔTmax directly governs cell-to-cell temperature uniformity, which is critical for preventing accelerated aging, capacity fade, and thermal runaway. In lithium-ion battery packs, minimizing temperature non-uniformity improves cell balancing, mitigates degradation, and enhances cycle life. Enhanced heat transfer *h* allows higher power density operation, it reflects the effectiveness of heat removal and is directly related to electrochemical efficiency and allowable discharge rates. Pressure-drop (ΔP) determines auxiliary pumping power, which impacts overall system energy efficiency**.** Minimization of *ΔP* reduces parasitic energy losses, improving overall energy efficiency of the storage system. Thus, the optimized objectives are directly linked to battery safety, lifetime, and energy efficiency.

Now Rao algorithms are applied to optimize the individual objective functions of minimizing the maximum temperature differential (ΔTmax), minimizing the pressure drop (ΔP), and maximizing the heat transfer coefficient (h). Table 6 indicates that all three Rao algorithms yield identical optimal solutions for both thermal and hydraulic objectives of the BTMS with NBLL channel. The thermal optimization converges to a boundary-dominated configuration (M = 2 g/s, S = 3 mm, W = 3 mm, $\alpha = 90^0$), resulting in the minimum temperature non-uniformity (ΔTmax = 0.75683) and the maximum heat-transfer coefficient (h = 4765.3). Likewise, pressure-drop minimization leads to the same interior optimum for all algorithms (M = 1.74324 g/s, S = 5 mm, W = 3 mm, $\alpha = 87.95^0$), achieving the minimum ΔP = 2756.69. The temperature constraint given by Eq. (14) is also satisfied. The complete agreement among Rao-1, Rao-2, and Rao-3 demonstrates the robustness and reliability of the optimization results and confirms that the identified optima are not artifacts of algorithmic variation.

Now for doing multi-objective optimization, the MO-Rao algorithms are applied to this 3-objectives optimization problem with a constraint. Dong *et al*. [23]

Table 6: Results of Individual Optimization of the Objectives by the Rao Algorithms for the BTMS with NBLL Channel

Algorithm	*M* *(ΔTmax)*	*S* *(ΔTmax)*	*W* *(ΔTmax)*	*A* *(ΔTmax)*	*ΔTmax* (min)	*h* (max)	*M (ΔP)*	*S* *(ΔP)*	*W* *(ΔP)*	*α* *(ΔP)*	*ΔP* (min)
Rao-1	2	3	3	90	**0.75683**	**4765.3**	1.74324	5	3	87.95	**2756.69**
Rao-2	2	3	3	90	**0.75683**	**4765.3**	1.74324	5	3	87.95	**2756.69**
Rao-3	2	3	3	90	**0.75683**	**4765.3**	1.74324	5	3	87.95	**2756.69**

used a population size of 50 and iterations of 200 and used the NSGA-II algorithm for getting the non-dominated solutions. For a fair comparison, the same population size and iterations are considered in the present work. Table **7** shows the 119 non-dominated solutions of the composite front formed by merging the best non-dominated and no-repeating solutions of MO-Rao algorithms. All these solutions satisfy the *Tmax* constraint. Each solution in this table is Pareto-optimal across both algorithms — no solution dominates these in all objectives.

Figure **8** shows the 3D plot of the composite front.

The 3D Pareto front shows a clear trade-off among temperature uniformity, heat-transfer enhancement, and pressure loss. Lower *ΔTmax* is achieved with higher *h*, but at the expense of increased *ΔP*, reflecting the inherent thermo-hydraulic coupling. The front forms a smooth, narrow surface, indicating strong correlation among objectives and good convergence. The mid-region of the front represents a knee zone where meaningful reductions in *ΔTmax* and gains in *h* are obtained with only moderate increases in *ΔP*, making these solutions the most practically attractive.

Now to find out the best compromise solution out of these 119 non-dominated solutions using BHARAT method, the decision-maker has to compare the weights of importance of the objectives. Suppose the decision-maker thinks *h* as the most important objective and assigns rank 1 to it. Suppose ranks 2 and 3 are assigned to the objectives of *ΔP* and *ΔTmax* respectively. Then, using R-method of the BHARAT method, the weights assigned to *h, ΔP* and *ΔTmax* are 0.40205, 0.30137, and 0.24657 respectively. The data of the objectives are normalized using the BHARAT method (considering *h* as the beneficial and *ΔP* and *ΔTmax* as non-beneficial objectives). Table **8** shows the data and the normalized data of the objectives, the scores given by the BHARAT method, and the ranks of the solutions.

From Table **8**, it can be understood for the considered weights 0.40205, 0.30137, and 0.24657 (to *h, ΔP* and *ΔTmax* respectively), the best compromise solution suggested is solution no. 25 with *ΔTmax* = 0.796644 ^{0}C, *h* = 4473.299, and *ΔP* = 3464.147 Pa, with *Tmax* = 30.52518 ^{0}C. The corresponding values of the design variables were *M* =1.966604 g/s, *S* =3.221331 mm, *W* =2.982462 mm, and *α* =88.1063^{0}. It may be noted that if equal weights of importance are

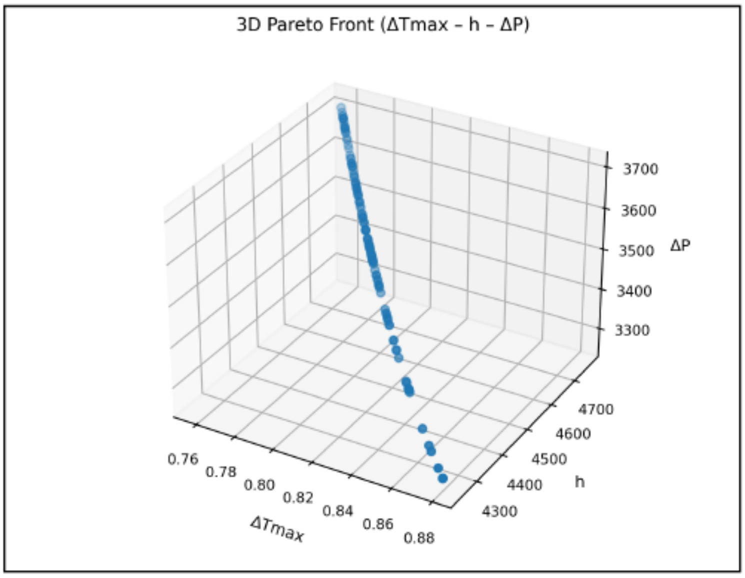

Figure 8: 3D Pareto front showing trade-offs among temperature uniformity (*ΔTmax*), heat-transfer enhancement (*h*), and pressure drop (*ΔP*) for energy-efficient BTMS operation.

Table 7: Non-Dominated Solutions of the Composite front Produced by MO-Rao Algorithms for the BTMS with NBLL Channel

S. No.	M	S	W	α	ΔTmax	h	ΔP	Tmax	Algorithm
1.	2	3	3	86.73571	0.79361	4604.275	3494.388	30.54264	MO-Rao-1
2.	2	3	3	86.2948	0.79869	4582.346	3471.112	30.568	MO-Rao-1
3.	2	3	3	86.87366	0.792025	4611.128	3501.911	30.53474	MO-Rao-1
4.	2	3	3	87.49645	0.784907	4642.012	3537.292	30.49928	MO-Rao-1
5.	2	3	3	89.89262	0.758017	4760.04	3695.12	30.36587	MO-Rao-1
6.	2	3	3	86.67316	0.794329	4601.167	3491.016	30.54623	MO-Rao-1
7.	2	3	3	88.63459	0.772035	4698.231	3607.962	30.43531	MO-Rao-1
8.	2	3	3	89.52434	0.762098	4741.982	3668.623	30.38606	MO-Rao-1
9.	2	3	3	82.7902	0.840027	4406.518	3327.581	30.77534	MO-Rao-1
10.	2	3	3	80.71779	0.865269	4301.273	3277.379	30.90281	MO-Rao-1
11.	2	3	3	88.71924	0.771085	4702.401	3613.529	30.43059	MO-Rao-1
12.	2	3	3	89.06048	0.767265	4719.195	3636.406	30.41165	MO-Rao-1
13.	2	3	3	85.80228	0.804397	4557.8	3446.491	30.59651	MO-Rao-1
14.	2	3	3	88.71663	0.771115	4702.272	3613.357	30.43074	MO-Rao-1
15.	2	3	3	89.75324	0.759559	4753.209	3684.996	30.37349	MO-Rao-1
16.	2	3	3	87.06242	0.789862	4620.497	3512.388	30.52396	MO-Rao-1
17.	2	3	3	87.77549	0.781734	4655.822	3553.899	30.48349	MO-Rao-1
18.	2	3	3	89.52699	0.762068	4742.112	3668.81	30.38591	MO-Rao-1
19.	2	3	3	85.42999	0.808733	4539.21	3428.845	30.6182	MO-Rao-1
20.	2	3	3	88.12997	0.77772	4673.34	3575.67	30.46354	MO-Rao-1
21.	2	3	3	89.75038	0.759591	4753.069	3684.79	30.37365	MO-Rao-1
22.	2	3	3	86.60171	0.795151	4597.615	3487.191	30.55033	MO-Rao-1
23.	2	3	3	88.16707	0.777301	4675.172	3577.992	30.46145	MO-Rao-1
24.	2	3	3	88.71613	0.77112	4702.248	3613.324	30.43077	MO-Rao-1
25.	2	3	3	89.43746	0.763063	4737.718	3662.49	30.39084	MO-Rao-1
26.	2	3	3	87.07727	0.789692	4621.234	3513.222	30.52311	MO-Rao-1
27.	2	3	3	86.27132	0.798961	4581.177	3469.906	30.56935	MO-Rao-1
28.	2	3	3	88.63418	0.77204	4698.211	3607.936	30.43533	MO-Rao-1
29.	2	3	3	89.67763	0.760397	4749.502	3679.553	30.37764	MO-Rao-1
30.	2	3	3	87.8075	0.781371	4657.405	3555.834	30.48169	MO-Rao-1
31.	2	3	3	86.42935	0.797137	4589.043	3478.092	30.56024	MO-Rao-1
32.	2	3	3	88.83674	0.769768	4708.186	3621.327	30.42406	MO-Rao-1
33.	2	3	3	88.53188	0.77319	4693.169	3601.266	30.44104	MO-Rao-1
34.	2	3	3	85.64372	0.806241	4549.886	3438.874	30.60574	MO-Rao-1
35.	2	3	3	85.65416	0.80612	4550.407	3439.371	30.60513	MO-Rao-1
36.	2	3	3	88.83112	0.769831	4707.91	3620.953	30.42437	MO-Rao-1
37.	2	3	3	86.38338	0.797667	4586.756	3475.695	30.56289	MO-Rao-1
38.	2	3	3	86.51104	0.796195	4593.107	3482.382	30.55554	MO-Rao-1
39.	2	3	3	88.41393	0.774517	4687.353	3593.653	30.44762	MO-Rao-1
40.	2	3	3	83.55843	0.830821	4445.292	3352.738	30.72901	MO-Rao-1
41.	2	3	3	88.2403	0.776474	4678.787	3582.6	30.45735	MO-Rao-1
42.	2	3	3	88.09968	0.778063	4671.844	3573.78	30.46524	MO-Rao-1
43.	2	3	3	87.68889	0.782718	4651.537	3548.695	30.48838	MO-Rao-1

44.	2	3	3	86.69367	0.794093	4602.187	3492.119	30.54505	MO-Rao-1
45.	2	3	3	88.23082	0.776581	4678.319	3582.001	30.45788	MO-Rao-1
46.	2	3	3	86.61925	0.794949	4598.487	3488.127	30.54932	MO-Rao-1
47.	2	3	3	84.079	0.824629	4471.492	3371.797	30.69789	MO-Rao-1
48.	2	3	3	89.48272	0.76256	4739.939	3665.679	30.38835	MO-Rao-1
49.	2	3	3	89.56114	0.761689	4743.788	3671.234	30.38403	MO-Rao-1
50.	2	3	3	88.01184	0.779056	4667.505	3568.331	30.47017	MO-Rao-1
51.	2	3	3	83.8075	0.827853	4457.835	3361.654	30.71409	MO-Rao-2
52.	2	3	3	84.71857	0.817072	4503.6	3397.439	30.65998	MO-Rao-2
53.	2	3	3	86.20867	0.799686	4578.057	3466.702	30.57297	MO-Rao-2
54.	2	3	3	85.76383	0.804844	4555.881	3444.63	30.59875	MO-Rao-2
55.	2	3	3	86.48356	0.796512	4591.74	3480.935	30.55712	MO-Rao-2
56.	2	3	3	89.60691	0.761181	4746.034	3674.493	30.38152	MO-Rao-2
57.	2	3	3	88.91228	0.768922	4711.904	3626.385	30.41986	MO-Rao-2
58.	2	3	3	89.75276	0.759564	4753.186	3684.962	30.37352	MO-Rao-2
59.	2	3	3	82.49554	0.84358	4391.612	3318.872	30.79325	MO-Rao-2
60.	2	3	3	87.26547	0.787541	4630.567	3523.898	30.51239	MO-Rao-2
61.	2	3	3	87.94783	0.779781	4664.342	3564.389	30.47378	MO-Rao-2
62.	2	3	3	85.60921	0.806643	4548.162	3437.236	30.60775	MO-Rao-2
63.	2	3	3	84.83364	0.815719	4509.367	3402.313	30.65319	MO-Rao-2
64.	2	3	3	82.84441	0.839375	4409.258	3329.24	30.77206	MO-Rao-2
65.	2	3	3	84.51805	0.819435	4493.543	3389.136	30.67183	MO-Rao-2
66.	2	3	3	87.77784	0.781708	4655.938	3554.041	30.48336	MO-Rao-2
67.	2	3	3	82.56541	0.842737	4395.148	3320.89	30.78899	MO-Rao-2
68.	2	3	3	84.8672	0.815324	4511.049	3403.75	30.65121	MO-Rao-2
69.	2	3	3	86.43441	0.797078	4589.295	3478.357	30.55995	MO-Rao-2
70.	2	3	3	88.12944	0.777726	4673.314	3575.637	30.46356	MO-Rao-2
71.	2	3	3	84.9687	0.814132	4516.133	3408.135	30.64524	MO-Rao-2
72.	2	3	3	84.52264	0.819381	4493.774	3389.323	30.67155	MO-Rao-2
73.	2	3	3	87.09936	0.789439	4622.33	3514.464	30.52185	MO-Rao-2
74.	2	3	3	86.31685	0.798435	4583.444	3472.249	30.56672	MO-Rao-2
75.	2	3	3	86.61552	0.794992	4598.302	3487.928	30.54954	MO-Rao-2
76.	2	3	3	87.29604	0.787192	4632.083	3525.652	30.51065	MO-Rao-2
77.	2	3	3	89.25761	0.765065	4728.885	3649.939	30.40075	MO-Rao-2
78.	2	3	3	86.33718	0.798201	4584.456	3473.299	30.56555	MO-Rao-2
79.	2	3	3	86.04555	0.801574	4569.93	3458.47	30.5824	MO-Rao-2
80.	2	3	3	85.90562	0.803197	4562.954	3451.536	30.59051	MO-Rao-2
81.	2	3	3	87.42115	0.785765	4638.282	3532.89	30.50355	MO-Rao-2
82.	2	3	3	79.90779	0.875296	4259.88	3264.762	30.95361	MO-Rao-2
83.	2	3	3	88.44332	0.774186	4688.803	3595.543	30.44598	MO-Rao-2
84.	2	3	3	88.03406	0.778805	4668.603	3569.705	30.46892	MO-Rao-2
85.	2	3	3	86.25212	0.799183	4580.221	3468.921	30.57046	MO-Rao-2
86.	2	3	3	89.74062	0.759699	4752.591	3684.086	30.37418	MO-Rao-2
87.	2	3	3	88.2837	0.775985	4680.929	3585.346	30.45491	MO-Rao-2
88.	2	3	3	86.81544	0.792694	4608.236	3498.722	30.53807	MO-Rao-2
89.	2	3	3	80.51411	0.867782	4290.878	3273.836	30.91553	MO-Rao-2
90.	2	3	3	84.65195	0.817857	4500.26	3394.654	30.66391	MO-Rao-2

91.	2	3	3	87.40115	0.785993	4637.291	3531.727	30.50468	MO-Rao-2
92.	2	3	3	89.80892	0.758942	4755.939	3689.027	30.37044	MO-Rao-2
93.	2	3	3	79.52631	0.88005	4240.336	3260.184	30.97773	MO-Rao-2
94.	2	3	3	86.86099	0.792171	4610.499	3501.215	30.53547	MO-Rao-2
95.	2	3	3	87.47805	0.785116	4641.101	3536.213	30.50032	MO-Rao-2
96.	2	3	3	89.42021	0.763255	4736.871	3661.278	30.39179	MO-Rao-2
97.	2	3	3	82.60873	0.842214	4397.34	3322.156	30.78636	MO-Rao-2
98.	2	3	3	87.96874	0.779544	4665.375	3565.674	30.4726	MO-Rao-2
99.	2	3	3	89.81398	0.758886	4756.187	3689.394	30.37017	MO-Rao-2
100.	2	3	3	84.73694	0.816856	4504.521	3398.212	30.65889	MO-Rao-2
101.	2	3	3	90	0.75683	4765.3	3703	30.36	MO-Rao-3
102.	2	3	3	87.26548	0.787541	4630.568	3523.899	30.51239	MO-Rao-3
103.	2	3	3	89.29785	0.764617	4730.862	3652.731	30.39853	MO-Rao-3
104.	2	3	3	81.30094	0.858106	4330.983	3288.9	30.86658	MO-Rao-3
105.	2	3	3	87.63412	0.78334	4648.827	3545.427	30.49148	MO-Rao-3
106.	2	3	3	85.90479	0.803207	4562.913	3451.495	30.59056	MO-Rao-3
107.	2	3	3	87.08466	0.789608	4621.601	3513.637	30.52269	MO-Rao-3
108.	2	3	3	89.54667	0.76185	4743.078	3670.206	30.38483	MO-Rao-3
109.	2	3	3	87.38707	0.786153	4636.594	3530.909	30.50548	MO-Rao-3
110.	2	3	3	83.78259	0.82815	4456.581	3360.746	30.71558	MO-Rao-3
111.	2	3	3	82.59801	0.842343	4396.798	3321.842	30.78701	MO-Rao-3
112.	2	3	3	87.08523	0.789601	4621.629	3513.669	30.52266	MO-Rao-3
113.	2	3	3	88.37112	0.774999	4685.242	3590.911	30.45002	MO-Rao-3
114.	2	3	3	88.64816	0.771883	4698.899	3608.852	30.43455	MO-Rao-3
115.	2	3	3	85.56845	0.807118	4546.127	3435.311	30.61012	MO-Rao-3
116.	2	3	3	84.08257	0.824586	4471.672	3371.933	30.69768	MO-Rao-3
117.	2	3	3	87.93209	0.779959	4663.564	3563.424	30.47466	MO-Rao-3
118.	2	3	3	89.14057	0.766371	4723.132	3641.876	30.40722	MO-Rao-3
119.	2	3	3	86.84559	0.792348	4609.734	3500.371	30.53635	MO-Rao-3

Table 8: Normalized Data of the Objectives, BHARAT Scores and the Ranks of the Non-Dominated Solutions for the Case of BTMS with NBLL Channels

S. No.	ΔTmax	h	ΔP	Tmax	ΔTmax (normalized)	h (normalized)	ΔP (normalized)	BHARAT score*	BHARAT score**
1.	0.79361	4604.275	3494.388	30.54264	0.953655	0.966209	0.937898	0.952587	0.906261
2.	0.79869	4582.346	3471.112	30.568	0.947589	0.961607	0.944187	0.951128	0.904811
3.	0.792025	4611.128	3501.911	30.53474	0.955563	0.967647	0.935883	0.953031	0.906703
4.	0.784907	4642.012	3537.292	30.49928	0.964229	0.974128	0.926522	0.95496	0.908624
5.	0.758017	4760.04	3695.12	30.36587	0.998435	0.998896	0.886948	0.961426	0.91509
6.	0.794329	4601.167	3491.016	30.54623	0.952792	0.965557	0.938804	0.952384	0.906059
7.	0.772035	4698.231	3607.962	30.43531	0.980305	0.985926	0.908374	0.958201	0.911862
8.	0.762098	4741.982	3668.623	30.38606	0.993088	0.995107	0.893354	0.960516	0.914178
9.	0.840027	4406.518	3327.581	30.77534	0.900959	0.92471	0.984913	0.936861	0.890752
10.	0.865269	4301.273	3277.379	30.90281	0.874676	0.902624	1	0.925766	0.879939
11.	0.771085	4702.401	3613.529	30.43059	0.981513	0.986801	0.906975	0.958429	0.91209
12.	0.767265	4719.195	3636.406	30.41165	0.9864	0.990325	0.901269	0.959331	0.912992
13.	0.804397	4557.8	3446.491	30.59651	0.940866	0.956456	0.950932	0.949418	0.903115
14.	0.771115	4702.272	3613.357	30.43074	0.981475	0.986774	0.907018	0.958422	0.912083

15.	0.759559	4753.209	3684.996	30.37349	0.996407	0.997463	0.889385	0.961085	0.914748
16.	0.789862	4620.497	3512.388	30.52396	0.95818	0.969613	0.933091	0.953628	0.907297
17.	0.781734	4655.822	3553.899	30.48349	0.968142	0.977026	0.922193	0.955787	0.909449
18.	0.762068	4742.112	3668.81	30.38591	0.993126	0.995134	0.893308	0.960523	0.914185
19.	0.808733	4539.21	3428.845	30.6182	0.935822	0.952555	0.955826	0.948068	0.901777
20.	0.77772	4673.34	3575.67	30.46354	0.973139	0.980702	0.916578	0.956806	0.910467
21.	0.759591	4753.069	3684.79	30.37365	0.996366	0.997433	0.889435	0.961078	0.914741
22.	0.795151	4597.615	3487.191	30.55033	0.951807	0.964811	0.939834	0.952151	0.905827
23.	0.777301	4675.172	3577.992	30.46145	0.973664	0.981087	0.915983	0.956911	0.910572
24.	0.77112	4702.248	3613.324	30.43077	0.981468	0.986768	0.907026	0.958421	0.912081
25.	0.763063	4737.718	3662.49	30.39084	0.991831	0.994212	0.89485	0.960298	0.91396
26.	0.789692	4621.234	3513.222	30.52311	0.958386	0.969768	0.93287	0.953675	0.907343
27.	0.798961	4581.177	3469.906	30.56935	0.947267	0.961362	0.944515	0.951048	0.904732
28.	0.77204	4698.211	3607.936	30.43533	0.980299	0.985921	0.908381	0.9582	0.911861
29.	0.760397	4749.502	3679.553	30.37764	0.995309	0.996685	0.8907	0.960898	0.914561
30.	0.781371	4657.405	3555.834	30.48169	0.968592	0.977358	0.921691	0.95588	0.909543
31.	0.797137	4589.043	3478.092	30.56024	0.949435	0.963012	0.942292	0.95158	0.90526
32.	0.769768	4708.186	3621.327	30.42406	0.983192	0.988015	0.905022	0.958743	0.912403
33.	0.77319	4693.169	3601.266	30.44104	0.978841	0.984863	0.910063	0.957923	0.911583
34.	0.806241	4549.886	3438.874	30.60574	0.938714	0.954795	0.953039	0.948849	0.902551
35.	0.80612	4550.407	3439.371	30.60513	0.938855	0.954905	0.952901	0.948887	0.902589
36.	0.769831	4707.91	3620.953	30.42437	0.983112	0.987957	0.905115	0.958728	0.912388
37.	0.797667	4586.756	3475.695	30.56289	0.948804	0.962532	0.942942	0.951426	0.905107
38.	0.796195	4593.107	3482.382	30.55554	0.950559	0.963865	0.941131	0.951852	0.90553
39.	0.774517	4687.353	3593.653	30.44762	0.977164	0.983643	0.911991	0.957599	0.91126
40.	0.830821	4445.292	3352.738	30.72901	0.910942	0.932846	0.977523	0.940437	0.894258
41.	0.776474	4678.787	3582.6	30.45735	0.974701	0.981845	0.914805	0.957117	0.910778
42.	0.778063	4671.844	3573.78	30.46524	0.972711	0.980388	0.917062	0.956721	0.910382
43.	0.782718	4651.537	3548.695	30.48838	0.966926	0.976127	0.923545	0.955533	0.909195
44.	0.794093	4602.187	3492.119	30.54505	0.953075	0.965771	0.938507	0.952451	0.906126
45.	0.776581	4678.319	3582.001	30.45788	0.974566	0.981747	0.914958	0.95709	0.910751
46.	0.794949	4598.487	3488.127	30.54932	0.952049	0.964994	0.939581	0.952208	0.905884
47.	0.824629	4471.492	3371.797	30.69789	0.917783	0.938344	0.971998	0.942708	0.89649
48.	0.76256	4739.939	3665.679	30.38835	0.992486	0.994678	0.894071	0.960412	0.914074
49.	0.761689	4743.788	3671.234	30.38403	0.99362	0.995486	0.892719	0.960608	0.914271
50.	0.779056	4667.505	3568.331	30.47017	0.971471	0.979478	0.918463	0.95647	0.910132
51.	0.827853	4457.835	3361.654	30.71409	0.914208	0.935478	0.974931	0.941539	0.89534
52.	0.817072	4503.6	3397.439	30.65998	0.926271	0.945082	0.964662	0.945338	0.899081
53.	0.799686	4578.057	3466.702	30.57297	0.946409	0.960707	0.945388	0.950835	0.90452
54.	0.804844	4555.881	3444.63	30.59875	0.940344	0.956053	0.951446	0.949281	0.902979
55.	0.796512	4591.74	3480.935	30.55712	0.950181	0.963578	0.941523	0.951761	0.905439
56.	0.761181	4746.034	3674.493	30.38152	0.994284	0.995957	0.891927	0.960723	0.914385
57.	0.768922	4711.904	3626.385	30.41986	0.984274	0.988795	0.903759	0.958943	0.912603
58.	0.759564	4753.186	3684.962	30.37352	0.9964	0.997458	0.889393	0.961084	0.914747
59.	0.84358	4391.612	3318.872	30.79325	0.897164	0.921581	0.987498	0.935415	0.889338
60.	0.787541	4630.567	3523.898	30.51239	0.961004	0.971726	0.930044	0.954258	0.907925
61.	0.779781	4664.342	3564.389	30.47378	0.970568	0.978814	0.919479	0.956287	0.909948
62.	0.806643	4548.162	3437.236	30.60775	0.938246	0.954434	0.953493	0.948724	0.902427
63.	0.815719	4509.367	3402.313	30.65319	0.927808	0.946292	0.96328	0.945793	0.89953
64.	0.839375	4409.258	3329.24	30.77206	0.901659	0.925285	0.984423	0.937122	0.891008
65.	0.819435	4493.543	3389.136	30.67183	0.9236	0.942972	0.967025	0.944532	0.898286

66.	0.781708	4655.938	3554.041	30.48336	0.968175	0.97705	0.922156	0.955794	0.909456
67.	0.842737	4395.148	3320.89	30.78899	0.898062	0.922323	0.986898	0.935761	0.889677
68.	0.815324	4511.049	3403.75	30.65121	0.928257	0.946645	0.962873	0.945925	0.89966
69.	0.797078	4589.295	3478.357	30.55995	0.949505	0.963065	0.942221	0.951597	0.905277
70.	0.777726	4673.314	3575.637	30.46356	0.973132	0.980697	0.916586	0.956805	0.910466
71.	0.814132	4516.133	3408.135	30.64524	0.929616	0.947712	0.961634	0.946321	0.900051
72.	0.819381	4493.774	3389.323	30.67155	0.923661	0.94302	0.966972	0.944551	0.898304
73.	0.789439	4622.33	3514.464	30.52185	0.958693	0.969998	0.93254	0.953744	0.907412
74.	0.798435	4583.444	3472.249	30.56672	0.947891	0.961837	0.943878	0.951202	0.904885
75.	0.794992	4598.302	3487.928	30.54954	0.951997	0.964955	0.939635	0.952196	0.905872
76.	0.787192	4632.083	3525.652	30.51065	0.96143	0.972044	0.929581	0.954352	0.908018
77.	0.765065	4728.885	3649.939	30.40075	0.989236	0.992358	0.897927	0.95984	0.913502
78.	0.798201	4584.456	3473.299	30.56555	0.94817	0.96205	0.943593	0.951271	0.904953
79.	0.801574	4569.93	3458.47	30.5824	0.94418	0.959002	0.947639	0.950273	0.903963
80.	0.803197	4562.954	3451.536	30.59051	0.942272	0.957538	0.949542	0.949784	0.903478
81.	0.785765	4638.282	3532.89	30.50355	0.963177	0.973345	0.927677	0.954733	0.908398
82.	0.875296	4259.88	3264.762	30.95361	0.864656	0.893938	1.003865	0.920819	0.87514
83.	0.774186	4688.803	3595.543	30.44598	0.977582	0.983947	0.911512	0.95768	0.911341
84.	0.778805	4668.603	3569.705	30.46892	0.971784	0.979708	0.918109	0.956534	0.910195
85.	0.799183	4580.221	3468.921	30.57046	0.947004	0.961161	0.944784	0.950983	0.904667
86.	0.759699	4752.591	3684.086	30.37418	0.996224	0.997333	0.889604	0.961054	0.914717
87.	0.775985	4680.929	3585.346	30.45491	0.975316	0.982295	0.914104	0.957238	0.910899
88.	0.792694	4608.236	3498.722	30.53807	0.954757	0.96704	0.936736	0.952844	0.906517
89.	0.867782	4290.878	3273.836	30.91553	0.872143	0.900442	1.001082	0.924556	0.878763
90.	0.817857	4500.26	3394.654	30.66391	0.925382	0.944381	0.965453	0.945072	0.898819
91.	0.785993	4637.291	3531.727	30.50468	0.962897	0.973137	0.927982	0.954672	0.908337
92.	0.758942	4755.939	3689.027	30.37044	0.997217	0.998036	0.888413	0.961222	0.914885
93.	0.88005	4240.336	3260.184	30.97773	0.859986	0.889836	1.005274	0.918365	0.872765
94.	0.792171	4610.499	3501.215	30.53547	0.955387	0.967515	0.936069	0.95299	0.906662
95.	0.785116	4641.101	3536.213	30.50032	0.963972	0.973937	0.926805	0.954905	0.908569
96.	0.763255	4736.871	3661.278	30.39179	0.991582	0.994034	0.895146	0.960254	0.913916
97.	0.842214	4397.34	3322.156	30.78636	0.89862	0.922784	0.986522	0.935975	0.889886
98.	0.779544	4665.375	3565.674	30.4726	0.970863	0.979031	0.919147	0.956347	0.910008
99.	0.758886	4756.187	3689.394	30.37017	0.99729	0.998088	0.888325	0.961234	0.914897
100.	0.816856	4504.521	3398.212	30.65889	0.926516	0.945275	0.964442	0.945411	0.899153
101.	**0.75683**	**4765.3**	**3703**	**30.36**	**1**	**1**	**0.885061**	**0.961687**	**0.915351**
102.	0.787541	4630.568	3523.899	30.51239	0.961004	0.971726	0.930044	0.954258	0.907925
103.	0.764617	4730.862	3652.731	30.39853	0.989816	0.992773	0.897241	0.959943	0.913605
104.	0.858106	4330.983	3288.9	30.86658	0.881977	0.908858	0.996497	0.929111	0.88319
105.	0.78334	4648.827	3545.427	30.49148	0.966157	0.975558	0.924396	0.955371	0.909034
106.	0.803207	4562.913	3451.495	30.59056	0.942261	0.957529	0.949554	0.949781	0.903475
107.	0.789608	4621.601	3513.637	30.52269	0.958489	0.969845	0.93276	0.953698	0.907366
108.	0.76185	4743.078	3670.206	30.38483	0.993411	0.995337	0.892969	0.960572	0.914234
109.	0.786153	4636.594	3530.909	30.50548	0.962701	0.972991	0.928197	0.95463	0.908295
110.	0.82815	4456.581	3360.746	30.71558	0.91388	0.935215	0.975194	0.94143	0.895233
111.	0.842343	4396.798	3321.842	30.78701	0.898482	0.92267	0.986615	0.935922	0.889834
112.	0.789601	4621.629	3513.669	30.52266	0.958497	0.969851	0.932751	0.9537	0.907368
113.	0.774999	4685.242	3590.911	30.45002	0.976556	0.9832	0.912687	0.957481	0.911141
114.	0.771883	4698.899	3608.852	30.43455	0.980498	0.986066	0.90815	0.958238	0.911898
115.	0.807118	4546.127	3435.311	30.61012	0.937694	0.954006	0.954027	0.948576	0.902281
116.	0.824586	4471.672	3371.933	30.69768	0.91783	0.938382	0.971959	0.942723	0.896505

117.	0.779959	4663.564	3563.424	30.47466	0.970346	0.978651	0.919728	0.956241	0.909903
118.	0.766371	4723.132	3641.876	30.40722	0.987551	0.991151	0.899915	0.959539	0.9132
119.	0.792348	4609.734	3500.371	30.53635	0.955174	0.967354	0.936295	0.952941	0.906613

The solution indicated in bold is the best compromise solution; *: For equal weights of objectives; **: For different weights of objectives.

assigned to the three objectives (*i.e.*, 1/3 to each objective), by sheer coincidence (because of its data) the best compromise solution will be solution no. 25 itself with $\Delta Tmax$ = 0.796644 ^{0}C, h = 4473.299, and ΔP = 3464.147 Pa, with $Tmax$ = 30.52518 ^{0}C.

Dong *et al*. [23] used NSGA-II algorithm but did not produce the non-dominated Pareto solutions table but reported that the optimal solution was: $\Delta Tmax$ =0.812^{0}C, h =4312.568, and ΔP =3234.564 Pa, respectively with $Tmax$ = 30.614^{0}C. The corresponding values of the design variables were M =1.978 g/s, S =4.955 mm, W =2.867 mm, and α =87.008^0. However, a close look at the best compromise value given by the composite front and a comparison with the optimal solution given by NSGA-II of Dong *et al*. [23] reveals that the best compromise solution dominates the solution of NSGA-II [23] in the case of two objectives $Tmax$ and h out of the three objectives. Thus, the composite front obtained from the MO-Rao algorithms has provided a much better solution thatn the NSGA-II used by Dong *et al*. [23]. It may be noted from Table **8** that some of the non-dominated solutions given by the composite front are clearly better than the optimal solution suggested by Dong *et al*. [23]. Thus, the solutions given by the proposed MO-Rao algorithms are highly competitive. Table **9** shows the comparison of optimization results.

Compared with the NSGA-II-based optimized configuration reported in the literature, the proposed framework achieves superior thermal uniformity and heat transfer, which translate into improved battery safety and usable power density, albeit at a moderate increase in pressure drop.

The identified optimal BTMS configurations are scalable to larger battery packs, where improved thermal uniformity reduces the need for conservative derating and enhances system-level efficiency and reliability in smart energy storage installations.

For fair comparison, identical RSM surrogate models and constraints are used in the present work, embedding the baseline and conventional designs. In the case of microchannel optimization, the microchannel without fins (η = 1) is taken as reference. The comparisons are made with the results of GA solutions from Nekahi *et al*. [22]. In the case of BTMS, the comparisons are made with the NSGA-II solutions from Dong et al. [23]. The framework complements smart energy systems at the design and planning level rather than replacing real-time controllers.

The next section presents conclusions of the present work.

5. CONCLUSIONS

Thermal engineering systems are integral to a wide array of industrial, residential, and transportation applications. These systems—such as heat exchangers, refrigeration cycles, boilers, condensers, thermal energy storage systems, and power plants—deal with the generation, transfer, storage, and dissipation of heat. Given their energy-intensive nature, optimization plays a pivotal role in enhancing their performance, reducing operational costs, and ensuring sustainability.

The present work has developed and described the working of two simple and powerful optimization algorithms named Rao algorithms. The results of the successful application of the proposed algorithms on the single objective optimization of a finned-microchannel heat sink are presented. The convergence is attained in fewer function evaluations, confirming computational efficiency.

Table 9: Multi-Objective Optimization Results for the BTMS with NBLL Channels

Optimization method	*M*	*S*	*W*	*α*	Δ*Tmax*	*h*	Δ*P*	*Tmax*	Remarks
NSGA-II [23]	1.978	4.955	2.867	87.008	0.812	4312.568	**3234.564**	30.614	The solution given by the composite front and the BHARAT method is better than NSGA [23].
Composite front+BHARAT method ($w_{\Delta Tmax}$ = 0.24657, w_h=0.40205, and $w_{\Delta P}$ = 0.30137)	2	3	3	90	**0.75683**	**4765.3**	3703	**30.36**	
Composite front+BHARAT method ($w_{\Delta Tmax}$ = w_h = $w_{\Delta P}$ = 1/3	2	3	3	90	**0.75683**	**4765.3**	3703	**30.36**	

The multi-objective versions of the Rao algorithms are named MO-Rao. These are successfully applied for multi-objective optimization of a finned-microchannel heat sink with 2 objectives, and a battery thermal management system with 3 objectives. The results are compared with the results of other algorithms such as multi-objective GA and NSGA-II. The performance of the MO-Rao algorithms is found competitive and good. The BHARAT method, in conjunction with the R-method, is found to find the best compromise non-dominated solution from amongst a large number of Pareto-front solutions.

Because the optimization is performed on fast-evaluating RSM models, the obtained Pareto fronts can be readily integrated into intelligent energy management frameworks, where real-time operating conditions can be mapped to pre-optimized design or control configurations.

Although optimization is performed at component level, the resulting gains propagate directly to system-level energy efficiency, safety, and sustainability. Improved thermal uniformity, reduced parasitic losses, and enhanced heat-transfer performance scale directly to battery packs, modular energy storage systems, and larger thermal systems. The proposed optimization framework leads to energy-efficient thermal management, enhanced safety and lifetime of energy storage systems, and sustainable design of thermal engineering systems, in addition to the methodological contributions.

The promising results achieved by the Rao algorithms and their multi-objective versions pave the way for extensive future research, including: Application to a broader range of real-world, high-dimensional, and computationally expensive problems, both constrained and unconstrained. The present work can be applied to the optimization problems of different thermal systems such as refrigeration and HVAC systems, thermal power plants (Rankine, Brayton, combined cycles), solar thermal collectors and storage, combustion chambers, electronic cooling devices, thermal storage systems (latent and sensible heat storage), thermally driven desalination units, etc. The applications in high-dimensional data and machine learning will further explore the scalability, adaptability, and domain versatility of the proposed algorithms. The Pareto-optimal solutions can be embedded in digital twins or supervisory controllers to enable adaptive or data-driven decision-making under varying operating conditions.

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