Motion Prediction and Risk Assessment for The Decision Making of Autonomous Vehicles

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Abstract: The last few years, the automotive industry sees the Autonomous Vehicles (AV) as a great opportunity to increase comfort and road safety. One of the most challenging tasks is to detect dangerous situations and react to avoid or, at least, mitigate accidents. This requires a prediction of the evolution of the traffic surrounding the vehicle. This paper is a survey of the methods used in Automotive engineering for predicting future trajectories and collision risk assessment. models of vehicles are classified from the simplest to the more complexes. These technologies aim to improve road safety by estimating the level of dangerousness of a situation to make decision to avoid collision or mitigate its consequences.

Keywords: Motion prediction, Collision risk, Autonomous vehicle.

1. INTRODUCTION

One of the fields that have seen the greatest evolution in recent years in the automotive industry is certainly Advanced Driver-Assistance Systems (ADAS), all these technologies aims to improve road safety and comfort of drivers and passengers.

To achieve such complexes task, 4 components are necessary:

- **Perception:** is a module that use multiples sensors (Camera, Radar, LiDAR, ultrasonic) to make a 3D map of the surrounding environment with road markings, traffic sign, cars, and also pedestrian and all obstacles.
- Decision Making: this module uses the 3D representation of the surrounding environment provided by the perception module, and also a preloaded map of the roads to decide which behavior to take. Like turn right or left, make an avoidance maneuver or change lane.
- **Motion Planning:** the purpose of this module is to draw a path that the vehicle must follow, and considers information of "Decision Making" and "Perception" modules to achieve that, according also to positions and velocity of objects in the road.

Control: this module controls the physicals parameters of the vehicle's actuators (engine speed, steering angle, break...) to make actual path matches the planned path.

To achieve such complexes task, decision-making modules needs models that can predict future trajectories of the ego- vehicle and all the surrounding vehicles, and an estimate of the risk's level of the situation. Risk can be defined by the likelihood and severity of damage that may occur to the vehicle.

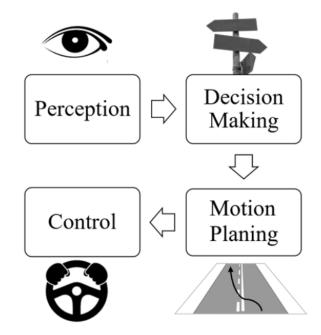


Figure 1: The 4 basic modules necessary to autonomous driving.

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This article summarizes the most used vehicle's models organized in two categories:

- Physics-based Motion Models are low-level models, and consider vehicle as an object ruled only by laws of physics.
- Maneuver-based Motion Models consider vehicle's motion as one or a series of maneuvers. In addition, some models also consider the interactions between vehicles. Also presented, most used methods for estimating degree of risk, even if the notion of risk is not always clearly defined, indeed, it depends on the context. Here, it qualifies the probability that a collision can occur and its dangerousness. This survey aims to resume the work done by S. Lefère, D. Vasquez, and C. Laugier [23] and update it.

2. PHYSICS-BASED VEHICLEMODELS

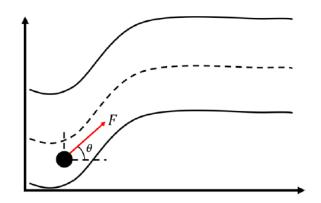
This type of model represents vehicles as dynamic entities ruled by physics laws, motion can be predicted by using dynamic or Kinematic models. Current states (e.g speed, steering angle), control inputs (e.g steering wheel, accel- eration) and vehicle proprieties (e.g weight) are considered to compute future states. Several works have been done on these kinds of models, and it is the most commonly used for short term trajectory prediction. The complexity of the models depends on how close to the reality it is. In this section we present two types of physics-based motion models, **dynamic models** and **kinematic models**. Dynamic models are classified by complexity.

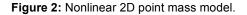
2.1. Dynamic Models

Dynamic models describe vehicle's motion according to Newton's second law of motion by considering different forces that affect it, such as the engine torque, breaks, lateral tire forces. The most complexes ones can involve many internal parameters or even combustion engine's dynamic.

2.1.1. 2D Point Mass Model

This model considers thevehicle as a single point in plane (center of gravity) with a mass [1], this model can be linear with a decoupled velocity and forces, as shown in Figure **2**.





The equations (1) represent the mathematical representation of the model.

$$\dot{x} = v_x
\dot{y} = v_y
\dot{v}_x = F_x/m
\dot{v}_y = F_y/m$$
(1)

With *x* and *y*, the longitudinal and lateral position, v_x and v_y the longitudinal and lateral speed, and, F_x and F_y the longitudinal and lateral Forces.

This model is relevant for motion on highways or urban arterial roads, but it become less precise when longitudinal dynamic is no longer the dominant one (non-straight road). Another representation can also be made by coupling the two forces. In this case, the model become nonlinear (Figure **3**).

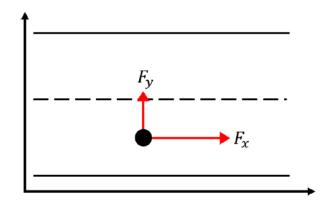


Figure 3: Linear 2D point mass model.

And the equations become:

$$\begin{aligned} \dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{v} &= F/m \\ \dot{\theta} &= \omega \end{aligned} \tag{2}$$

where v and θ are the speed and the heading of the point, *m* and ω the masse and the yaw rate.

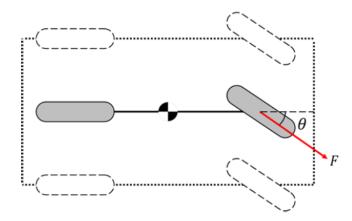


Figure 4: Dynamic bicycle model.

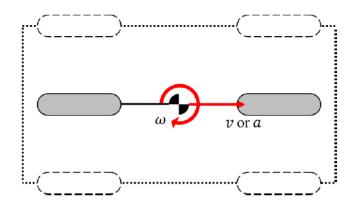


Figure 5: Constant Turn Rate and Velocity (CTRV) and Constant Turn Rate and Acceleration (CTRA) models.

2.1.2. Dynamic Bicycle Model

The dynamic bicycle models merge left and right wheels to obtain a layout where the two resultants wheels are disposed at the center front and rear, and equidistant from the gravity center of the vehicle [2]. Those models are more realistic then the point mass models, indeed, it reflects the side-slip angle. They offer a good compromise between complexity and realism.

2.1.3. Quadricycle Planar Model

With this model, the vehicle is modeled with four wheels, and the dynamic for each one is considered independently [3]. Quadricycle planar model can be too complex to compute for path planning and motion prediction, knowing that models like bicycle model has enough accuracy for these kinds of application. However, it can be relevant for control applications such as the Electronic Stability Control System (ESC).

2.1.4. 3D Models

There are more complex models that con-sider pitching and rolling [4] by integrating the suspension and the distribution of the masses, it is used to study chassis motion in order to improve the comfort of passengers or minimizing rolling in case of sport cars. Other model can also integrate the dynamic of the engine (and transmission) for usages like cruise control. Or even tires [5] (*e.g.* for traction control).

2.2. Kinematic Models

Kinematic models are a physical representation of the vehicle that considers only configuration variables and their velocities, and ignore different forces that can affect it motion.

Friction forces are neglected, and the entire car is considered as one body (dynamic of wheels is not considered independently) they are more popular than dynamic models for trajectory prediction because they are simpler, faster to compute in real-time, and, most of the time, enough accurate. In addition, the internal parameters of the vehicle needed by dynamic models (e.g. torque) are not observable by exteroceptive sensor. There by, it is meaningful to use these models for cars surrounding the vehicle of interest. Schubert *et al.* [6] have done a comparison of kinematics models for vehicles from the less to the more complex.

The simplest is the Constant Velocity (CV) and Constant Acceleration (CA) models (Figure 6). Both considers straight motion in longitudinal and later axes for vehicles. The Constant Turn Rate and Velocity (CTRV) and Constant Turn Rate and Acceleration (CTRA) models (Figure 7) take into consideration rotation around the Z-axis with the yaw angle and yaw rate.

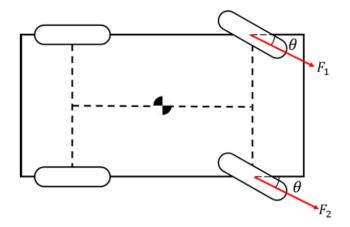


Figure 6: Quadricycle planar model.

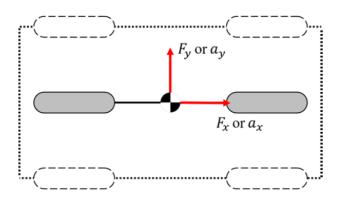


Figure 7: Constant Velocity (CV) and Constant Acceleration (CA) models.

The complexity remains low as the velocity and yaw rate are decoupled. By considering the steering angle instead of the yaw rate in the state variables, we obtain a "bicycle" representation, which takes into account the correlation between the velocity and they a wrate. From this representation, the Constant Steering Angle and Velocity (CSAV) and the

Constant Steering Angle and Acceleration (CSAA) can be derived.

3. TRAJECTORY PREDICTION OF SURROUN-DING VEHICLES

In the precedent section we presented different manner to model vehicle motion, however it is useless without an evolution model for trajectory prediction. The ones presented below differs in the handling of uncertainties.

3.1. Single Trajectory Simulation

An evident method to predict future trajectories is to simply ignore uncertainties and apply an evolution model to the current state of a vehicle supposing that the current state is perfectly known, and the evolution model and its parameters are sufficiently accurate. This method can be used with dynamic models or kinematic models [7-9].

The advantage of this method is calculation time, which makes it suitable for applications with strong real-time constraints. However, this method does not consider the uncertainties on the current states, neither the evolution of model and as a result the predicted trajectories are not reliable for long term prediction.

3.2. Linear Probabilistic Simulation

One assumption can be made about noise and uncertainty on measurements with modeling them by a

normal distribution [10, 11]. Thus, a Kalman filter can be used to predict the trajectory of the vehicle.

Kalman filtering is a technique for recursively estimating a vehicle's states from noisy and relatively uncertain measurements. It is a special case of Bayesian filtering where the evolution model is linear, and noise and uncertainty are considered using a normal distribution.

The advantage of these techniques is that they represent uncertainty on the predicted trajectory. However, it is not always possible to make assumption on the gaussianity of noises. A mixture of Gaussian can be used with Switching Kalman Filter (SKF) [12, 13], they rely on a bank of Kalman Filters to represent the possible evolution models of a vehicle ands witch between them.

3.3. Nonlinear Simulation

The weakness of Kalman filter is that it can only be used with linear models, however, Ex- tended Kalman Filter (EKF) offers a solution by linearizing the nonlinear model around the actual estimation at each iteration. It can be improved by using the unscented trans- formation (UT) and it is known as the Unscented Kalman Filter (UKF).

Sometimes, it is not possible to presume on the linearity of the models or the gaussianity of the uncertainties, in this case Monte Carlo method can be used.

The principle of this method is to randomly simple the evolution of the model from the input variables to generate possible future trajectories. Weight can be applied to penalize ones that do not respect the constraints of the road layout, or, the trajectories that exceed physical limitation of the vehicle [14, 15].

Limitation

Since these models rely on physical properties, Estimation depend on previous observations. They cannot provide an accurate prediction beyond one or two seconds. indeed, if the dynamic of the vehicle change estimation can differ (*e.g.* tun at an intersection).

To get a better estimation of future trajectories, model that can understand behavior of the vehicle is needed, this can be achieved for example by maneuvers classification.

4. MANEUVER-BASED MOTIONMODELS

The idea of these models is to represent vehicles as independent entities that can perform a finite number of maneuvers [16] (or behavior) depending on the road layout, and independently from other vehicles. The path of the vehicles can be described as a series of maneuvers, consequently, this type of modeling can perform a better prediction in long term compared to physics-based models. One method is to consider the possible trajectories into a finite set cluster, each one corresponds to specific behavior (Figure **8**).

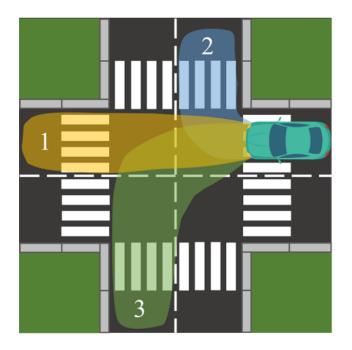


Figure 8: Example of clusters for a maneuver-based motion model (1. Straight, 2. Turn right, 3. Turn left).

These clusters can be learned from data previously observed, with variables like the physical state of the vehicle (*e.g.* position, speed, yaw rate, turn signal) and road information (topology of the road, speed limits, Traffic light).

In theory, any clustering algorithm can be used to achieve this task, like Multi-Layer Perceptron (MLP) [17], Logistic Regression, Relevance Vector Machines (RVM) [18], Support Vector Machines (SVM) [19].

Recent works tend to use Gaussian Processes to represent motion patterns [24, 29, 30], it can be seen as a generalization of Gaussian probability distributions. the trajectories in the learning data-set are sample functions from a Gaussian Process. Thus, the learning consists in fitting a Gaussian distribution over these functions. The main advantages are that GPs are robust to noise and their ability to represent the variation in the trajectories in a probabilistic manner.

Another popular method to recognize behaviors is to represent them as a series of consecutive events in a Hidden Markov Model (HMM) [16, 20], and the transition probabilities between events can be learned from data.

The limitation of this kind of modeling is that in practice, the assumption that the vehicles moves independently from each other on the road and do not interact is false. In fact, vehicles continually interact with each other, for example in intersections to decide which car should pass first, or for lane changing.

To over take this issue, some models considers that vehicles can influence each other's trajectory or behavior. Assuming that cars can interact between them provide a more realistic modeling, and a more "human" approach to predict their motion. Two methods are presented below, models based on trajectory prototypes, and based on Dynamic Bayesian Networks.

Models based on trajectory prototypes: This method is similar to maneuver-based motion models, with the difference that it take into account the mutual influence during the matching phase, the algorithm will penalize the pairs of trajectories causing a collision, assuming that drivers will take a safer path the majority of the time [21, 22].

Models based on Dynamic Bayesian Networks: most interaction-aware motion models are based on Dynamic Bayesian Network (DBN) [23]. The interaction between pair of cars can be modeled by Coupled Hidden Markov Models (CHMM). However, the number of inter-dependencies can grow quickly with the number of possible entities.

A solution to simplify the model is to consider asymmetric dependencies. This model has been used in number of works, especially for lane changing and overtaking maneuvers [24].

5. COLLISION RISKASSESSMENT

This section presents various methods to estimate the risk of a situation in real time, for this task it is, most of the time, necessary to estimate future trajectories of the cars surrounding the ego-vehicle using one the models presented in the precedent section or a combination of them. The notion of risk is not clearly defined, many definitions can be found depending on the context. In Intelligent Transportation Systems (ITS), it is generally qualified by the dangerousness of a situation for the passengers, which can cause physical injuries.

5.1. Binary Collision Prediction

The principle is simple, future trajectory is computed by solving differential equations of the vehicle model for the ego-vehicle and the other vehicle in the scene. It is supposed that trajectories can be computed with enough accuracy (good model and precise measurements). One manner to detect collision is by defining a threshold on the distance between two points (from two trajectories at the same time step) [23].

Another method is the "unavoidable collisions", the algorithm will assign the value 0 or 1 depending on whether there exists a collision-free maneuver that the driver can perform. It is done by computing escape maneuvers and check which are feasible (with "feasible" meaning that the steering, braking or accelerating does not exceed the physical limitations of the vehicle).

5.2. Collision Risk Based on Indicators

One of the most popular methods for estimating collision risk is to calculating indicators:

- "Time-To-X": is an indicator of time where X correspond to an event in relation to the collision [23], like the time remaining to the collision it-self (Time-To-Collision), and this time can be compared to the expected time needed to stop the vehicle or can be used as an indication of which action should be taken. It also can be used in human driving situation to warn the driver, in this case, the driver reaction time should be added to the time to stop the vehicle.
- Another time indicator that are closely related to the TTC is the "Time-To-React", which correspond to the remaining time to act before the collision becomes inevitable. in this case, the reaction time of the driver must be considered.
- Predicted Object Minimum Distance (PMD): it is de- fined by the minimum distance between the vehicle and a potential obstacle (static or dynamic), if PMD=0 the collision is forecasted, and an emergency maneuver must be realized.

By opposition, higher is this value, less dangerous is the considered obstacle.

5.3. Probabilistic Collision Prediction

When the future motion of a vehicle is represented by a probability distribution on sample trajectories, probabilistic estimation of risks can be used by detecting collision between all possible pair of trajectories, more of collision is detected, higher is the risk [25]. This approach provides a lot of flexibility in the handling of uncertainties, and can be adapted for any model mentioned, for a Maneuver-based motion model for example, the estimation can be done with both the maneuvers and their executions or presume that the maneuvers are known and sum on the possible executions only.

Geometrical Method

This method uses Kalman filtering to compute the future trajectory of the ego-vehicle and other vehicles in the scene, the result is ellipses that represent the possible future positions of the vehicles. And more the time horizon of the prevision is long, more the ellipses are larges (Figure **9**).

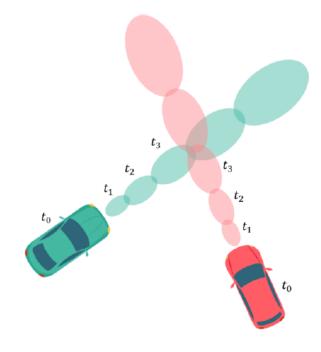


Figure 9: Risk assessment with Kalman filter's ellipses.

The estimation is based on how many ellipses of the ego vehicle cross ellipses of other vehicles (at the same time horizon) [26]. And more ellipses are implicated in the collision, more dangerous it is.

5.4. Collision Risk Assessment with Path Planners

Motion planners are algorithms that decide the path the vehicle must follow considering various constraints like time, traffic conditions, roads conditions, and different obstacle that can be encountered on the road. Some algorithms can adapt the path in real time if an unexpected event occur and give an alternate trajectory to follow. Two of them are presented below, potential field and elastic bands method.

- Potential field: This algorithm considers the vehicle as moving particle in a plan subject to a potential field which is the sum of various fields generated by different elements in the road (lane, road, car...) [27]. obstacles, other vehicles and road boarder have repulsive field, while the road has attractive field in the direction that must be followed. The advantage is that many purposes can be achieved with this single algorithm, like choosing the right lane in highway (where the traffic is more fluid), obstacle avoidance, lane keeping. And the risk of a maneuver can be defined as the average value of the field for the followed path.
- "Elastic bands": is inspired by mobile robotics and is used to compute an emergency path to avoid a collision with an obstacle. The trajectory is represented by springs connected to each other by nodes. if an object is detected, the path will be deformed around it, respecting physical constraints of the springs [28]. This method brings a smooth trajectory to avoid obstacle and allow to return to the original path.
- In addition to that, springs can be added by connecting them to the road edges and the already existing nodes to prevent an off-road path. Risk assessment can be achieved by estimate the force applied to the springs, and this value can be used as an indicator. Higher it is, more the situation is dangerous.

6. CONCLUSION

This Paper is a survey that resume the main models used ADAS for motion prediction, they were organized in three main categories.

Physics-based motion models represents vehicle by the mathematical equation of motion. It includes various variable such as speed, acceleration or yawrate. Ego-vehicle is mostly described by dynamic models since more information are available, like engine torque or breaks forces. For thesur- rounding vehicle, Kinematic models are preferred precisely because this information is not available. The drawback of these models is that it is not reliable after 2 seconds because the evolution of some variables is unpredictable.

Maneuver-based motion models represent vehicles as in- dependent entities and represent future motion like one or a series of behavior like "turn left", "go straight" or "change lane" This can be achieved with clustering algorithms and machine learning. The weakness of this method is that the assumption that the vehicles moves independently from each other on the road and do not interact is false. Interaction- aware motion models are similar to Maneuver-based motion models, except that it takes it into consideration.

The second part is dedicated to risk estimation, different methods and metrics were presented from the simplest to the more complexes. The choice of the method depends on which vehicles model is used and also on the situation (road intersections, highways...).

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