Virtual Sensor Design for Linear and Nonlinear Dynamic Systems

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Abstract: The problem of virtual sensors design in linear and nonlinear systems is studied. The problems is solved in three steps: at the first step, the linear model invariant with respect to the disturbance is designed; at the second step, a possibility to take into account the nonlinear term and to estimate the given variable is checked; finally, stability of the observer is achieved. The relations allowing to design virtual sensor of minimal dimension estimating prescribed component of the state vector of the system are obtained. The theoretical results are illustrated by practical example.

Keywords: Linear systems, Nonlinear systems, Virtual sensors, Reduced order models, Observers.

1. INTRODUCTION

Different sensors are an integral part of modern complex technical systems, in particular, they are used for measuring the state vector components to solve the problems of control and fault diagnosis. Clearly, the more components are measured, the simpler solution can be obtained. The use of additional physical sensors may result in extra expenses and not always can be realized in practice. Besides, such sensors are of not high reliability. In this case, virtual sensors are of the most interest. Besides, virtual sensors can be used to replace the faulty sensors and in sensorless control.

There are many papers considering different problems in design and application of virtual sensors [1-13, 15, 18-20]. Most of these papers consider different practical applications of virtual sensors: for health monitoring of automotive engine [1], for active reduction of noise in active control systems [3], for hiding the fault from the controller point of view [6], in walking legged robots [7], for failure diagnosis in aircraft [8], in the process of fault detection in industrial motor [9], for fault detection, isolation, and data recovery in a bicomponent mixing machine [10], in the sensor-cloud platform [15], for a tunnel furnace [20]. A new architectural paradigm for remotely deployed sensors whereby a sensor's software is separated from the hardware are presented in [18]. In [2, 13], different theoretical aspects of using virtual sensors in linear systems are considers; in [19], virtual sensors are used for fault tolerant control in linear descriptor systems. Detailed procedure to design virtual sensors of full dimension for linear systems is suggested in [4].

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The main contribution of the present paper is that a procedure to design virtual sensors of minimal dimension for nonlinear systems estimating prescribed components of the state vector is developed. This allows to reduce complexity of the virtual sensors in comparison with cited above papers where such sensors of full order are constructed. Besides, the limitations imposed on the initial system are relaxed that allows to extend a class of systems for which the virtual sensors can be constructed. The suggested solution is based on the reduced order model of the original system. The reduced model may have different properties with respect to the faults and disturbances. When the model is sensitive to the faults and insensitive to the disturbances, the problem of exact fault identification can be solved. If the reduced model is sensitive both to the faults and the disturbances, the problem of approximate fault identification can be solved only.

The set of the prescribed components depends on the problem of control or fault diagnosis under consideration. Note that in [16] the problem of estimating the prescribed components was solved based on sliding mode observers [5, 17]. Virtual sensors can be useful to simplify this problem solution.

The rest of the paper is organized as follows. In Section 2, virtual sensor is designed for linear systems. Section 3 considers such a problem for general discrete-time nonlinear systems. In Section 4, the problem is solved by so-called logic-dynamic approach. Practical example is considered in Section 5. Section 6 concludes the paper.

2. LINEAR SYSTEMS

2.1. The Main Models

Consider system described by linear dynamic model

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$$\begin{array}{rcl}\dot{x}(t) & = & Fx(t)+Gu(t)+L\rho(t),\\ y(t) & = & Hx(t),\end{array} \tag{1}$$

where $x(t)\in R^n$, $u(t)\in R^m$, $y(t)\in R^l$ are vectors of state, control, and output, F, G, H, and L are known constant matrices, $\rho(t)\in R^p$ is the unmatched disturbance, it is assumed that $\rho(t)$ is an unknown bounded function of time.

The problem is as follows: given $y_v(t)=H_v x(t)$ for known matrix H_v, construct virtual sensor of minimal dimension estimating the variable $y_v(t)\in R^p$. The problem is solved by using Luenberger observer invariant with respect to the disturbance estimating the variable $y_v(t)$. In addition to $y_v(t)$, such an observer should estimate some output variable $y_*(t)$ for generating the residual $r(t)$ to achieve stability of the observer. The observer is based on the model of the the original system given by

$$\begin{array}{rcl}\dot{x}_v(t) & = & F_*x_v(t)+G_*u(t)+J_*H_0x(t),\\ y_v(t) & = & H_{*v}x_v(t)+Qy(t),\\ y_*(t) & = & H_*x_v(t),\\ r(t) & = & R_*y(y)-y_*(t),\end{array} \tag{2}$$

where $x_v(t)\in R^k$, $k\le n$, is the state vector, F_*, G_*, J_*, H_*, H_{*v}, Q, and R_* are matrices to be determined,

$$H_0=\begin{pmatrix} H\\ H_v\end{pmatrix},\quad y_0=H_0x=\begin{pmatrix} y\\ y_v\end{pmatrix}.$$

The problem is solved in three steps: initially, the model of minimal dimension estimating the variable $y_*(t)$ is designed, then a possibility to estimate the variable $y_v(t)$ is checked; finally, the matrix K_* ensuring stability of the observer is found.

To implement the first step, introduce the matrix Φ such that $x_v(t)=\Phi x(t)$. It is known [22] that matrices describing the model satisfy the conditions

$$\begin{array}{rcl}R_*H & = & H_*\Phi,\quad \Phi F=F_*\Phi+J_*H_0,\\ \Phi G & = & G_*,\quad \Phi L=0.\end{array} \tag{3}$$

The additional condition appears due to the equation

$$y_v(t)=H_vx(t)=H_{*v}x_v(t)+Qy(t);$$

since $x_v(t)=\Phi x(t)$ and $y(t)=Hx(t)$, it follows

$$H_v=H_{*v}\Phi+QH. \tag{4}$$

Previously, it is necessary to check a possibility to design the model invariant with respect to the disturbance. Introduce the matrix L_* of maximal rank such that $L_*L=0$. Since the condition of invariance is of the form $\Phi L=0$, then $\Phi=NL_*$ for some matrix N. Replace the matrix Φ in the equation $R_*H=H_*\Phi$ by NL_*: $R_*H=H_*NL_*$, or

$$(R_*-H_*N)\begin{pmatrix} H\\ L_*\end{pmatrix}=0.$$

Clearly, the last equation has a solution if

$$rank\begin{pmatrix} H\\ L_*\end{pmatrix}<rank(H)+rank(L_*). \tag{5}$$

Similar replacing in $\Phi F=F_*\Phi+J_*H_0$ and $H_v=H_{*v}\Phi+QH$ gives $NL_*F=F_*NL_*+J_*H_0$ and $H_v=H_{*v}NL_*+QH$; these equations are solvable if

$$rank\begin{pmatrix} L_*F\\ L_*\\ H_0\end{pmatrix} < rank(L_*F)+\begin{pmatrix} H_0\\ L_*\end{pmatrix}, \tag{6}$$

$$rank\begin{pmatrix} H\\ L_*\end{pmatrix} = rank\begin{pmatrix} H\\ L_*\\ H_v\end{pmatrix}, \tag{7}$$

respectively.

If (5) or (6) or (7) is not satisfied, the model invariant with respect to the disturbance does not exist and one has to use robust methods [22]. Assume that (5)-(7) are satisfied and construct the model.

2.2. Model Design

The matrices F_* and H_* are sought in the canonical form

$$F_*=\begin{pmatrix} 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \dots & \dots & \dots & \dots & \\ 0 & 0 & 0 & \dots & 0\end{pmatrix},\ H_*=(\ 1\ \ 0\ \ 0\ \ \dots\ \ 0\ ). \tag{8}$$

Clearly, this is always possible if (F_*,H_*) in (2) is observable. If (F_*, H_*) is unobservable, system (2) can be transformed into observable canonical form and then the matrices describing the observable part of this form can be presented in the above-described canonical form of less dimension.

Using these matrices, one obtains from (3) equations for rows of the matrices Φ and J_*:

$$\begin{array}{rcl} \Phi_1 & = & R_* H, \quad \Phi_i F = \Phi_{i+1} + J_{*i} H_0, \\ i & = & 1,...,k-1, \quad \Phi_k F = J_{*k} H_0, \end{array} \tag{9}$$

where Φ_i and J_{*i} are i-th rows of the matrices Φ and J_*, $i = 1,...,k$. As is shown in [22-24], equations (9) can be transformed into the single equation

$$(\, R_* \quad -J_{*1} \quad ... \quad -J_{*k} \,) W^{(k)} = 0, \tag{10}$$

where

$$W^{(k)} = \begin{pmatrix} HF^k \\ H_0 F^{k-1} \\ ... \\ H_0 \end{pmatrix}.$$

The condition $\Phi L = 0$ of invariance with respect to the disturbance can be taken into account in the form [22-24]

$$(\, R_* \quad -J_{*1} \quad ... \quad -J_{*k} \,) L^{(k)} = 0,$$

where

$$L^{(k)} = \begin{pmatrix} HL & HFL & ... & HF^{k-1}L \\ 0 & H_0 L & ... & H_0 F^{k-2} L \\ ... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{pmatrix}.$$

The last equation and (10) result in single equation

$$(\, R_* \quad -J_{*1} \quad ... \quad -J_{*k} \,)(W^{(k)} \; L^{(k)}) = 0. \tag{11}$$

Equation (11) has a nontrivial solution if

$$rank(W^{(k)} \; L^{(k)}) < l + (l+1)k. \tag{12}$$

To construct the model, find from (12) the minimal dimension k and the row $(\, R_* \quad -J_{*1} \quad ... \quad -J_{*k} \,)$ satisfying (11). Then calculate the rows of the matrix Φ based on (9).

At the second step, a possibility to estimate the variable $y_v(t)$ is checked based on (4). Rewrite it in the form

$$H_v = (H_{*v} \; Q)\begin{pmatrix} \Phi \\ H \end{pmatrix}. \tag{13}$$

This equation has a solution if

$$rank\begin{pmatrix} \Phi \\ H \end{pmatrix} = rank\begin{pmatrix} \Phi \\ H \\ H_v \end{pmatrix}. \tag{14}$$

If (14) is satisfied, the variable $y_v(t)$ can be estimated by observer. Otherwise, one finds another solution of (11) with former or incremented dimension k. If (14) is not satisfied for all $k \le n$, the problem cannot be solved.

Assuming that (14) is satisfied for some k, we find the matrices H_{*v} and Q from (13) and set $G_* = \Phi G$. As a result, the model of minimal dimension invariant with respect to the disturbance and estimating the variable $y_v(t)$ has been designed in the form

$$\begin{array}{rcl} \dot{x}_v(t) & = & F_* x_v(t) + G_* u(t) + J_* y_0(t), \\ y_v(t) & = & H_{*v} x_v(t) + Q y(t), \\ y_*(t) & = & H_* x_v(t). \end{array} \tag{15}$$

2.3. Observer Design

Introduce the estimation error $e(t) = \Phi x(t) - x_v(t)$ and write down the equation for $e(t)$ taking into account relations (3) and feedback $K_* r(t)$:

$$\begin{array}{rcl} \dot{e}(t) & = & \Phi F x(t) + \Phi G u(t) \\ & & -(F_* x_v(t) + G_* u(t) + J_* y_0(t) + K_* r(t)) \\ & = & (\Phi F - J_* H_0) x(t) - F_* x_v(t) - K_*(R_* y(t) - y_*(t)) \\ & = & F_* \Phi x(t) - F_* x_v(t) - K_*(R_* H x(t) - H_* x_v(t)) \\ & = & F_* e(t) - K_*(H_* \Phi x(t) - H_* x_v(t)) \\ & = & (F_* - K_* H_*) e(t). \end{array}$$

It follows from (8) that the pair (F_*, H_*) is observable, therefore the matrix K_* exists such that $F_* - K_* H_*$ is stable matrix. Set $K_* = (K_1 \, K_2 \, ... K_k)^T$, then

$$F_* - K_* H_* = \begin{pmatrix} -K_1 & 1 & 0 & ... & 0 \\ -K_2 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & \\ -K_k & 0 & 0 & ... & 0 \end{pmatrix}.$$

It is known that if λ_1, λ_2, ..., λ_k are desirable eigenvalues of the observer, then

$$\begin{aligned} K_1 &= -(\lambda_1 + ... + \lambda_k), \\ K_2 &= \lambda_1\lambda_2 + ... + \lambda_{k-1}\lambda_k, \\ &... \\ K_k &= (-1)^k \lambda_1 ... \lambda_k. \end{aligned}$$

3. NONLINEAR SYSTEMS. GENERAL SOLUTION

For simplicity, consider discrete-time systems since a solution for continuous-time case is more complex. Let the system be described by nonlinear dynamic model

$$\begin{aligned} x(t+1) &= f(x(t), u(t), \rho(t)), \\ y(t) &= h(x(t)). \end{aligned} \tag{16}$$

Here $x \in X \subseteq R^n$, $u \in R^m$, $y \in R^l$ are vectors of state, control, and output; $\rho(t) \in R^s$ is the disturbance, it is assumed that $\rho(t)$ is unknown bounded function of time, f and h are nonlinear functions, the function f may be non-differentiable with respect to x.

The problem is to design virtual sensor insensitive to the disturbance and estimating the variable $y_v(t) = h_v(x(t)) \in R^p$ for the prescribed function h_v. Such a sensor is based on the model

$$\begin{aligned} x_*(t+1) &= f_*(x_*(t), u(t), y(t)), \\ y_v(t) &= h_{v*}(x_*(t), y(t)), \\ y_*(t) &= h_*(x_*(t)), \end{aligned} \tag{17}$$

where $x_* \in R^k$ is a state vector, $y_*(t) \in R$ is a variable such that $y_*(t) = \psi(y(t))$ for some function ψ; f_*, h_{v*}, h_*, and ψ are functions to be determined.

The approach to design the model (17) is based on special mathematical technique, algebra of functions, developed in [21]. The elements of algebra of functions are vector functions with the domain X; its main ingredients are relation of partial preorder $\leq$, two binary operations $\times$ and $\oplus$, binary relation Δ, and operators m and M; these ingredients are described in [21]. Note that the relation $\alpha \leq \beta$ can be treated as follows: the function α contains information about states from X no less than β.

To design the model, we assume that the function ϕ exists such that $x_*(t) = \phi(x(t))$. It was shown in that the function ϕ satisfies the condition [21]

$$h \times \phi \leq M(\phi). \tag{18}$$

Note that the condition (18) describes the first equation in (17).

The best solution is the model insensitive to the disturbance. To design such a model, introduce minimal (in the sense of relation $\leq$) function α^0 containing maximal number of functionally independent components such that the function $\alpha^0(f(x,u,\rho))$ does not depend on ρ. Define for $i \geq 1$ the sequence of non-decreasing functions $\alpha^0 \leq \alpha^1 \leq ...$ using the formula

$$\alpha^{i+1} = \alpha^i \oplus m(\alpha^i \times h), \quad i = 0, 1, ...$$

By results in [11, 21], there exists a finite k such that $\alpha^{k+1} \cong \alpha^k$. Define $\phi := \alpha^k$.

Theorem. [11, 21] The function ϕ is minimal (containing maximal number of independent components) satisfying the conditions (18) and $\alpha^0 \leq \phi$.

It follows from definitions of the operator M and relation $\leq$ that the condition (18) implies existence of the function f_* such that

$$f_*((\phi \times h)(x(t)), u(t)) = \phi(f(x(t), u(t), \rho(t))); \tag{19}$$

in addition, the condition $\alpha^0 \leq \phi$ means that the disturbance $\rho(t)$ does not affect the function f_*.

To design the dynamic part of the model (17), write down the relation

$$x_*(t+1) = \phi(x(t+1)) = \phi(f(x(t), u(t), \rho(t)))$$

and transform its right hand side based on (19); as a result, one obtains the dynamic part of (17).

To design the functions h_* and h_{v*}, write down the relations $y_* = h_*(x_*)$ and $y_v = h_{v*}(x_*, y)$ as follows:

$$\psi(h(x)) = h_*(\phi(x)), \quad h_v(x) = h_{v*}(\phi(x), h(x)),$$

that is equivalent to the functional inequalities

$$\phi \oplus h \neq const, \quad \phi \times h \leq h_v. \tag{20}$$

If these conditions are true for the function ϕ, the functions h_* and h_{v*} can be obtained based on (20) and definition of the relation $\leq$. As a result, the model (17) insensitive to the disturbance has been designed. Otherwise, the estimate of the variable $y_v(t)$ is approximate only.

To transform the model (17) into the observer, its stability should be ensured. This can be achieved by known methods [14].

4. LOGIC-DYNAMIC APPROACH

4.1. Model Design

If ϕ is a linear function, the problem can be solve by simpler method. To achieve this goal, the nonlinear system should be described as follows:

$$\begin{aligned} \dot{x}(t) &= Fx(t)+Gu(t)+C\Psi(x(t),u(t))+L\rho(t), \\ y(t) &= Hx(t). \end{aligned} \quad (21)$$

This model differs from (1) by nonlinear term $\Psi(x,u)$:

$$\Psi(x,u)=\begin{pmatrix} \phi_1(A_1x,u) \\ \ldots \\ \phi_q(A_qx,u) \end{pmatrix}, \quad (22)$$

where A_1, ..., A_q are constant matrices, ϕ_1, ..., ϕ_q are nonlinear functions. It is assumed that the function $C\Psi(x,u)$ satisfies Lipschitz condition about x uniformly for u:

$$\|C\Psi(x,u)-C\Psi(x',u)\| \le M\|x-x'\|, \quad (23)$$

where $M>0$ is some constant.

The problem of the virtual sensors design for nonlinear systems is solved now according to so-called logic-dynamic approach [22-24] which is based on the solution for linear part of the system [22]. As a result, a solution is similar to that for linear system with only difference that at the second step a possibility to express the nonlinear term via linear solution is checked additionally.

The nonlinear observer is given by

$$\begin{aligned} \dot{x}_v(t) &= F_*x_v(t)+G_*u(t)+J_*y_0(t) \\ &\quad +C_*\Psi(x_v(t),y_0(t),u(t))+K_*r(t), \\ y_v(t) &= H_{*v}x_v(t)+Qy(t), \\ y_*(t) &= H_*x_v(t), \\ r(t) &= R_*y(t)-y_*(t), \end{aligned} \quad (24)$$

where $C_*=\Phi C$,

$$C_*\Psi(x_v,y_0,u)=\begin{pmatrix} \phi_{i_1}(A_{*1i_1}x_v+A_{*2i_1}y_0,u) \\ \ldots \\ \phi_{i_k}(A_{*1i_k}x_v+A_{*2i_k}y_0,u) \end{pmatrix}, \quad (25)$$

A_{*1i_1}, A_{*2i_1}, ..., A_{*1i_k}, A_{*2i_k} are matrices satisfying the conditions

$$A_i=(A_{*1i}\ A_{*2i})\begin{pmatrix} \Phi \\ H_0 \end{pmatrix}, \quad i=i_1,\ldots,i_k; \quad (26)$$

$i_1,\ldots,i_k$ are nonzero columns of the matrix C_*.

In addition to (5) and (6), a possibility to design the nonlinear model invariant with respect to the disturbance can be checked previously. Rewrite (26) in more general form with $\Phi=NL_*$

$$A=A_*\begin{pmatrix} NL_* \\ H_0 \end{pmatrix}$$

that is equivalent to the condition

$$rank\begin{pmatrix} H \\ L_* \end{pmatrix}=rank\begin{pmatrix} H \\ L_* \\ A \end{pmatrix},$$

where $A=\begin{pmatrix} A_1 \\ \ldots \\ A_q \end{pmatrix}$. If this condition is not satisfied, the nonlinear model invariant with respect to the disturbance does not exist, and the robust methods should be used [22].

Similar to the linear case, one has to solve (11) for minimal k, find from (9) rows of the matrix Φ and check the condition (14); we assume it is satisfied.

To construct the nonlinear term, set $C_*:=\Phi C$, calculate (25) and check the condition

$$rank\begin{pmatrix} \Phi \\ H_0 \end{pmatrix}=rank\begin{pmatrix} \Phi \\ H_0 \\ A_i \end{pmatrix}, i=i_1,\ldots,i_k. \quad (27)$$

If it is satisfied, set $G_*:=\Phi G$; the matrices A_{*1i} and A_{*2i}, $i=i_1,\ldots,i_k$, are found from (26). If (27) is not satisfied, one finds another solution of (11) with former or incremented dimension k. If (27) is not satisfied for all $k \le n$, the nonlinear model cannot be designed.

As a result, the model (15) takes the form

$$\begin{aligned}\dot{x}_v(t) &= F_* x_v(t) + G_* u(t) + J_* y_0(t) \\ &\quad + C_* \Psi(x_v(t), y_0(t), u(t)), \\ y_v(t) &= H_{*v} x_v(t) + Q y(t), \\ y_*(t) &= H_* x_v(t).\end{aligned}$$

4.2. Stability Analysis

The equation for $e(t)$ in the nonlinear case takes the form

$$\dot{e}(t) = F_{**} e(t) + \Delta\Psi(t),$$

where $F_{**} = F_* - K_* H_*$,

$$\Delta\Psi(t) = C_*(\Psi(\Phi x(t), y_0(t), u(t)) - \Psi(x_v(t), y_0(t), u(t))).$$

Since the function Ψ satisfies the condition (23), then $\Delta\Psi(t)$ satisfies similar condition as well:

$$\|\Delta\Psi(t)\| \le M_* \|e(t)\| \tag{28}$$

for some $M_* > 0$. Since the matrix F_{**} is stable due to the choice of the matrix K_*, the symmetric positive definite matrices P and W exist such that

$$F_{**}^T P + P F_{**} = -W. \tag{29}$$

Consider Lyapunov candidate function $V = e^T P e(t)$ and take its derivative using (28) and (29):

$$\begin{aligned}\dot{V}(t) &= (F_{**}e(t) + \Delta\Psi(t))^T P e + e^T(t) P (F_{**} e(t) + \Delta\Psi(t)) \\ &= e^T(t)(F_{**}^T P + P F_{**}) e(t) + 2 e^T(t) P \Delta\Psi(t) \\ &\le -e^T(t) W e(t) + 2 e^T(t) P \Delta\Psi(t) \\ &\le -\|e(t)\|^2 \lambda_{min}(W) + 2\|e^T(t) P \Delta\Psi(t)\| \\ &\le -\|e(t)\|^2 \lambda_{min}(W) + 2\|e(t)\|^2 \lambda_{max}(P) M_*.\end{aligned}$$

Clearly, if

$$M_* < \frac{\lambda_{min}(W)}{2\lambda_{max}(P)}, \tag{30}$$

then $\dot{V}(t) < 0$, the observer is stable, and there is no need to use a feedback. Note that this approach is considered in [14]. It follows from (30) that the approach imposes severe conditions on the function Ψ since as a rule, $M_* < 1$ in (30). Only $k = 1$ implies $\lambda_{min}(W) = 2PK_*$ and $M_* < K_*$, that is K_* always can be chosen to satisfy the condition $M_* < K_*$ for arbitrary Lipschitz function.

5. EXAMPLE

Consider the control system

$$\begin{aligned}\dot{x}_1 &= a_1 u_1 / \vartheta_1 - a_2 a_4 \sqrt{x_1 - x_2}, \\ \dot{x}_2 &= a_3 u_2 / \vartheta_2 + a_2 a_4 \sqrt{x_1 - x_2} - a_5 \sqrt{x_2 - x_3}, \\ \dot{x}_3 &= a_5 \sqrt{x_2 - x_3} - a_6 \sqrt{x_3 - \vartheta_7} - \rho, \\ y_1 &= x_1, \quad y_2 = x_2.\end{aligned} \tag{31}$$

The equations (31) constitute a modified model of the well-known example of three-tank system (Figure **1**). The system consists of three consecutively united tanks with areas of the cross-section ϑ_1, ϑ_2, and ϑ_3. The tanks are linked by pipes with areas of the cross-section ϑ_4 and ϑ_5. The liquid flows into the first and the second tanks and follows from the third one through the pipe with area of the cross-section ϑ_6 located at height ϑ_7; ϑ_8 is the gravitational constant. The levels of liquid in the tanks are x_1, x_2, and x_3, respectively. Assume for simplicity that $a_1 = a_2 = \ldots = a_6 = 1$, $\vartheta_7 = 0$.

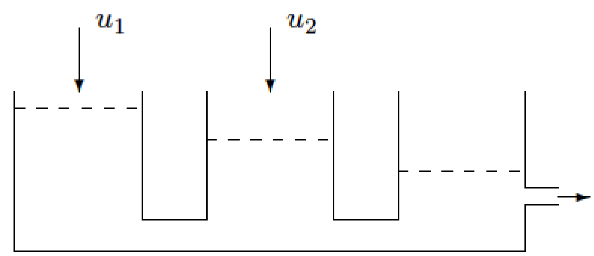

Figure 1: Three tank system.

Clear, $F = 0$ in the model (31). To overcome this difficulty, transform (31) by entering formal addends $-(x_1 - x_2) + (x_1 - x_2)$, $((x_1 - x_2) - (x_2 - x_3)) - ((x_1 - x_2) - (x_2 - x_3))$ and $(x_2 - x_3 - x_3) - (x_2 - x_3 - x_3)$ in the first, second, and third equations, respectively. As a result, the system is described by matrices and nonlinearities as follows:

$$\begin{aligned}F &= \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, G = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \\ H &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, L = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ \Psi(x) &= \begin{pmatrix} -\sqrt{A_1 x} + A_1 x \\ \sqrt{A_1 x} - \sqrt{A_2 x} - (A_1 x - A_2 x) \\ \sqrt{A_2 x} - \sqrt{A_3 x} - (A_2 x - A_3 x) \end{pmatrix}, \\ A_1 &= (1 -1 0), A_2 = (0 1 -1), A_3 = (0 0 1).\end{aligned}$$

Assume that $H_v = (1\,0\,0)$. One can show that the condition (12) is satisfied for $k = 1$ but (14) is not satisfied therefore set $k = 2$. The matrix $(V^{(2)}\,L^{(2)})$ is of the form

$$(V^{(2)}\,L^{(2)}) = \begin{pmatrix} -3 & 6 & -4 & 0 & 1 \\ 1 & -4 & 5 & 1 & -2 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

This equation has several solutions, consider the first one:

$$(R_* - J_{*1} - J_{*2}) = (1\,0\,2\,-1\,0\,-1\,0\,1).$$

It gives $R_* = (1\,0)$,

$$J = \begin{pmatrix} -2 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}, \quad \Phi = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Since $\Phi C = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, then $j_1 = 2$ and $j_2 = 1$,

$$A' = \begin{pmatrix} A_2 \\ A_1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix}.$$

Clearly, the condition (27) is satisfied, and (26) yields the solution $A_{*1} = (0\,1\,-1\,0\,0)$ and $A_{*2} = (0\,0\,1\,-1\,0)$.

As a result, the model is given by

$$\begin{aligned} \dot{x}_{v1} &= x_{v2} - 2y_1 + y_2 + u_2 + \sqrt{x_{v2} - y_1} - \sqrt{y_1 - y_2} \\ &\quad -(x_{v2} - y_1 - (y_1 - y_2)) \\ &= u_2 + \sqrt{x_{v2} - y_1} - \sqrt{y_1 - y_2}, \\ \dot{x}_{v2} &= -y_v + y_1 + u_1 - \sqrt{x_{v2} - y_1} + x_{v2} - y_1 \\ &= -x_{v2} + u_1 - \sqrt{x_{v2} - y_1} + x_{v2} \\ &= u_1 - \sqrt{x_{v2} - y_1}, \\ y_* &= x_{v1}, \quad y_v = x_{v2}, \end{aligned}$$

The equation for the error $e(t)$ takes the form

$$\dot{e}(t) = \begin{pmatrix} -K_1 & 0.5(x_{v2} - y_1)^{-1/2} \\ -K_2 & -0.5(x_{v2} - y_1)^{-1/2} \end{pmatrix} e(t).$$

Set $\lambda_1 = \lambda_2 = -1$, then

$$K_1 = 2 - \frac{1}{2\sqrt{x_{v2} - y_1}}, K_2 = \frac{4(x_{v2} - y_1) + 1}{2\sqrt{x_{v2} - y_1}} - 2.$$

Final description of the virtual sensor is given by

$$\begin{aligned} \dot{x}_{v1} &= u_2 + \sqrt{x_{v2} - y_1} - \sqrt{y_1 - y_2} + K_1 r, \\ \dot{x}_{v2} &= u_1 - \sqrt{x_{v2} - y_1} + K_2 r, \\ y_* &= x_{v1}, \quad y_v = x_{v2}, \\ r &= y_1 - y_*. \end{aligned} \tag{32}$$

Note that the approach suggested in [16] does not allow to identify the fault in the first sensor because of the condition $R_* D \neq 0$ in this case. Since the virtual sensor (32) gives identical matrix H_0, the problem of sensor fault identification can be solved for all sensors successfully.

For simulation, consider system (31) and the observer (32) with the controls $u_1(t) = 1$, $t \geq 1$, $u_2(t) = 0.5$, $t \geq 5$, $\rho_1(t) = -0.3$, $t \geq 6$, $\rho_2(t) = -0.4$, $t \geq 10$. Simulation results are shown in Figure **2**, where the functions $x_1(t)$ and $y_v(t)$ are presented.

6. CONCLUSION

In this paper, the problem of virtual sensor design has been studied for systems described by linear and nonlinear models under the disturbance. The suggested approach allows to obtain virtual sensor of minimal dimension estimating prescribed components of the state vector and insensitive to the disturbance.

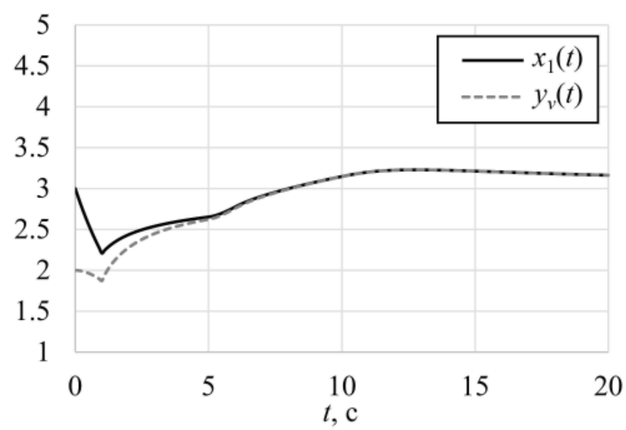

Figure 2: Estimation of the functions $x_1(t)$ and $y_v(t)$

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