Virtual Sensor Design for Replacement the Faulty Physical Sensors

Alexey Zhirabok[1,2,*], Alexander Zuev[2] and Vladimir Filaretov[3]

[1]Department of Automation and Robotics, Far Eastern Federal University

[2]Department of Intelligent Control, Institute of Marine Technology Problems

[3]Department of Robotics, Institute of Automation and Control Processes

Abstract: The paper considers the problem of virtual sensor design for nonlinear dynamic systems with non-smooth nonlinearities described by continuous-time models for faulty physical sensor replacement. It is assumed that to solve the problem, the system is equipped by diagnostic system allowing detecting and isolating the faulty sensor. For every such a sensor, the virtual sensor generating estimate replacing the faulty sensor is designed. To solve the problem, so-called logic-dynamic approach is used which does not guarantee optimal solution but uses only methods of linear algebra to solve the problem for systems with non-smooth nonlinearities. The virtual sensor can be designed in the identification canonical form or Jordan canonical form. The advantage of the first form is a standard procedure of the virtual sensor design while Jordan form allows obtaining a simpler solution. The relations allowing designing the virtual sensor both in identification and in Jordan canonical form are derived.

Keywords: Dynamic systems, Continuous-time, Non-smooth nonlinearities, Physical sensors, Faults, Virtual sensors, Identification canonical form, Jordan canonical form.

1. INTRODUCTION

Complex technical systems, as a rule, are equipped by different physical sensors to measure their performances. In addition, virtual sensors can be used for this purpose. They are based on the readings of physical sensors and produce estimates of unmeasured performances of the system. Besides, virtual sensors can be used to replace the faulty physical sensors.

The problems of virtual sensors design and application are considered in many papers [1-4, 6-11]; some applications of virtual sensors are analyzed in [15]. Detailed procedure to design virtual sensors of full dimension for linear systems is suggested in [1].

The main contribution of the present paper is that a procedure to design virtual sensors for replacing the faulty sensors for systems described by dynamic models with non-smooth nonlinearities is developed. To solve this problem, the method to design the virtual sensors of minimal dimension is suggested. This allows reducing complexity of the virtual sensors in comparison with cited above papers where such sensors of full order are constructed. Besides, the limitations imposed on the initial system are relaxed that allows extending a class of systems for which the virtual sensors can be constructed. The suggested solution is based on the reduced order model of the original system.

To solve the problem for systems with non-smooth nonlinearities, so-called logic-dynamic (LD) approach is used. This approach was used to solve the problems of fault diagnosis [12] and to analyze observability and controllability of nonlinear systems [13]. The LD approach does not produce optimal solution of the problem in the sense of the virtual sensor dimension but uses only methods of linear algebra to solve the problems for systems with non-smooth nonlinearities.

2. THE MAIN MODELS

Consider the system described by nonlinear differential equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + P\psi(Gx(t),u(t)), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $y(t) \in R^l$, $l > 1$, are state, control, and output vectors, *A, B, C*, and *P* are known matrices; for simplicity, we assume that the only type of nonlinearity described by the term $\psi(Gx(t),u(t))$ is in the system, *G* is the matrix, the function ψ may be non-smooth. Note that that matrix *C* describes physical sensors of the system.

We assume that the system is equipped by the diagnostic system which allows detecting and isolating

*Address correspondence to this author at the Department of Automation and Robotics, Far Eastern Federal University, Vladivostok, Russia,
E-mail: zhirabok@mail.ru

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the faulty sensor. Besides we assume that at some instant of time this system makes a decision that the j-th sensor has been failed. This means that the variable $y_j(t)$ yields faulty information about the system states, and the virtual sensor estimating the variable $v(t)=y_j(t)=C_jx(t)$ should be designed where C_j is the j-th row of the matrix *C*.

The problem is solved by designing the nonlinear functional observer estimating the variable $v(t)$. Such an observer is described by the equations

$$\begin{aligned}
&\dot{z}(t)=A_*z(t)+J_*y(t)+B_*u(t)+P_*\psi_*(z(t),y(t),u(t))+Kr(t),\\
&y_*(t)=C_*z(t),\\
&v(t)=C_vz(t)+Qy_0(t),\\
&r(t)=R_*y_0(t)-y_*(t),
\end{aligned} \tag{2}$$

where $z(t)\in R^k$ is the observer state vector, k is the observer dimension, A_*, J_*, B_*, P_*, R_*, C_*, C_v, Q, and K are matrices to be determined; $P_*\psi_*(z,y,u)$ is the nonlinear term; $y_0(t)=C_0x(t)$, C_0 is the matrix *C* without the j-th row. Note that the variable $y_*(t)$ in (2) is necessary to generate the residual $r(t)$ used in the feedback to provide stability of the observer. The variable $y(t)$ in the first equation contains in the j-th position not the faulty sensor readings but the variable $v(t)$ estimating the readings.

Remark 1. In contrast to a model suggested in [15], the model (2) does not contain information about the faulty sensor readings.

The observer (2) assumes that the matrices A_* and C_* are in the identification canonical form (ICF):

$$A_*=\begin{pmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ 0 & 0 & 0 & \cdots & 0\\ \cdots & \cdots & \cdots & \ddots & \cdots\\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix},\quad C_*=(\ 1\ \ 0\ \ 0\ \ \cdots\ \ 0\ ). \tag{3}$$

It is known [14] that to design the observer, Jordan canonical form (JCF) of the matrix A_* can be used as well:

$$A_*=\begin{pmatrix} \lambda_1 & 0 & \cdots & 0\\ 0 & \lambda_2 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_k \end{pmatrix}. \tag{4}$$

It is assumed that the eigenvalues $\lambda_1,...,\lambda_k$ in (4) are negative therefore the matrix A_* is stable by construction. In this case, the observer (2) is simplified:

$$\begin{aligned}
&\dot{z}(t)=A_*z(t)+J_*y(t)+B_*u(t)+P_*\psi_*(z(t),y(t),u(t)),\\
&v(t)=C_vz(t)+Qy_0(t).
\end{aligned} \tag{5}$$

Remark 2. Note that stability of the observer (5) is insured by the negative eigenvalues $\lambda_1,...,\lambda_k$. Since the variable $y(t)$ in (5) contains $v(t)$, this results in the feedback; therefore, special analysis of stability may be required.

According to the LD approach, the ICF-based solution is performed in three steps. At the first step, the nonlinear term is removed from (1) and the linear model is designed:

$$\begin{aligned}
&\dot{z}(t)=A_*z(t)+J_*y(t)+B_*u(t),\\
&y_*(t)=C_*z(t).
\end{aligned} \tag{6}$$

Then based on the relation

$$v(t)=C_vz(t)+Qy_0(t), \tag{7}$$

the possibilities of the variable $v(t)$ estimation and the nonlinear term $P_*\psi_*(z,y,u)$ construction are checked. Finally, the matrix K is designed. Let us consider these steps in detail.

3. ICF-BASED MODEL DESIGN

We assume that the matrix Φ exists such that $z(t)=\Phi x(t)$. It is known that the matrices describing the model (6) meet the following equations [11, 12]:

$$R_*C_0=C_*\Phi,\ \ \Phi A=A_*\Phi+J_*C,\ \ B_*=\Phi B. \tag{8}$$

The first step solution is based on the equation [11, 12]

$$(\ J_{*k}\ \ \cdots\ \ J_{*1}\ \ -R_*\ )U^{(k)}=0, \tag{9}$$

where

$$U^{(k)}=\begin{pmatrix} C\\ CA^{k-1}\\ \vdots\\ C_0A^k \end{pmatrix},\ \ k=1,\ 2,\ ...\ .$$

Equation (9) has a nontrivial solution if

$$rank(U^{(k)})<lk-1. \tag{10}$$

To design the model, one finds from (10), starting from $k=1$, the minimal *k*, from (9) the row $(\ J_{*k} \ \cdots \ J_{*1} \ -R_* \ )$, finally, based on the relations for rows Φ_i and J_{*i} of the matrices Φ and J_*:

$$R_*C_0=\Phi_1,\ \Phi_iA=\Phi_{i+1}+J_{*i}C,\ i=\overline{1,k-1},\ \Phi_kA=J_kC,$$

the matrices Φ and $B_*=\Phi B$ are found.

To perform the second step, rewrite (7) with $v(t)=C_jx(t)$ in the form

$$C_jx(t)=C_v\Phi x(t)+QC_0x(t),$$

that results in

$$C_j=C_v\Phi+QC_0=(\ C_v \quad Q\ )\begin{pmatrix}\Phi\\ C_0\end{pmatrix}. \tag{11}$$

This equation has a solution if

$$rank\begin{pmatrix}\Phi\\ C_0\end{pmatrix}=rank\begin{pmatrix}\Phi\\ C_0\\ C_j\end{pmatrix}. \tag{12}$$

If (12) is satisfied, then the matrix C_j can be expressed via $(\Phi^T \ \ C_0^T)^T$ and the model estimates the variable $v(t)=C_jx(t)$; the matrices C_v and Q are found from (11). If (12) is not satisfied, one has to find another solutions of (9) with former or incremented *k*.

If $P_*=\Phi P=0$, the final model is linear; to transform it into the observer, one chooses the eigenvalues $\lambda_1,...,\lambda_k$ and finds the feedback matrix K [11]:

$$K_1=-(\lambda_1+\lambda_2+...+\lambda_k),\quad K_2=\lambda_1\lambda_2+\lambda_1\lambda_3...+\lambda_{k-1}\lambda_k,\quad \ldots,$$
$$K_k=(-1)^k\lambda_1\lambda_2...\lambda_k.$$

the case $P_*\neq 0$ is considered in Section 5.

4. JCF-BASED MODEL DESIGN

By analogy with the ICF, the JCF-based solution is performed in two steps. At the first step, the linear model is designed:

$$\dot z(t)=A_*z(t)+J_*y(t)+B_*u(t). \tag{13}$$

Then the possibilities of the variable $v(t)$ estimation and the nonlinear term $P_*\psi_*(z,y,u)$ construction are checked. The linear observer stability is insured by the canonical form of the matrix A_*.

The matrices describing the model (13) meet the following equations [11, 12]

$$\Phi A=A_*\Phi+J_*C,\ B_*=\Phi B.$$

Based on (4), the first equation can be transformed into k independent equations:

$$\Phi_iA=\lambda_i\Phi_i+J_{*i}C,\ i=\overline{1,k},$$

which are presented in the form

$$(\Phi_i \quad -J_{*i})\begin{pmatrix}A-\lambda_iI_n\\ C\end{pmatrix}=0,\ i=\overline{1,k}, \tag{14}$$

where I_n is the identity matrix.

One has to choose $\lambda_i<0$ and find from (14) the minimal number of the matrix Φ rows satisfying the conditions (12) and find the matrices C_v and Q from (11); finally, set $B_*=\Phi B$. If $P_*=\Phi P=0$, the observer has been designed. The case $P_*\neq 0$ is considered in Section 5.

Remark 3. Since the variable $y_*(t)$ is not estimated, the suggested approach allows reducing the dimension of the observer.

5. THE NONLINEAR CASE

It is assumed that $P_*\neq 0$. In this case the relations (8) and (13) are supplemented by $P_*=\Phi P$ and

$$G=G_*\begin{pmatrix}\Phi\\ C\end{pmatrix}. \tag{15}$$

Equation (15) has a solution if

$$rank\begin{pmatrix}\Phi\\ C\end{pmatrix}=rank\begin{pmatrix}\Phi\\ C\\ G\end{pmatrix}. \tag{16}$$

To check the possibility of transformation of the linear model into the nonlinear model, one has to check the condition (16); if it is satisfied, construct the nonlinear term

$$\psi_*(z,y,u)=\psi(G_*\begin{pmatrix}z\\ y\end{pmatrix},u),$$

where the matrix G_* is found from (15); the nonlinear term $\psi_*(z,y,u)$ is added to the linear model (6) or (13). The nonlinear model has been designed.

If (16) is not satisfied, one has to find another solution of (9) with former or incremented *k* (for the ICF-based model) or to find another solution of (14) (for the JCF-based model). When the system has several different nonlinearities, a solution can be obtained by analogy with [11, 15]. Actually, each nonlinearity is considered independently of one another.

Analysis of stability for nonlinear observer can be fulfilled by methods described in [5, 14, 15].

6. EXAMPLE

Consider the control system

$$\begin{aligned}
&\dot{x}_1(t) = u_1(t)/\vartheta_1 - b_1\sqrt{x_1(t)-x_2(t)},\\
&\dot{x}_2(t) = u_2(t)/\vartheta_2 + b_1\sqrt{x_1(t)-x_2(t)} - b_2\sqrt{x_2(t)-x_3(t)},\\
&\dot{x}_3(t) = b_2\sqrt{x_2(t)-x_3(t)} - b_3\sqrt{x_3(t)-\vartheta_3},\\
&y_1(t) = x_2(t),\quad y_2(t) = x_3(t).
\end{aligned} \tag{17}$$

The equations (17) constitute a modified model of the well-known example of three-tank system (Figure **1**). The levels of liquid in the tanks are x_1, x_2, and x_3, respectively; is it assumed that cross-sections of tanks and pipes and controls $u_1(t)$ and $u_2(t)$ are such that $x_1(t) \ge x_2(t) \ge x_3(t)$ for all $t \ge 0$.

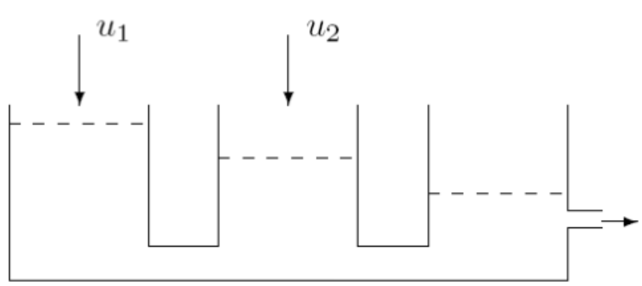

Figure 1: Three tank system.

Assume that the first sensor has failed; as a result, set $C_1 = (0\ \ 1\ \ 0)$ and construct the virtual sensor with $C_0 = (\ 0\ \ 0\ \ 1\ )$ and $y_0(t) = y_2(t)$. For simplicity, one assumes that $\vartheta_1 = \vartheta_2 = 1$, $\vartheta_3 = 0$, $b_1 = b_2 = b_3 = 1$.

The system can be described by matrices and nonlinearities as follows [15]:

$$A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{pmatrix},\ B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$P = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix},\ \psi(x,u) = \begin{pmatrix} -\sqrt{G_1x} + G_1x \\ -\sqrt{G_2x} + G_2x \\ -\sqrt{G_3x} + G_3x \end{pmatrix},$$

$G_1 = (\ 1\ \ -1\ \ 0\ )$, $G_2 = (\ 0\ \ 1\ \ -1\ )$, $G_3 = (\ 0\ \ 0\ \ 1\ )$.

One can show that the ICF does not produce a solution since the condition (12) is not satisfied; as a result, the JCF will be used. Equation (14) becomes

$$(\Phi_i\quad -J_{*i})\begin{pmatrix} -1-\lambda_i & 1 & 0 \\ 1 & -2-\lambda_i & 1 \\ 0 & 1 & -2-\lambda_i \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 0.$$

Set $\lambda_1 = -1$ and obtain $\Phi_1 = (\ 1\ \ 0\ \ 0\ )$ and $J_{*1} = (\ 0\ \ 1\ )$. With $\lambda_2 = -2$ we obtain $\Phi_2 = (\ 1\ \ -1\ \ 0\ )$ and $J_{*2} = (\ -1\ \ 1\ )$, that results in

$$B_* = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix},\ P_* = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \end{pmatrix}.$$

One can check that the condition (12) is satisfied, the solution of (11) is $C_v = (\ 1\ \ -1\ )$, $Q = 0$, that is $v = z_1 - z_2$, $z_1 = \Phi_1 x$, $z_2 = \Phi_2 x$.

The linear model is described by the equations

$$\begin{aligned}
&\dot{z}_1(t) = -z_1(t) + v(t) + u_1(t) = -z_2(t) + u_1(t),\\
&\dot{z}_2(t) = -2z_2(t) + v(t) - y_2(t) + u_1(t) - u_2(t) =\\
&\qquad = -3z_2(t) + z_1(t) - y_2(t) + u_1(t) - u_2(t),\\
&v(t) = z_1(t) - z_2(t).
\end{aligned}$$

One can check that the condition (16) is satisfied, the solution of (15) is

$G_{*1} = (\ 0\ \ 1\ \ 0\ \ 0\ )$, $G_{*2} = (\ 1\ \ -1\ \ -1\ \ 0\ )$.

The nonlinear term is described by the expression

$$P_*\psi_*(x_*,y,u)=\begin{pmatrix} -\sqrt{z_2}+z_2 \\ -2(-\sqrt{z_2}+z_2)-(-\sqrt{z_1-z_2-y_2}+z_1-z_2-y_2) \end{pmatrix}=$$

$$=\begin{pmatrix} -\sqrt{z_2}+z_2 \\ -2\sqrt{z_2}+\sqrt{z_1-z_2-y_2}-z_1+3z_2+y_2 \end{pmatrix}.$$

The nonlinear model is described by the equations

$$\dot{z}_1(t)=-\sqrt{z_2(t)}+u_1(t),$$
$$\dot{z}_2(t)=-2\sqrt{z_2(t)}+\sqrt{z_1(t)-z_{z2}(t)-y_2(t)}+u_1(t)-u_2(t),$$
$$v(t)=z_1(t)-z_2(t). \quad (18)$$

It can be shown that the observer is stable; therefore, it can replace the faulty first sensor.

Simulation results of the system (17) and the observer (18) are presented on Figure **2** with $u_1(t)=0{,}5$, $u_2(t)=0{,}2$ and the initial conditions $x(0)=(5\quad 3\quad 1)^T$, $x_*(0)=(1\quad 4)^T$. Clearly, the variable $z(t)$ converges to $x_2(t)$.

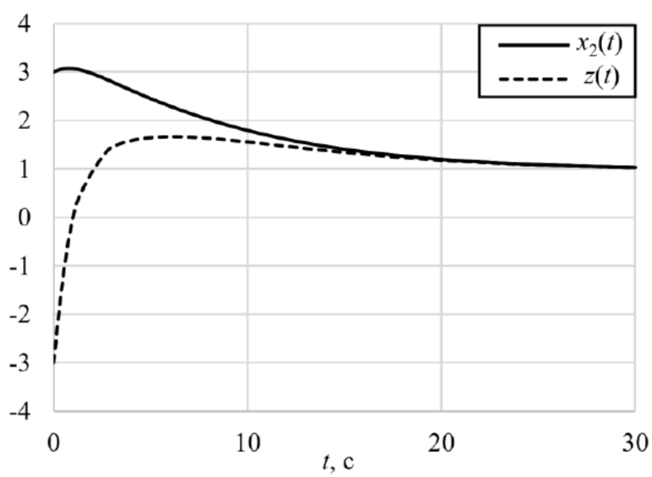

Figure 2: Graphs of the functions $x_2(t)$ and $z(t)$.

7. CONCLUSION

In this paper, the problem of virtual sensor design for nonlinear dynamic systems with non-smooth nonlinearities described by continuous-time models for faulty physical sensor replacement has been considered. The virtual sensors have been designed based on the identification and Jordan canonical forms. To solve the problem for nonlinear systems, so-called logic-dynamic approach has been used which does not guarantee optimal solution but uses only methods of linear algebra to solve the problem for systems with non-smooth nonlinearities. The synthesized virtual sensors provide a possibility for system to continue performing its functions with faulty physical sensors.

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