A Review on Variation Modeling of Aircraft Assembly

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Abstract: *Purpose;* The purpose of this paper is to provide a state-of-art review on variation modeling and propagation in aircraft assembly based on process-oriented strategy. And the main focus is on classifying and delineating different approaches, methods, and techniques with critical appraisals of their theories, applications, and limitations, based on which future research directions and corresponding methodologies are proposed.

Design/methodology/approach; To facilitate understanding of the background, this paper starts with a brief description of aircraft assembly. Afterwards, this paper presents a comprehensive review of practical solutions in aircraft variation modeling. Their characteristics are summarized, which serves as a basis for the discussion of deviation control strategies. Thereafter, the possible trends are discussed to facilitate assembly quality control in future research.

Keywords: Aircraft assembly, Variation modeling, Influence coefficients, Numeric methods, Deformation theories.

1. INTRODUCTION

With the development of aircraft industries, it is prevailing for enterprises to out-source components rather than manufacturing them internally, which requires benchmarking of suppliers' manufacturing processes. Correspondently, the focus of interchangeability has been changed from product to process, which means the shift from product-based design to assembly process analysis [1]. And one of the essential aspects of process analysis is the geometrical quality assurance during the assembly process, also called tolerance research.

In terms of tolerance research, tolerance synthesis and tolerance analysis are two typical ideas to incorporate the manufacturing uncertainties. The former is to conduct the modeling process in a top-down approach. The latter is to take into account the deviation of individual parts in a bottom-up approach, also termed as "variation analysis," through which the generation, accumulation, and propagation of variation can be illustrated explicitly.

To fulfill the need for process variation analysis, the concept of Skin Models was proposed [2-4], which incorporates the deviations brought by manufacturing and assembly processes and is defined as a "model of the physical interface between the workpiece and its environment." And SMS (Skin Model Shape) is a finite approximation of the infinite Skin Model. Recently, new frameworks like part digital twin-driven model [5] have been proposed for assembly quality control. Based on the framework, Wang [6] established an effective digital twin-based assembly precision model combining the fusion knowledge (SMS, design knowledge and geometry information) and traditional assembly variation modeling, which improved the accuracy and reliability of the analytical results. In the digital twin-based framework, one of the key aspects is constructing the physical model of assembly object, which strongly demonstrated the potential use of variation modeling. This paper primarily focuses on the generation and propagation of geometrical deviation in aircraft assembly stages, which is a typical use case of general SMS based variation analysis.

In aircraft assembly, the key indexes of geometrical quality primarily consist of gap and step difference in the seam areas, form errors of the skins, coordination deviations of the key joining holes such as the hole concentricity, hole distance, hole position deviation and hole symmetry. The factors affecting the geometrical deviations of a subassembly generally can be classified into three categories as Table **1** shows [7]. And this paper intends to introduce potential solutions incorporating those factors into modelling partially or completely.

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Table 1: Factors Affecting Geometrical Deviations of a Subassembly

Categories	Specifications
Part variation	Part variation, collisions between parts, material properties, gravity
Fixturing	Positions of locators and support points, number of support points, fixture deviation, clamping sequence, clamping forces, clamp size, fixture stiffness
Joining process	Sequence, position and number of joining points, joining tool variation, joining force

Many of the researches related to variation modeling of aircraft assembly focus on drilling [8-10] and measurement [11-13] process. Usually, the discrete deviation transfer model is used to predict the displacements of target points [14, 15]. However, this kind of model isn't appropriate for describing the deviation of continuous features such as the aerodynamic shape of panel skin. Moreover, if only the connection and measurement stages being considered, it is impossible to determine the effects of various deviation sources on the final product after multi-process assembly, which results in the absence of a theoretical framework for such process optimization strategies as deviation coordination and feedback compensation. Therefore, it is necessary to analyze assembly variation in the whole assembly process, simultaneously considering ingaitial part deviation, assembly deformation, and deviation accumulation and propagation.

This paper intends to systematically review recent advancements of aircraft variation modeling in methodological and practical contributions for assembly process analysis so that potential solutions and applications for the assembly of new aircraft can be clarified. Section 2 introduces the general employment of digital technologies in aircraft assembly, pointing out the difficulties challenging the precise descriptions of the assembly variation. The main focus is on Sections 3, discussing the appealing methodological improvements in aircraft variation modeling, including the disadvantages and problems to be solved remaining in the existing researches. The specific methods available can be divided into three categories according to different strategies of deformation analysis, namely, linear theories based on the superposition of discrete deviations and nonlinear deformation analysis. Section 4 focuses on the deviation control methodologies available in practical aircraft assemblies. Based on former discussions, some conclusions and recommendations for future research are presented in Section 5 and Section 6.

2. AIRCRAFT DIGITAL ASSEMBLY

2.1. The Deployment of Digital Aircraft Assembly

Demands for aircraft with better performance, higher reliability, robustness, lower cost, and risk are ever-increasing. To improve competitiveness, aircraft manufacturers have abandoned the traditional manual assembly method, which is difficult to ensure product quality stability and assembly efficiency. Instead, the strategy of digital assembly based on 3D digital models is widely adopted. For example, Boeing reached an average production of 52 B737 per month in 2018 using digital assembly systems [16].

Typically, each stage of the aircraft assembly needs to go through several steps: measurement, positioning, drilling, and connection. Correspondently, large scale metrology, numerical control locating technology, and automatic drilling and riveting technology are regularly used to ensure efficient and stable assembly. Furthermore, the digital coordination technology is proposed to control the geometrical quality throughout the assembly process, whose basis is this review's topic – modeling of the assembly variation.

Consistent with design intentions, aircraft structures can be decomposed into large sub-assemblies (such as fuselage, wing), sections (such as fuselage sections and wing-boxes), components (such as fuselage panels), and parts (such as stringers and frames). In contrast to the decomposition, the aircraft's assembly is conducted in a bottom-up way with tolerance and other constraints. Firstly, parts are assembled into components, then parts and components are assembled into sections. Further, section units are built into large sub-assemblies, and sub-assemblies can be finally completed into a whole aircraft. The step-by-step assembly process of the aircraft is shown in Figure **1**.

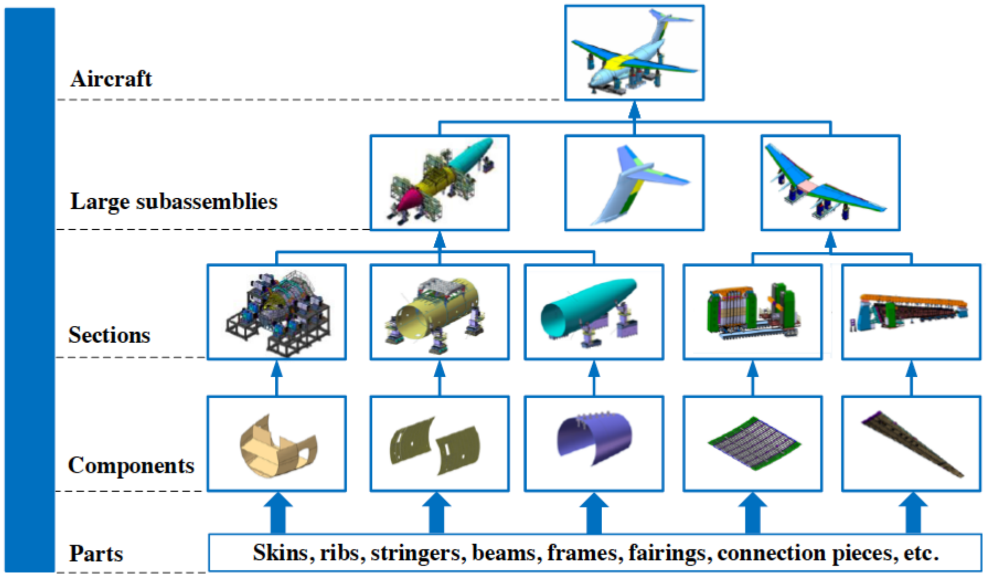

Figure 1: Multilevel assemblies of aircraft.

2.2. Challenges Posed on Variation Modeling of Aircraft

Due to the numerous parts involved and their characteristic of weak rigidity, assembly deformation is easy to occur during the positioning, clamping and joining processes, which can cause surface waviness of the skins, as well as the gaps and flushes between adjacent components, directly affecting the aerodynamics of the aircraft. Furthermore, the additional variation sources originate from the transmissions between different stages also pose difficulties on the modeling of variations throughout the assembly processes. If assembly deviations accumulate from lower levels, it will be hard to be corrected or compensated in subsequent assembly processes. The residual stress will accumulate inside the structures, which may cause local damages and significantly affect flight safety.

3. ADVANCES IN VARIATION MODELING OF AIRCRAFT ASSEMBLY

As aforementioned, aircraft assembly is complicated. It is susceptible to manufacturing deviation, positioning error, gravitational deformation, connection force, and environmental effects. To study the generation and transmission mechanism of aircraft assembly variation, parts' key geometrical characteristics should be first mathematically described. Sources of deviations such as fixture error, assembly force, and external load during the assembly need to be analyzed and determined. When the relationship between the input deviation and the variation of product characteristics is established, an assembly deviation transfer model representing the displacements of target points deviating from their theoretical positions can be constructed.

Generally speaking, the modeling of assembly variation dates back to the 1990s. The concept of key characteristic (KC)[17] was proposed to represent the product's process characteristics. And kinematics [15] was introduced into the modeling process to describe deviations' transmission. Henceforth, various modeling strategies have been proposed to adapt to different situations, such as the state-space model [18, 19], stream-of-variation analysis [20-22] and vector loop model [23-25], etc. The concept of stream-of-variation was proposed to analyze the flow of assembly variation comprehensively. The mathematic state-space model regards the discrete assembly stations as state variables and is universally appropriate for the simulation of deviation transmission among multi-stations. And vector loop model represents assembly variation with position vector changes, which is effective in rigid body simulation. Those methods were designed regardless of assembly objects and could be applied universally to different kinds of assembly.

To illustrate the state of the art clearly, this review focuses on the challenge of deformation description of aircraft components and divided the researches into three categories according to different strategies of deformation analysis, namely, linear theories based on the superposition of discrete deviations and nonlinear deformation analysis. Specifically, the method of influence coefficients (MIC), interpolation method based on measurement, statistical numerical simulations, experimental and finite element based virtual assembly, simplified models based on repeated substructures, and analytical models based on beam-shell mechanics have been exploited in aircraft assembly modeling. Particularly, the illustrations of deformation generated in the joining process are introduced for the integration of modeling solutions. And this review primarily focuses on the application of these typical strategies in aircraft assembly, as well as their pros and cons. The overall literature is illustrated in Figure **2**.

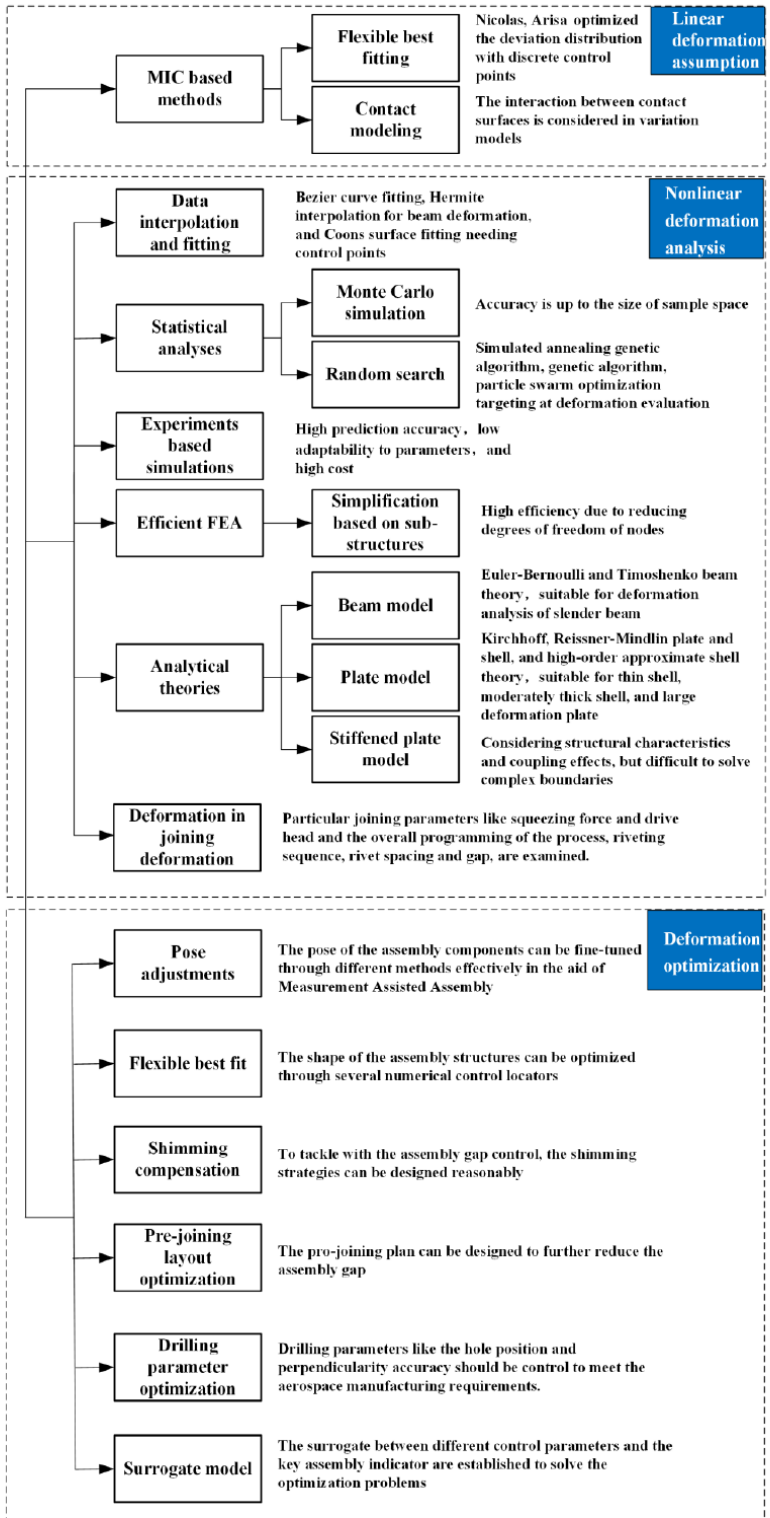

Figure 2: Classification of the overall literature.

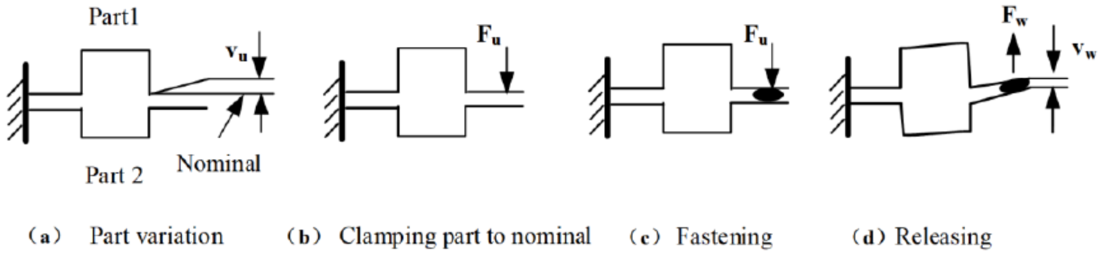

Figure 3: Typical processes for variation modeling using MIC [26].

3.1. Strategies Based on the Method of Influence of Coefficients

3.1.1. Method of Influence Coefficients

Unlike the rigid body assumption and considering the efficiency of simulation, the Method of Influence Coefficients (MIC) was proposed by [26] based on the linear deformation theory. The sensitivity matrix was constructed by separately exerting unit displacements to discrete control points in the finite element model and obtaining the displacement at the product's characteristic points. After analyzing the process of PCFR (Positioning, Clamping, Fastening, and Releasing), as shown in Figure **3**, the variation transfer model of sheet-metal assembly is established based on the sensitivity matrix. Though this model has extensive applications in sheet-metal or other thin-walled structure assemblies, it does not consider the contact between workpieces. It is not suitable for analyzing nonlinear deformation.

As composite structures, especially CFRP (Carbon Fiber Reinforced Plastic) structures, play an increasingly important role in aircraft manufacturing, which is formed through generic additive manufacturing, typically including lay-up, curing, and de-moulding processes [27]. Their mechanical properties are quite different from metals. And it is necessary to study the performance of CFRP in the assembly stages.

Recently, some new progress has been achieved in the assembly variation modeling of CFRP structures. Based on the linear relationship between key control points and global deformation, Nicolas [28] realized the deformation control of the CFRP structures and partially compensated the original geometric deviation. Similarly, Arista [29] proposed the idea of the flexible best fitting, taking the gap between the skins as the optimization goal. The compliant assembly of the fuselage section panels was completed by controlling the slight deformation of the panels. The specific scheme of the system configuration and the distribution of control points are shown in Figure **4** and Figure **5**. Ramirez [30] designed a flexible positioning tool suitable for composite panel assembly realizing multi-point control of the panel shape based on the MIC method.

Focusing on the curing process, Dong [31, 32] established a variation model based on regression analysis and combined thermal stress analysis to simulate the deviation in the curing process, which provided an active control for manufacturing variation optimization. Similarly, Jareteg [33] considered the difference of thermal expansion coefficients between the composite matrix and reinforced fibres and used the MIC method to model the spring-back deformation during the curing process.

Figure 4: Fuselage section assembly station of A350 XWB [29].

Figure 5: Distribution of control points for the fuselage panel [29].

The method of coefficients is first proposed and exploited in the area of sheet-metal assembly, especially in the compliant assembly of automobiles. Typically, this method is suitable for high flexibility objects like the auto-bodies and skin panels of aircraft fuselage sections. This method's basis is small deformation assumption and linear superposition principle regardless of the mechanical properties of assembly objects. This limitation is not noteworthy if only metal structures are involved. However, for the assembly of composite structures, the stress distribution and assembly damage tolerance are of great concern, which means MIC simplification is not appropriate.

To further improve the accuracy of nonrigid variation simulation of thin-walled structures, some researchers considered contact interaction [34-41] and local surface deformation [42] to avoid adjacent part penetrating each other, which is neglected in the conduction of original MIC. Although there are no direct applications of these improved aircraft assembly methods, they indicate that it's of a great necessity to consider the contact deformation in future research.

Moreover, the local deformation calculation becomes complicated once composite materials get involved due to their anisotropy characteristic. Zhang [43] considered the contact effects on dynamic responses of composite structures to tackle this problem. They focused on the small scale and simplified the rough surface as isotropic – *i.e.*, only resin involved. However, the isotropy assumption of the contact surface is not appropriate for the composite structure assembly process simulation.

To sum up, an overview of the methods based on MIC is shown in Table **2**

3.2. Modeling Based on Numeric Methods

3.2.1. Interpolation and Fitting Based on Measured Data

In aircraft shop floors, it's usually hard or high costs to acquire sufficient geometrical data, making the variation analysis more challenging.

To study the influence of the pre-connection on the internal clearance between the panel skin and the skeleton, Qu[44], [45] regarded the stiffness at the connection points as the superposition of the stiffness of each part $k^{-1} = k_1^{-1} + k_2^{-1}$ (see Figure **6**). The variable stiffness matrix was constructed based on the MIC method to obtain the changes of normal gaps in different pre-connection schemes. Significantly, the initial gap used for optimization was acquired by the Bezier curve fitting, which was demonstrated to be an effective strategy to represent the practical deformation. Other curve fitting methods such as Taylor expansion, Hermite polynomial [46], and Chebyshev polynomial [47] interpolation are also frequently utilized to describe the deformation of one-dimensional beams. Specifically, Gerstmayr and Shabana [48] used polynomial fitting to represent the two-node flexible beam element's deformation. And the undetermined coefficients in the fitting expressions

Table 2: An Overview of MIC Based Methods

Reference	Method	Description	Remark
[26]	First proposed MIC	Linear superposition of stiffness at each key point	No consideration of material property and surface contact
[28], [29], [30]	Flexible best fitting based on MIC		Manufacturing variation optimization for CFRP composites
[31], [32], [33]	Regression analysis/MIC		Modeling the effects of curing process on assembly variation
[34], [35], [36], [37], [38], [39], [41]	Improved MIC		Surface contact considered
[42]	Improved contact modeling		Local deformation precisely calculated
[43]		The small scale of manufacturing deviation-isotropic contact surface	Designed for contact modeling of CFRP

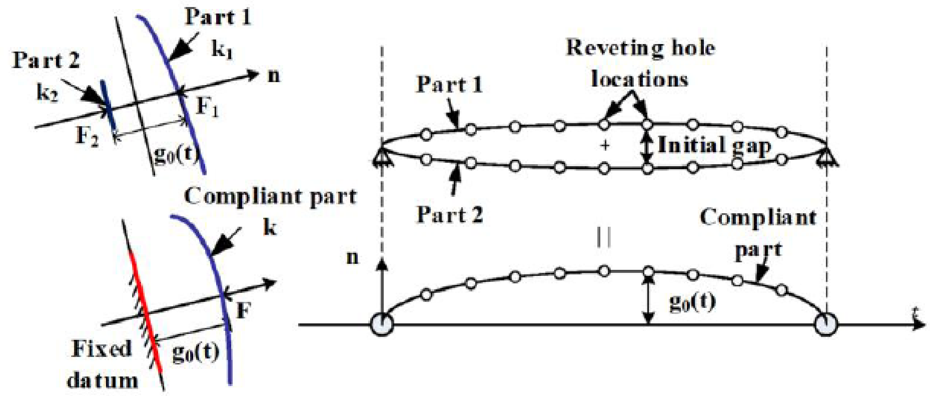

Figure 6: Equivalent clearance model at pre-connected points [44, 45].

were solved according to the nodal displacement and force equilibrium equations.

Similarly, deformation can also be fitted by surface fitting methods. However, it is confined to situations where only small and local surfaces are considered because the amount of control point would be too huge if large-area surfaces are involved. For example, Bi [49] used the Coons surface interpolation method to fit the panel skin's local shape. Based on the fitting result, the surface normal could be calculated, then used to correct the holes' deviations.

In this part, several modeling strategies based on interpolation and fitting theories are briefly introduced, whose characteristics are summarized in Table **3**. It can be seen that these methods are effective to condense the input data while keeping the accuracy of analysis at a pretty high level.

3.2.2. Numerical Simulations

Statistical analysis and process optimization

Apart from the determinate analysis of geometrical quality, there also exist demands for assembly tolerances' statistical design.

Cai [50] combined Taylor series expansion and MCS (Monte Carlo Simulation) to predict flexible panels' assembly dimensions. The surface contact avoiding penetration was considered in the methodology as well. Corrado and Polini [51] used MCS to investigate the effect of the tolerance design of the tail beam's five components on the interface gap among the parts, taking the mutual mechanical interaction into account. The accuracy of the MCS depends on the sample space's size, and it costs increasingly to achieve higher accuracy [52]. Besides, based on probabilistic analyses, the manufacturing process control parameters could be optimized. For example, Abdelal [53] analyzed the panel deformation. They discovered that such parameters as the design of riveting die, riveting forces, distribution of riveting holes, and reinforcement materials could influence the panel's dimension accuracy. And Vichare [54] proposed a general framework for utilizing in-process measurement information. In their case studies, the influence of sequences on assembly quality had also been discussed through simulations.

However, it is often time-consuming or impossible to construct a complete finite element analysis (FEA) model. So Wang [55] used the Orthogonal Experimental Design (OED) method to analyze

Table 3: An Overview of Interpolation and Fitting

Reference	Method	Description	Remark
[44], [45] [46] [47]	Bezier curve fitting Hermite polynomial interpolating Chebyshev polynomial interpolating	Global deformation and initial coordination error can be fitted by discrete input of sample nodes	It turned out to be adequate to represent the practical deformation
[48]	polynomial fitting		
[49]	Coons surface interpolation		

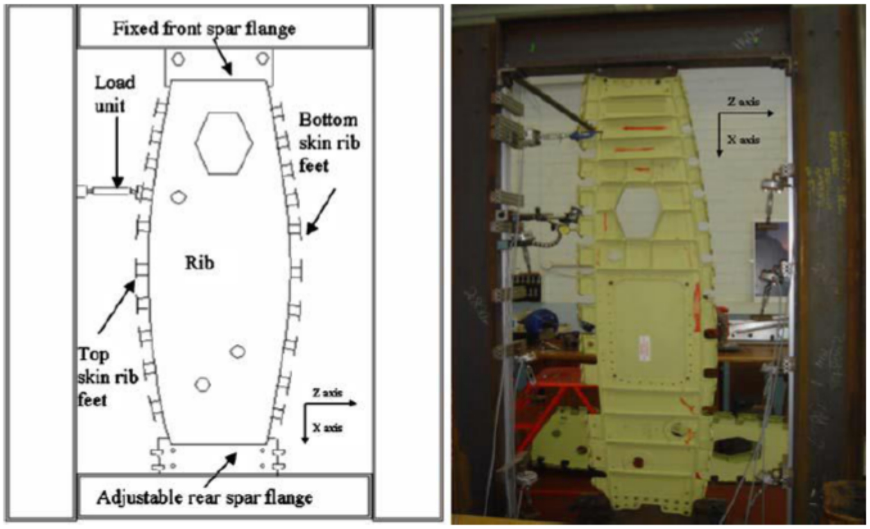

Figure 7: (**a**) Schematic drawing (**b**) The experiment rig [60].

assembly deformation with the combination of modes caused by various deviation sources. However, the number of OED sample points increases exponentially with the problem's dimension, called "the curse of dimensionality." Therefore, Camelio [56] combined the Principal Component Analysis (PCA) and MIC to obtain the input deviation's covariance matrix. The eigenvectors of the covariance matrix represent the deformation modes aroused by the deviations. And the corresponding eigenvalues indicate the contribution that particular mode makes on the whole deformation. The modes of smaller contributions are omitted, which tremendously reduces the calculation cost required to simulate assembly variation.

The methods of random search are frequently employed as well for deformation evaluation and assembly process optimization. To describe the assembly deformation of panels, Bi [57] determined the principles for evaluation point selection based on the adaptive simulated annealing genetic algorithm to minimize the estimation error. Abedini [58] established a position deviation transfer model. They used genetic algorithms to obtain the optimal layout of the 3-2-1 positioning strategy to minimize the drilling holes' position deviation. Liu [59] used particle swarm optimization to determine the distribution of support points in the N-2-1 positioning scheme during the panels' automatic drilling and riveting process.

Experiments based simulations

To simulate accurately, Saadat [60] conducted experiments to simulate the wing-box assembly's physical conditions. The deviations of unmeasured rib feet were estimated based on the measurement data's interpolation by applying tension/pressure to one of the rib feet and measuring other key positions on the rib. The experiment rig is shown in Figure **7**. Combing the experiment results and FEA, they revealed the connection deformation between the wing ribs and the skin.

Simplification based on repeated sub-structures

There are plenty of local features in the FEA model of complex aircraft structures, which significantly affect the convergence and computational cost. Therefore, Lin [61] proposed a simplified sub-structure based variation propagation model given the existence of repeated structures in aircraft panels, which needs no extra FEA runs, unlike the classical MIC based methods. The core of the strategy is to generate other substructures by the transformation of a chosen one, as shown in Figure **8**. This method is suitable for structures containing multiple interchangeable parts. And the prediction efficiency can be improved because of the drastically reduced degrees of freedom of discrete nodes in the model.

As depicted in this part, numerical simulations have been universally applied to modeling the aircraft assembly. Apart from the typical Monte Carlo simulations, the semi-physical simulations combining the experiments and virtual simulation methods like FEA are robust in modeling the variation. The characteristics of these methods are demonstrated in Table **4**.

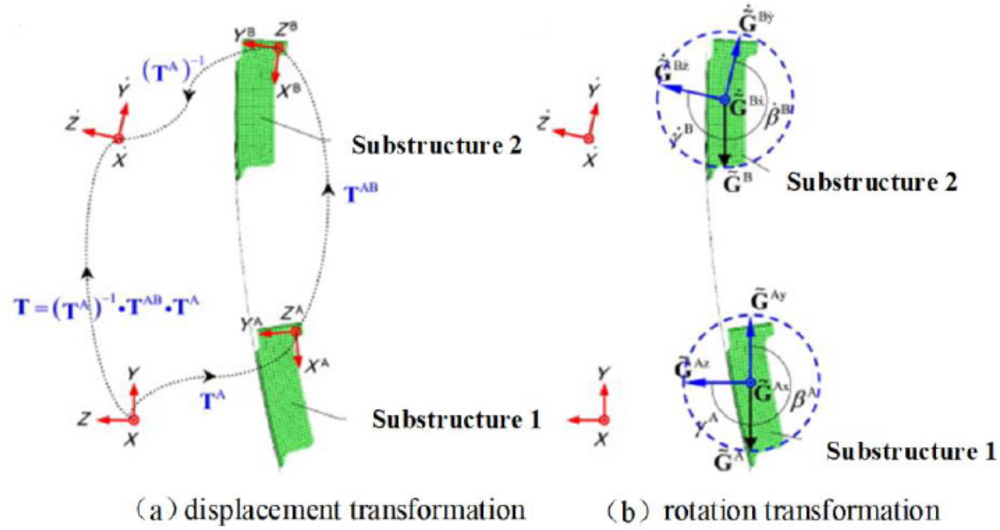

Figure 8: Transformations of displacement and rotation between sub-structures [61].

Table 4: An Overview of Methods Based on Numerical Simulation

Reference	Method	Description	Remark
[50], [51], [52], [53], [54],	MCS simulation	Simulation with samples from global variable space	The accuracy and cost depend on the size of the sample space
[55]	Orthogonal Experimental Design (OED)	Deformation based on FEA	Sample points increase exponentially with the complexity of the problem
[56]	Combining the Principal Component Analysis (PCA) and MIC	Obtaining the covariance matrix of the input deviation and ignoring the modes of a more negligible contribution	Tremendously reduction of the calculation cost comparing to the OED
[57], [58], [59]	The methods of random search	Process optimization based on variation modeling	-
[60]	Semi-physical simulation	FEA combined with experiments	High prediction accuracy，low adaptability to parameters，and high cost
[61]	Simplification of based on repeated sub-structure	Simplifying the FEA model through analyses assembling if repeated sub-structures exist	High efficiency due to reducing degrees of freedom of nodes

3.3. Deformation Models Based on Beam and Plate Theories

Most of the previous nonlinear modeling strategies are FEA-based, making it hard to incorporate process parameters into variation analysis. In other words, they can't support process optimization schemes, such as variation feedback and compensation [62]. Thus, much interest has been attracted to establishing explicit nonlinear variation models for component assembly deformation.

3.3.1. Beam-Based Theory

Beam-like parts such as stringers are critical constituents in fuselage panels. The size of its cross-section is much smaller than the length. Generally, the stringers are modeled with a one-dimensional space curve based on beam theories, including Euler-Bernoulli [63] and Timoshenko beam [64, 65] theory, etc. Whether to consider the transverse shear deformation and the different expressions of the stress-strain relationship [66] are the main differences among those theories.

To deal with the deformation of curved beams with initial curvature, Liu [67] derived the strain energy of the system during stretching and bending of curved beam elements (as shown in Figure **9**) based on Absolute Nodal Coordinate Formulation (ANCF). The Green-Lagrangian strain tensor relaxed the small

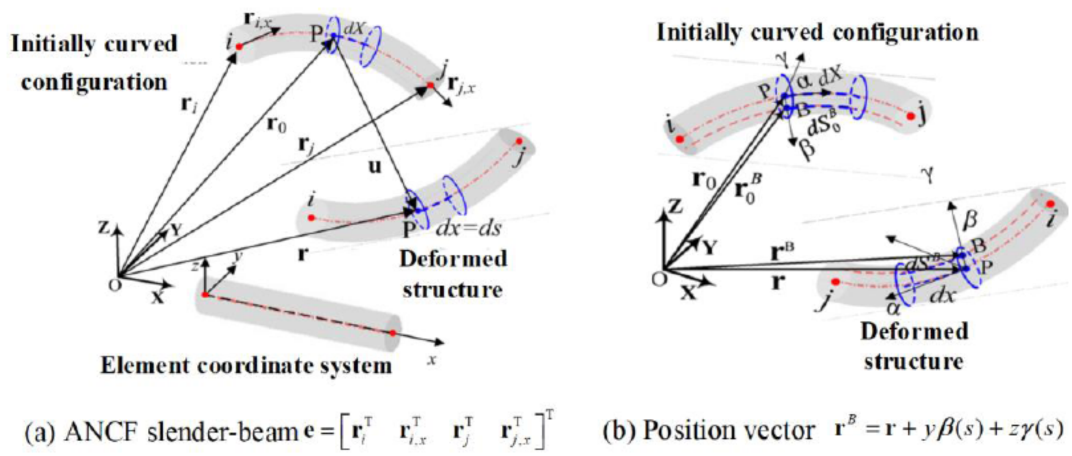

Figure 9: Vector expressions of a curved beam and its cross-section [67].

deformation assumption. However, the traditional ANCF method still has gradient deficiency, which will cause apparent shear locking effects.

Considering the non-coincidence of the shear center and the center of the area, Jafari and Mahjoob[68] derived the precise stiffness matrix of curved beams with nonuniform cross-section. Several kinds of beams with typical cross-sections have been tested to validate the method. Biondi and Caddemi[69] modeled the Euler-Bernoulli beam with singularities caused by concentrated forces and abrupt changes in the cross-section. The unit step function and Dirac function δ were introduced into the static governing equations to derive the expressions of beam deformation.

However, simple beam based theories have limited applications. For more complex structures, the beams need to be constructed into an equivalent system.

Su and Cesnik [70] proposed a simplified model of wing panels with a strain-based geometrically nonlinear beam formulation (shown in Figure **10**). It requires fewer degrees of freedom to model the arbitrary complex beam deformation compared to the displacement-based or mixed form formulations.

For the simulation of fuselage sections, Cheng [71] simplified the complex structure into a system of combined beams (displayed as Figure **11**). They established the relationship between the deviation of the key measurement points and the deviation of the key control points and realized the prediction of

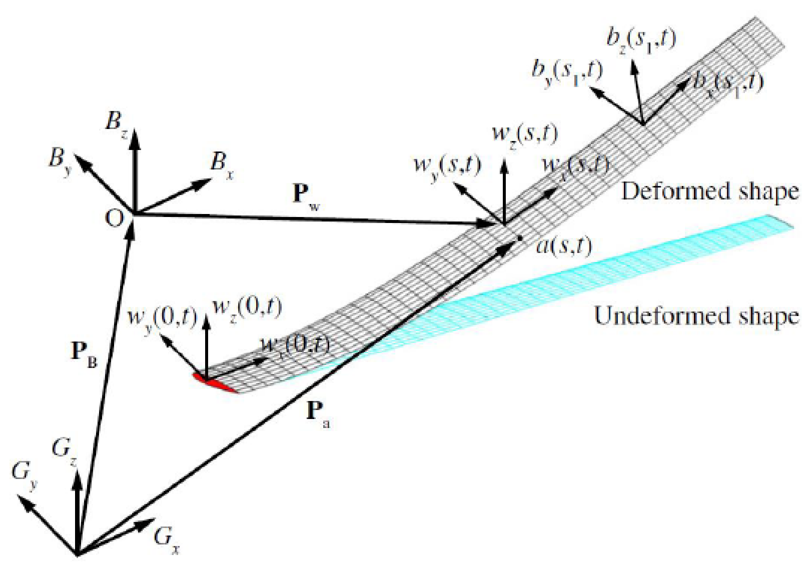

Figure 10: Simplified wing panel structure using geometrically nonlinear beam elements [70].

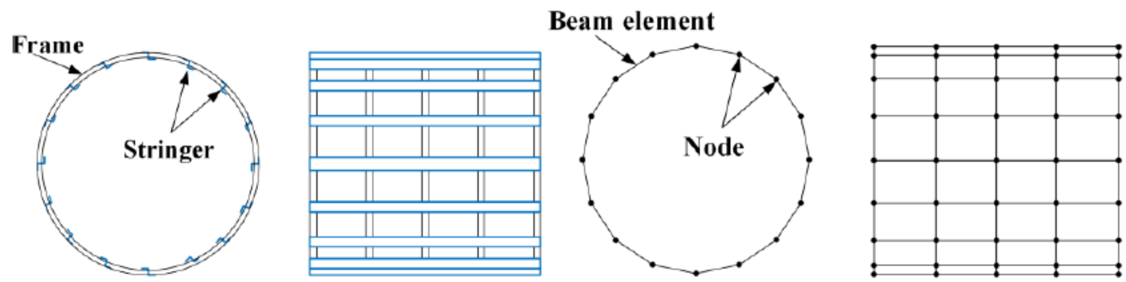

Figure 11: (**a**) Physical model of the fuselage section (**b**) Simplified model of the fuselage section Simplification of the fuselage section [71].

assembly deviation. Further, they constructed a deviation transfer model [72] based on discrete systems, which revealed the essence of deviation transmission between multi stages of aircraft fuselage assembly.

3.3.2. Shell-Based Theories

In panel-like structures, the thickness is much smaller than the curvature radius of the mid-plane so that two-dimensional curved surfaces can be exploited to describe the structures. Different theories such as Kirchhoff shell theory, Reissner-Mindlin plate and shell theory [73], and higher-order approximate shell theory [74] can be utilized to analyze the deformation. For example, based on the Kirchhoff-Love shell theory, Alijani [75] adopted the extended Kantorovich theory to obtain the closed-form solution for the deformation analysis of a cylindrical shell under clamping boundary conditions.

Generally, the deformation analysis based on the shell theory can be concluded as following steps:

Step 1: Establishing a balance equation based on the principle of minimum potential energy or differential elements;

Step 2: Combining the strain-displacement relationship and the constitutive equation to establish a higher-order partial differential governing equations of displacement;

Step 3: Substituting boundary conditions into governing equations to derive the explicit expressions of deformation displacements.

3.3.3. Hybrid Beam-Shell Theory

In addition to the beam theory or shell theory, the hybrid beam-shell theory that both considers the characteristics of beams and shells is also frequently used in the deformation analysis of complex structures.

A hybrid connection model of one-dimensional beam elements, two-dimensional shell elements, and three-dimensional solid elements [76] was proposed to analyze the joined fuselage-wing structure. However, the constructed global stiffness matrix is quite huge, which may cause a high computational cost.

Silva [77] proposed a Wave Finite Element method (WFE) to improve efficiency, which constructs the dynamic stiffness matrix of the periodic structures based on the substructure method, see Figure **12**.

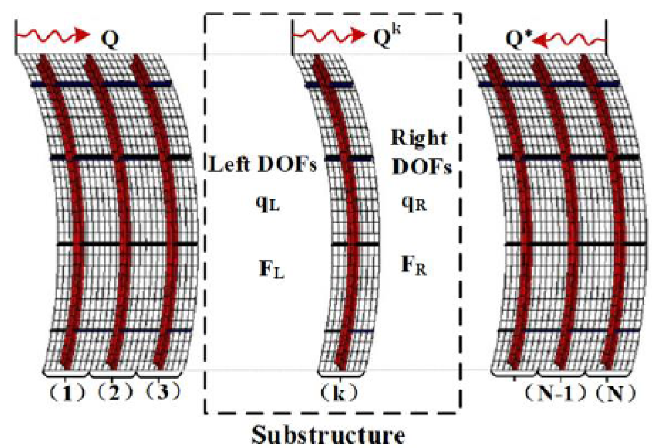

Figure 12: Substructures of periodic structure [77].

However, in those hybrid models, the boundary elements between the panel skin and the reinforcements are considered rigid bodies. The complex composite structure is simplified as an entirety. This strategy cannot reflect the inter-part deformations resulted from discrete connection points, which severely overestimates the connection boundary effect. For example, Pacheco[78] analyzed the aero-elastic deformation of stiffened aircraft panel based on the Mindlin theory and the Timoshenko beam model. It was found that when the overlapping boundary between the ribs and the panel was assumed as the rigid connection (Figure **13**), the amplitude of the deformation was lower than the actual state.

To consider the connection effect between structures properly, Sapountzakis and Mokos [79] set up a separation surface between the beams and the plate. Under the assumption that plate and beam could slip in all directions without separation, the coupling effect between the beam and the plate was introduced into the deformation model as traction forces and continuous boundary conditions. The general solution to the nonlinearly coupled deflection equations of the stiffened plate with an asymmetrical section was obtained based on the boundary element method. The positions of the traction forces in the model are shown in Figure **14**.

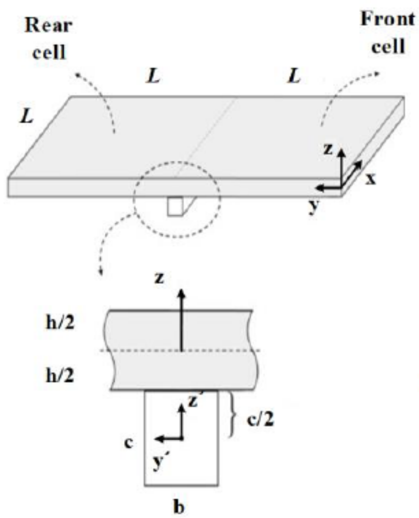

Figure 13: Stiffener and plate joined as rigid bodies [78].

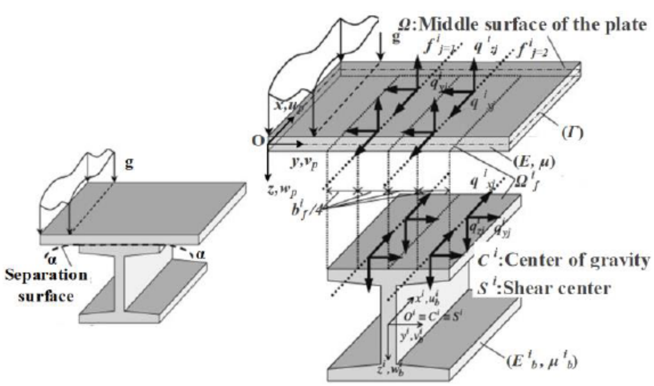

Figure 14: Composite structure of beam-plate considering boundary coupling relationship [79].

3.3.4. Solutions to the Established Models

Apart from the different modeling strategies, solving the models under practical boundary conditions is another challenge.

Wesley [80] employed the Lagrange multiplier method and the double-exponential integration method to solve deformation models of one- and two-dimensional structures. And The element-free Galerkin method [81] was adopted by Tamijani and Kapaniaused to solve the governing equations for buckling and static analysis of simply supported plates with different stiffener structures. Moreover, Ahmad [82] used the Rayleigh-Ritz method to solve the high-order governing differential equations of stiffened and freely vibrating plates, separately under the conditions of fully clamped, free, and cantilevered support.

Besides, for the boundary value problem of the deformation governing Partial Differential Equations (PDE), the partial fraction expansion method , Fourier transform [83], and extended Kantorovich method [84] can also be exploited.

The above solutions to the boundary value problems can be classified into two categories:

1) Assume that the displacement solution is of a particular form (like the Fourier series expansion) satisfying part of the boundary conditions. And then substitute the displacement solution containing unknown coefficients into the governing PDE to eliminate trigonometric terms. By solving the acquired ordinary differential or linear equations, the general form of the deformation solution is obtained, and a set of algebraic equations with unknown constant coefficients can be generated according to the remaining boundary conditions;

2) Employing the method of separating variables, the displacement function is expressed as a product of multi-variates, substituted into the governing equation for gradual decoupling. Then the boundary conditions are introduced to reduce the order of a single variable involved equation, thereby obtaining an explicit expression of the deformation.

However, unlike the above ideal boundary conditions (free, simply supported, and fully clamped boundaries), the positioning and clamping edges existing in practical aircraft assembly can be arbitrary forms, making the problem more complicated.

To embrace the challenge, Wang [85] proposed a theoretical model for predicting stringer deformation resulting from the positioning and clamping process based on extended Euler–Bernoulli beam theory. Innovatively, the transformation between the ideal one-dimensional feature and the three-dimensional entity

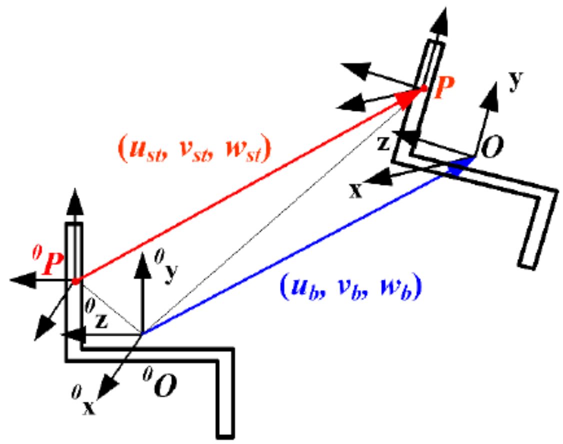

Figure 15: Coordinate transformation of anchor points on the same cross-section of stringer [85].

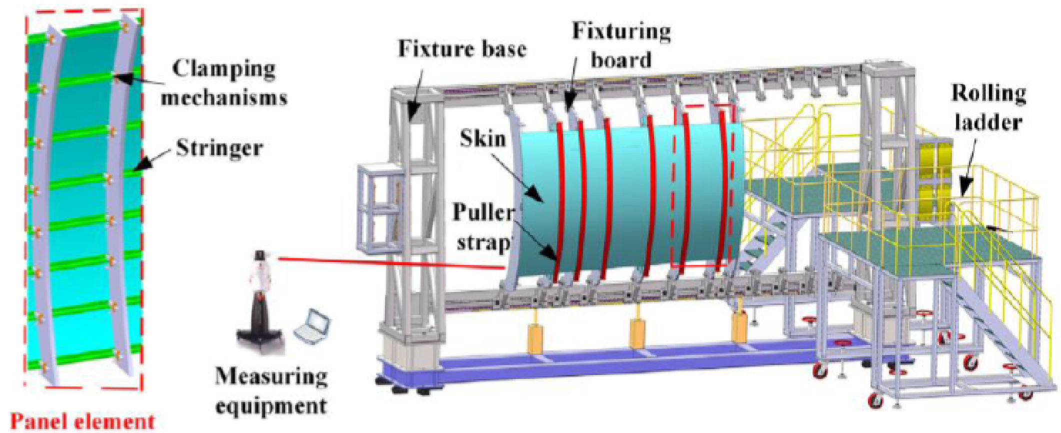

Figure 16: Assembly system of the fuselage panel [86].

has been introduced (see Figure **15**). An explicit deformation expression of the three-dimensional stringer can be obtained. Furthermore, they established an analytical model describing an aircraft thin-walled panel (see Figure **16**) under positioning variation and clamping force [86]. After combining the separate part variation models with process analysis, the effects of connection coupling between stringer and skin were integrated into the variation modeling [87].

According to the characteristics of aircraft components, several analytical theories, summarized as Table **5** shows, have been proposed to model the aircraft assembly, which can easily incorporate process parameters into consideration compared to the FEA based methods.

3.4. Deformation Models Involved in Joining Process

Above studies focus mainly on the manufacturing deviation, deformation under complex boundary conditions and the deviation transfer between assembly processes. However, the drilling and joining processes are simplified and the caused deformation is neglected. Comparing to joining process, the deformation caused by the drilling process is small enough to be neglected. Therefore, this section primarily introduces the modeling of deformation caused by the joining process.

Riveting is the main joining technology in the assembles of aeronautical thin-walled structures, such as fuselage panels and wing panels. For riveting quality control, Muller [88] investigated the influence of rivet squeezing force on driven head dimension and the rivet/hole interference for the joining of aluminum alloy and composite structures, and the effect of squeeze force on the fatigue life of structures was analyzed as well.

Similarly, De Rijck [89] studied the relationship between the rivet squeezing force and the driven head

Table 5: An Overview of Models Based on Beam and Shell Theories

Reference	Method	Description	Remark
[63] [64], [65] [66], [67], [68], [69],	Euler-Bernoulli, Timoshenko and other beam theories	-	Beam-like parts such as stringers
[70],		Strain-based geometrically nonlinear beam	a simplified model of wing panels
[71]		Simplifying the fuselage sections into composite beam structure	-
[85]		extended Euler–Bernoulli beam theory	the transformation between the ideal one-dimensional feature and the three-dimensional entity has been introduced
[73], [74], [75]	Kirchhoff shell theory Reissner-Mindlin plate and shell theory higher-order approximate shell theory	-	-
[86]		Deformation of aircraft thin-walled panel under positioning variation and clamping force	Complex boundary conditions considered
[76]	Combination of Beam and shell theory – *i.e.*, the stiffened plate model	Target at describing the joined fuselage-wing structure	The global stiffness matrix is quite huge
[78]		The overlapping boundary is regarded as rigid	The connection boundary effect severely overestimated
[79]		Considering the coupling effect between the beam and the plate	The coupling effect considered but challenging to be solved under complex boundaries
[87]			

dimension. Furthermore, they concluded that the diameter and height of driven head can be used to evaluate the effect of squeeze force on the quality of riveted joints and the fatigue characteristics of joints. Further, Atre and Johnson [90] studied the effects of sealants and adhesives on the induced stress resulting from the riveting process, which revealed the behavior of sealant on the structure fatigue damage under different conditions. And the influence of squeeze force, rivet geometry and installation flushness on the residual stress were studied in detail [91]. Rans [91] analyzed the four selected factors affecting the joint quality, including squeezing deformation level, interference pin insertion, the material property and sheet package thickness, thorough comparison experiments. Cheraghi [92] constructed a FEA model to investigate the effect of process parameters such as squeeze force, rivet length, rivet diameter, hole diameter tolerance and countersink depth on the joint quality.

In riveting deformation control, da Cunha [93] proposed a methodology to capture residual strains in riveted aerospace structures. First, a single rivet deformation model was established using the finite element method, collecting the displacement at the edge of the rivet hole after riveting process. Then, combining the single rivet deformation and linear extrapolation model, the deformation of a row rivets after sequential riveting could be obtained. The predicted results are close to the experimental results, and the method can be extended to adapt the assembly process of larger aircraft panels. Cheng [94] established a variation model covering both the pre-joining stage and joining stage, incorporating the joining process into the whole process modeling. The total eight sub-processes are presented in Figure **17**.

Aman [95] investigated the effect of riveting sequence, rivet spacing and gap between connected parts on riveting quality. Cheng [96] assumed that the single rivet deformation was caused by internal pressure in the rivet hole. And a riveting mechanics equivalent unit was constructed based on the principal stress and the thick-walled cylinder theory. Then, an optimal riveting sequence for the horizontal stabilizer was obtained from the simplified simulation. Masters [97], based on the local/global approach mapping simulation method, evaluated the effect of the localized distortion on the overall assembly deformation formed during the installation of individual rivets. Chang[98] presented a successive calculation model to simulate the panel deformation after multiple rivets assembled.

The riveting deformation of panel is determined by a large number of riveted local deformations under

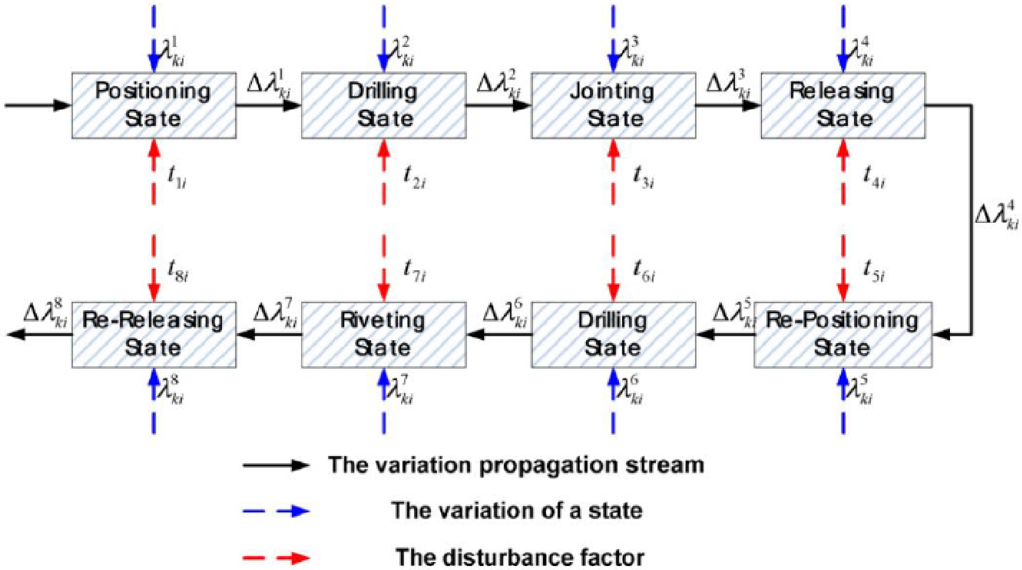

Figure 17: The variation propagation in multi-state riveting [94].

certain riveting sequence, which meant the numerical simulation of the discrete riveting process was computationally intensive. Therefore, it is necessary to improve the computational efficiency with high accuracy in balance through reasonable equivalent assumptions. The highly nonlinear rivet deformation was equivalent to axial pressure combined with radial pressure exerted on the riveted panel [99], which significantly improved the computational efficiency of the panel deformation. Chang [100] proposed that the local deformation of riveting holes be obtained using 3D solid model, whose deformation then was integrated into the simple shell model of the panel and stringers. It turned out that computation time could be significantly reduced comparing to the full 3D FEA model.

4. DEVIATION CONTROL BASED ON VARIATION MODELLING

Those variation modeling strategies are developed to adapt different hierarchies of aircraft assembly. After the reconstruction of the deviation, different controlling methodologies should be exploited to realize the geometrical assurance. In traditional aircraft assembly, the complex conformal fixtures are universally utilized to guarantee the geometrical quality, which is of high cost both in economy and efficiency. As the process automation is strengthened in arcraft industries, new facilities are exploited and the corresponding deviation control methods are upgraded either.

At present, most of the researches focus on the stages of component assembly and section assembly, and the strategies involved are listed as following.

4.1. Coordination and Deviation Optimization Based on Pose Adjustments

Assisted with the advanced concept of MMA (Measurement Assisted Assembly), the poses of the aircraft parts can be determined by the registration between the measurement data of key characteristic points and their nominal values.

Traditionally, the least squares model and some classical non-iterative algorithms can be exploited to solve the pose registration problem, such as the singular value decomposition (SVD) algorithm [101], the unit quaternion method [102], the double quaternion method [103] and the linear subspace method [104]. Eggert [105] compared the accuracy and stability of these four algorithms.

Recently, some other pose evaluation methods have been proposed, which consider some engineering constraints on the basis of the least squares model. Yu [106] while solving the general point matching problem, incorporate the constraints of colinearity, coplanarity, and symmetry before and after the transformation. A similar approach was proposed by Yuan [107] and applied to the joining assembly of the aircraft fuselage sections. Since the data obtained by the measurement contains certain errors, it will affect the quality of the matching. With this in consideration, Wang [108] introduced a thermal deformation factor in the assembly of large aircraft components to compensate for the matching errors caused by temperature changes. Additionally, laser tracking requires a transfer between stations during the measurement process, thus the thermal deformation of the reference points would cause certain errors. Yu [109] proposes a compensation method for transferring station errors based on the thermal deformation of components. Besides, Wang [110] proposed a method taking the accumulation of concentricity deviations as an evaluation criterion while assembling the cylindrical components. By adjusting the rotation angle about the center axis of each cylindrical component, the final cumulative coaxial error is minimized. Similarly, Wang [111] developed a two steps alignment method to adjust the components for controlling the gaps between the skeleton structure and skins, as shown in Figure **18**.

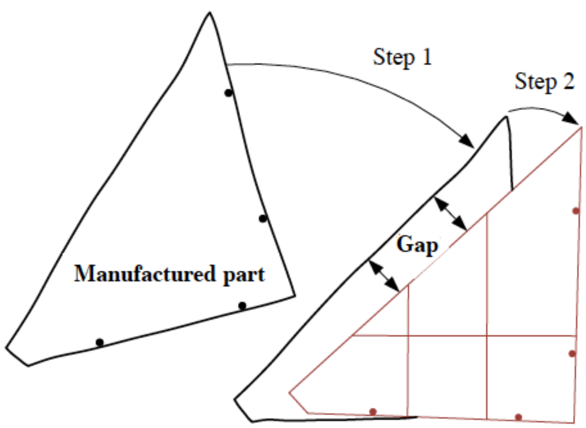

Figure 18: Step alignment of the panel to skeleton frame [111].

4.2. Flexible Best Fit for Shape Control

In the postioning process, Nicolas [28] partially compensated the original geometric deviation by controlling the deformation of the assembled object. Similarly, Arista and Falgarone [29] proposed a flexible best-fit method to optimize the gap between panel skins and completed the compliant assembly of the fuselage section's panels by controlling the small deformation of the separate panels. Ramirez and Wollnack [30] proposed a flexible positioning tooling, as shown in Figure **19**, applicable to the assembly of composite panels to achieve multi-point control of the panel shape. Airbus [112] designed an automated wing box assembly system using industrial robots to perform such process operations as positioning and joining of wing ribs and skins.

Figure 19: Holding fixtures for CFRP-structures [30].

Because the aircraft components are usually positioned by several numerical control locators distributed discretely in space, the deformations of large fuselage or wing panels are unavoidable due to its weak-stiffness and low-rigidity. Bi [113] obtained the relationship between the displacement of the control point, *i.e*, the displacements of numerical control positioner, and the position error of the detection points through orthogonal simulation when studying the panel's deformation and correction. After several iterations of correction, the distribution of panel deviation had been improved obviously.

4.3. Shimming Compensation

Shimming is the main process compensation method to achieve the assembly gap control. Different kinds of shims are used to fill the gaps, so that the assembly joint surface can fit together, thus improving the structural integrity and load-transmitting capacity. The shim used in aircraft assembly gap compensation generally include liquid resin shims, metal shims and composite shims. Smith [114] designed a prototype system for liquid shim compensation, using a specially designed toling to squeeze out the excess liquid. The system was used in the F35 fuselage assembly, as shown in Figure **20**.

For controlling the gap between the composite panel and metal frames, Wang [115, 116] first optimize the gap distribution between skeleton frame and panel skin by tansforming the pose of the composite panel, and then a reasonable shim scheme could be designed based on the residual gap values. Moreover, Cheng, on the other hand, conducted an experimental and simulation study on the mechanical properties of the overall structure after shimming compensation for the wing panels. Particularly, the influence of shimming

Figure 20: The shimming scheme of F35 forward fuselage [114].

strategies, including shim materials and shimming percentages, etc, on the structural strength of composite components has been studied through comparison experiments [117].

4.4. Layout Optimization in Pre-joining Process

Apart from the researches on riveting process mentioned in section 3.4, the scheme of pre-joining also affect the joining deformation. Zaitseva [118] developed a special program to simulate the riveting process during the docking of the A350 outer wing box and the central wing box. Combined with the MIC method, the distribution and number of rivets were designed in the pre-connection stage so that the gap distribution is optimized after pre-connection. Then, the optimized results were imported into the finite element software and the stress distribution of the assembled object was obtained by the post-processing. This method was also extended to be applied in the assembly of other fuselage structures of the A350 and A320 [119]. A similar approach was introduced in optimizing the gap distribution between the stringer and skin in pre-connection stage for an aircraft fuselage panel [45].

4.5. Parameter Optimization in Drilling Process

As known, the deformation caused by drilling process can be neglected comparing to the joining process. But the actual shape of aircraft structures can deviate from the nominal values due to the accumulated assembly deviation and the position and normal direction of drilling holes can influence the position and direction of joining forces. Therefore, it is necessary to control the position and normal of the joining holes except from the specific machining parameters.

Dong [120] used the feedback of the grating ruler to control the hole position and perpendicularity accuracy in order to meet the aerospace manufacturing requirements. Innovatively, Zhu [121] constructed a bilinear interpotation surface in the drilling area using the reference holes' position to achieve linear interpolation compensation. The shematic diagram of the interpolation is presented in Figure **21**. Similarly, Dong [122] used a binomial curve boundary to fit the ideal local surface and constructed the Coons error function, which had a higher compensation accuracy for hole positions comparing to the bilinear interpolation method. Furthermore, in order to accurately represent the cubic surface at the joining area of the fuselage sections, Bi [49] introduced surface normal vector into the hole position error compensation model. In addition, Wang [123] considered the edge distance between the constrained holes and key features such as the skeleton groove. And the positions of the remaining holes were corrected using Shepard interpolation to meet the requirements for the edge distance while retaining the distribution characteristics of the original holes.

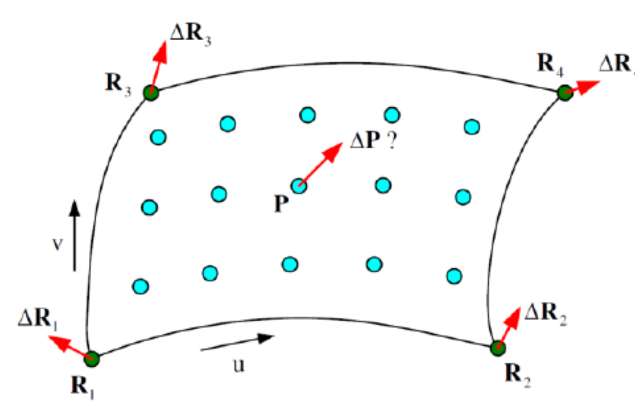

Figure 21: The bilinear interpolation surface with four reference holes [121].

Apart from the hole position compensation, the normal correction techniques in automatic drilling are also essential for the process optimization. Xue and Zhang [124] achieved normal adjustment of end-effector based on a three-point contact displacement sensor. Zhang and Wang [125] proposed a four-point spherical fitting algorithm to estimate the normal vector on the maching surface with the normal vector on a sphere, which simplifies the local drilling area as a sphere. Thus, the method is not applicable to structures with complex shapes. In recent years, non-contact measurement technology has been applied extensively in aircraft manufacturing. Ju [126] adjusted the normal direction by four laser displacement sensors installed at the same height, but this method puts high requirements on the installing accuracy of the sensors, which confined the extension of the method.

4.6. Deviation Control and Coordination Based on Surrogate Modeling

Recently, innovated by the improvement in the artificial intelligence, the data-driven methodologies, especially, the surrogate models are gaining momentum. A surrogate model, also known as a proxy model or emulator, is a simplified mathematical or computational model that approximates the behavior or output of a more complex or computationally expensive model.

Surrogate models aim to capture the essential features and relationships of the original model while reducing the computational effort required for predictions or optimizations. They provide a faster and more efficient alternative by using a smaller number of evaluations or simulations. Surrogate models are commonly used in various fields, including engineering, optimization, and computer simulations.

In aircraft assembly, Yue and Shi [127] used a validated FEM platform [128] as the basis, generated deformation data samples through simulation, and proposed a surrogate model based on the universal Kriging model. They established a relationship model between the deviation of each error source point and the assembly deviation of composite components. By using a feedforward control strategy to optimize and adjust various control nodes, they achieved the optimal distribution and active control of assembly deviations. Furthermore, they considered the uncertainty of the model [129], including the uncertainty of actuators, parts, model, and unquantified uncertainty, to improve the prediction accuracy of the model and the effectiveness of the feedforward control strategy. During the joining of aircraft fuselage sections, the issue of deformation-induced gaps and step differences that severely affect the structural strength of the fuselage was studied by Du [130]. The optimal distribution scheme for actuators' deformation at the end of the fuselage section was investigated. Constraints were imposed on the magnitude of external forces applied by each actuator to avoid damage to the composite components. Due to the sparsity of deformation control problems in aircraft assembly, where the number of control variables significantly exceeds the number of observations, they proposed a sparse sensing model and solved the convex optimization problem using the alternating direction method of multipliers. Additionally, for the minimization of the maximum gap [131], which greatly determines the assembly quality during fuselage section joining, they improved the optimization objective and specific solution method based on this sparse optimization model, significantly reducing the maximum gap.

5. CONCLUSIONS

Variation modeling methods for aircraft assemblies and their advances are reviewed in this paper. Several common modeling approaches are discussed in detail, and their characteristics in different applications are summarized.

The vector-loop model uses a 3D vector to represent the position changes of the assembly objects, suitable for high rigidity objects. MIC based methods presume that the assembly deformation is small enough to be approximated with a linear combination of unit influence on various deviation sources. Interpolation and fitting use a limited number of discrete points to reconstruct the deformation of a particular area, whose accuracy depends on the sampling rate of the key characteristic points. Statistical variation analysis can be conducted with different simulation methods such as MCS and PCA. Their results can directly facilitate process optimization. Experiments and virtual assembly combines the experiment results and FEA to analyze the deformation of a complex structure like an aircraft wing-box, which make them feasible for industries. The sub-structure method was proposed to simplify the simulation model, decreasing the calculation complexity caused by repeated structures to improve efficiency. Finally, deformation models based on different mechanics can provide an explicit expression of nonlinear assembly deformation of panels and components, making it easier to incorporate process parameters into variation analysis.

In addition, the deformation models focus on the joining process, especially riveting technology, have been discussed in detail. Those separately developed methodologies can be combined with above modeling strategies to fulfill the integrity of analyses.

The practical solutions to deviation control have been introduced comprehensively after the modeling illustration, which reflects the primary concerns about aircraft digital assembly and can provide inspirations for the future researches ranging from process improvements to the equipment design.

6. CHALLENGES AND RISKS

Process control is increasingly being introduced into manufacturing to ensure assembly quality (reliability and robustness), and new materials and manufacturing processes are exploited. Some obstacles still need to be overcome in the field of variation analysis and modeling. Based on the authors' investigation, it has been found that the researches on the following aspects are relatively limited and may attract more interest in future research.

There are two main areas, namely, assembly mechanistic models for introduced anisotropic materials and data-driven intelligent assembly methodologies.

1) Skin model shapes combined with contact modeling

Skin model shapes take form deviations into consideration, making it possible to precisely model the surface contact. And local deformation has turned out to have significant influence upon the modeling accuracy [42, 43].

Thus, it is meaningful to consider both global and local deformation resulting from the manufacturing and locating errors. Moreover, composite materials have the characteristic of anisotropy making it more challenging to give the explicit form of deformation under complex assembly conditions

2) Assembly variation modeling considering mechanical performance

From the traditional point of views, the geometric accuracy of the assembly mainly affects the aerodynamic performance of the aircraft, so the accuracy indicators such as clearance, step difference and shape deviation are proposed in the assembly process.

So far, the assembly technology focuses primarily on controlling the geometric deviations. And it is believed that as long as the various types of geometric errors are controlled, the quality assembly can be achieved. For panel deformation control, Bi [113] assumed that the smaller the geometric deviation of panel components, the lower the stress level of the structure, and thus the higher the strength of the structure is realised. Besides, Cheng [132] studied the effect of residual gaps on the tensile and fatigue strength of the metal structures after the control and compensation of assembly gaps.

Particularly, for composite structures, damages like delamination would occur if accumulated internal stress exceeds the limited value. Therefore, the stress state of during the assembly process need to be monitored in the subsequent study so as to establish the direct relationship between the assembly accuracy and the mechanical properties of the structure. Then the strength of the assembly can be quantitatively assessed or predicted.

3) Integration of multi-source heterogeneous data

With multi-metrology introduced on the shop floor, new approaches like data fusion [133] are needed for incorporating such inspection results as point cloud data into modeling input. Moreover, the raw data can't be directly exploited. The information of key manufacturing and coordination features need to be extracted accurately. For example, Manohar [134] designed the optimal measurement solution by learning from historical assembly gap data, which reduced the total amount of measurement data to 3% of the original with the accuracy of the gap prediction as high as 99%.

Moreover, aircraft assembly process is quite complex involving many kinds of equipment. In addition to geometric errors, there are a variety of dynamic and static data. An integration framework suitable for heterogeneous data is needed before the data processing.

4) Development of realistic and real-time simulation model

The essence of variation modeling is to create the physical representatives for assembly objects under specific process parameters, which means it can be integrated into intelligent systems with the increasing digitalization in manufacturing. Digital twin provides a method to link theoretical and physical models and realize the real-time process monitoring and optimal control. By introducing the digital twin into assembly geometry assurance, the root causes of the geometrically related variations can be identified and well managed [4]. Furthermore, multi-source heterogeneous data like loads and operations can be integrated in such digital twins to facilitate the real time assembly monitoring.

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FINDINGS

This study critically reviews methods to model the relationship between the deviation variables and the deviation distribution of assemblies based on process-oriented strategy, considering manufacturing deviation, locating variation, and part deformation in assembly stations. And the practical deviation control methodologies are discussed subsequently. Finally, the future challenges and risks in aircraft variation modeling are briefly introduced.

RESEARCH LIMITATIONS/IMPLICATIONS

The study is limited to deviation modeling and variation propagation problems in aircraft assembly stations to predict geometrical quality and instructions for process improvement.

ORIGINALITY/VALUE

This paper presents a comprehensive review of aircraft variation modeling.

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