

Opposition-Based Learning Equilibrium Optimizer with Application in Mobile Robot Path Planning

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Abstract: The objective of mobile robot path planning (MRPP) is to devise the shortest obstacle-free path for autonomous mobile robots based on a given terrain. Numerous MRPP methods have been extensively researched. This paper presents a novel approach called Opposition-based Learning Equilibrium Optimizer (OEO) for generating smooth paths for mobile robots. The fundamental idea behind OEO is to introduce an opposition-based learning mechanism while maintaining the overall framework of the basic EO algorithm. This modification alleviates the susceptibility of the basic EO algorithm to local optima. The OEO algorithm is employed to provide smooth paths for autonomous mobile robots, and the results are compared with several classical metaheuristic algorithms. Comparative analysis across different environments demonstrates that the proposed OEO-based path planning method consistently yields the shortest and most collision-free paths with superior stability.

Keywords: Equilibrium optimizer, Opposition-based learning, Artificial intelligence algorithm, Robot path planning.

1. INTRODUCTION

With the advent of the AI era, the intelligence and automation level of Autonomous Mobile Robots (AMRs) has witnessed remarkable advancements. AMRs find widespread application in cutting-edge fields like smart homes, intelligent logistics, and self-driving cars [1]. Path planning constitutes a vital component of automated mobile robot systems, tasked with generating feasible, safe, and smooth routes from starting to destination points within known or unknown environments. Intelligent path planning serves as an indispensable tool across various domains, including robot path planning, unmanned combat vehicles (UCVs), vehicle routing problems (VRPs), transportation system navigation, military command systems, cruise missile trajectory planning, unmanned aerial vehicle (UAV) trajectory planning, fire escape, and automated guided vehicles (AGVs) [2]. Given their extensive use, MRPP problems have garnered significant attention from researchers, leading to the application of numerous optimization algorithms to address the challenges. However, as AMR applications continue to expand in scale and diversity, existing path planning methods face constraints arising from complexity and nonlinearity, preventing them from fully meeting all requirements. As an intriguing research hotspot, several studies have focused on improving its effectiveness and efficiency [3].

Traditional MRPP algorithms encompass Best First Search Algorithm [4], Dijkstra's Algorithm [5], A* Algorithm [6], Jump Point Search Algorithm [7], Breadth First Search Algorithm [8], Trace Algorithm [9], Rapidly Exploring Random Trees Algorithm [10], Probabilistic Roadmap Algorithm [11], among others. Nevertheless, Dijkstra's algorithm's inefficiency stems from the need to traverse a large number of nodes and its inability to handle negative edge problems. Best First Search Algorithm bears similarities to the Dijkstra Algorithm but employs a heuristic function to expedite target node guidance.

In recent years, a multitude of bio-inspired algorithms have found widespread use in solving MRPP problems, such as Particle Swarm Optimization (PSO) [12], Genetic Algorithm (GA) [13], Ant Colony Optimization (ACO) [14], Salp Swarm Algorithm [15] and more. Swarm intelligence algorithm combines stochastic algorithms with local search and exhibits impressive performance in addressing highly nonlinear and multimodal optimization problems. Nonetheless, swarm intelligence algorithm faces challenges like local optimization and slow convergence [16]. GA simulates evolutionary principles found in the biological world, displaying potent global optimization capabilities. However, its need for a substantial population and extensive search space may lead to local optimization and slow convergence during the search process [17]. In PSO, particles' flight process serves as the individual search process, with flight speed dynamically adjusted based on the individual's historical optimal position and the population's historical optimal position. PSO is prone to early convergence when tackling complex

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optimization problems [18]. In the realm of MRPP, ACO divides the search space into a grid and leverages state transition probabilities and pheromone updating methods for resolution [19]. Nevertheless, ACO's convergence rate might be slow, necessitating substantial computational efforts. Known for their slow local convergence, Firefly Algorithm (FA) [20-21], Artificial Bee Colony (ABC) [22-23], and Salp Swarm Algorithm (SSA) [24], while GWO demonstrates deficiencies in population diversity and slow late convergence. In the field of MRPP, PSO, ABC, GWO, and FA are commonly employed comparison algorithms. This paper also conducts a comparative analysis of these widely used algorithms in mainstream practice.

EO, originally introduced by Faramarzi in 2019 [25], is a physically based metaheuristic algorithm. It boasts a straightforward framework, requires minimal parameter adjustments, and exhibits superior optimization performance. The algorithm has been successfully applied in diverse fields like feature selection [26], photovoltaic solar parameter extraction [27], medical image segmentation [28], and medical image fusion [29]. As population intelligence algorithms are stochastic optimization techniques, randomness significantly influences the search process. Maintaining an optimal balance between exploration and exploitation is critical for their effectiveness during the search.

Despite EO's excellent performance in function optimization and various real-world applications, certain limitations persist, including poor local search ability and susceptibility to local optima. Moreover, although researchers from different fields have favored EO, its application to MRPP problems remains unexplored.

To address these issues, this paper proposes a path planning algorithm called Opposition-based Learning EO (OEO). By integrating a reverse learning mechanism into the basic EO, OEO overcomes the tendency to converge to local optima. Furthermore, the OEO algorithm is applied to the path planning of AMRs. Comparative tests are conducted to evaluate the OEO-based MRPP method in different environments against several classical metaheuristic algorithms. The results demonstrate that the proposed MRPP method provides shorter obstacle-free paths for AMRs.

The primary contributions of this study are as follows:

(a) The proposal of an enhanced EO algorithm to tackle the limited exploration capabilities of the basic EO.

(b) Application of the improved EO algorithm to offer optimal accessibility paths for AMRs.

(c) Systematic experiments demonstrate that the proposed OEO algorithm outperforms its competitors for the MRPP problem.

The structure of this paper is as follows: Section I presents the background of the research. Section II delves into the relevant literature. Section III provides an introduction to the basic EO. Section IV introduces the MRPP method based on the enhanced EO. Section V compares the proposed method with other algorithms to validate its effectiveness in MRPP. The final Section presents the conclusion.

2. RELATED WORK

MRPP commonly employs traditional algorithms such as the A-star algorithm [30], artificial potential field approach [31], and neural networks [32]. However, these algorithms often struggle with poor convergence performance when dealing with complex environments. The A-star algorithm's reliance on prior environmental knowledge hampers its efficiency in large-scale and intricate conditions. Artificial potential field methods tend to stagnate and suffer from local optimization issues amid obstacles in complex environments. On the other hand, neural networks possess real-time environmental awareness, rendering them suitable for dynamic scenes. Yet, in complex obstacle-laden settings, the computational burden of processing large-scale networks becomes a major challenge. Although traditional MRPP algorithms excel in specific conditions, they fail to adapt and converge swiftly in complex environments, lacking the required adaptability.

To address the MRPP problem, it can be transformed into an optimization problem with the search for the optimal path represented as an objective function. Swarm intelligence algorithms have become popular for solving discontinuous, non-smooth, and discrete variable problems, thanks to their strong stochastic nature, lack of gradient information, and absence of a priori knowledge of the optimization problem. In recent years, an increasing number of population intelligence algorithms have been applied to MRPP problems. Researchers continuously enhance

and explore algorithms with greater planning efficiency, optimization power, and robustness.

For example, in [33], Wang *et al.* proposed an improved MRPP method based on the modified SSA algorithm. Firstly, they enhanced the adaptability of the basic SSA by incorporating a dynamic learning mechanism. Secondly, an improved orthogonal counter-learning strategy was employed to enhance the algorithm's capability to escape local optima. The developed improved SSA algorithm was applied to solve the MRPP problem. The performance of the proposed method was tested in several different environments and compared with some classical metaheuristic algorithms. The experimental results demonstrated that the MRPP method based on the improved SSA outperformed its competitors in terms of path length and obstacle avoidance.

In [34], another MRPP method based on SSA was introduced. They combined orthogonal experimental design with quasi-opposition learning to form an orthogonal opposition learning mechanism. By integrating this mechanism into the basic SSA, the algorithm's population diversity was enriched, thus improving its ability to escape local optima. The designed improved SSA algorithm was applied to MRPP and its performance was tested in three terrains with different characteristics. The results indicated that the SSA-based MRPP method developed in their study could generate shorter obstacle-free paths for AMRs compared to classical metaheuristic algorithms.

In [35], a novel MRPP method based on metaheuristic algorithms was proposed. Firstly, they defined the concept of velocity based on particle movement. Secondly, a velocity clamping mechanism was designed and introduced into SSA to help particles explore the solution space adequately. Additionally, a decay factor was incorporated into the basic SSA to enhance the algorithm's convergence performance. Finally, an adaptive mechanism was embedded in the particle's position movement process to help maintain a balance between exploration and exploitation. The enhanced SSA algorithm introduced in their study was applied to MRPP, and the results were compared with several classical swarm intelligence algorithms. Based on the comparison results in various terrains, the SSA-based MRPP method developed in their study demonstrated better scalability and robustness.

In [36], an ABC algorithm based on fractional-order calculus (FOABC) was introduced to overcome the

drawback of insufficient convergence accuracy in the basic ABC algorithm. To evaluate the performance of the FOABC algorithm, it was used to solve the MRPP problem. The comparative results with various classical MRPP methods demonstrated the remarkable performance of FOABC in MRPP problems.

In [37], a hybrid algorithm based on CS and SCA was proposed. This mixed method effectively retained the advantages of each respective algorithm and compensated for their shortcomings. The proposed hybrid algorithm was applied to the MRPP problem, and its superior performance was observed in terms of path length, turning frequency, collision avoidance, and path smoothness. The method proved to be effective for multi-robot cooperative systems.

The aforementioned studies present various MRPP methods and their applications, demonstrating their potential to address the challenges of path planning for AMRs in complex environments. These methods showcase improvements in convergence, adaptability, and robustness, making them valuable contributions to the field of robotics and swarm intelligence-based optimization techniques.

Although swarm intelligence algorithms have been widely applied to MRPP problems and have demonstrated satisfactory results due to their stochastic nature and excellent optimization characteristics, no researchers have explored the performance of the EO algorithm on MRPP problems since its proposal. Since its inception, the EO algorithm has proven its effectiveness in various real-world problems. Based on empirical evidence, the EO algorithm is expected to perform well in the context of MRPP problems. In this study, we make the first attempt to apply the EO algorithm to MRPP problems. To validate its performance, the EO-based MRPP method is compared with several state-of-the-art metaheuristic methods in various environments.

The EO algorithm has shown promising results in various real-world applications, indicating its potential for addressing complex optimization problems. Considering its capabilities, we investigate the application of the EO algorithm to the MRPP problem for the first time. In order to assess its performance, we conduct comparative analyses between the EO-based MRPP method and several cutting-edge metaheuristic approaches in diverse environments.

This study marks the first exploration of the EO algorithm in the context of MRPP problems, and the

results obtained from this research can provide valuable insights into the algorithm's adaptability and efficiency in solving path planning challenges for AMRs. The findings may contribute to the advancement of swarm intelligence-based optimization techniques and their application in the field of robotics.

3. OVERVIEW OF THE EO

The standard EO starts its search process by randomly initializing a set of particles in the solution space. The mathematical model for this phase is as follows:

$$C_i^{initial} = C_{min} + rand_i(C_{max} - C_{min}) \quad i = 1, 2, \dots, N \quad (1)$$

where $rand_i$ is an element of a random vector between $[0, 1]$, C_{max} and C_{min} are the upper and lower bounds of the search space, N is the population size.

Each particle in the population updates its concentration according to the following equation:

$$C = C_{eq} + (C - C_{eq}) \cdot F + \frac{G}{\lambda V} (1 - F) \quad (2)$$

where C_{eq} is the concentration of the equilibrium candidate, λ is an element of a random vector between $[0, 1]$. The equilibrium candidate was randomly selected from the equilibrium pool, and the equilibrium pool was constructed as described below.

$$C_{eq, pool} = \{C_{eq(1)}, C_{eq(2)}, C_{eq(3)}, C_{eq(4)}, C_{eq(ave)}\} \quad (3)$$

In Eq. (3), $C_{eq(1)}$, $C_{eq(2)}$, $C_{eq(3)}$, and $C_{eq(4)}$ represent the four best particles identified thus far, with $C_{eq(ave)}$ denoting their average. The utilization of these top four particles contributes to enhanced exploration capabilities for the EO process, while the incorporation of their average values fosters exploitation. It is worth noting that employing fewer than five candidates may compromise the performance of this method in the context of multi-modal and composite functions; however, it may yield improved outcomes for single-modal functions. Conversely, utilizing more than five candidates would yield contrary effects. Each particle updates its concentration by selecting an equilibrium candidate in the equilibrium pool with equal probability.

The exponential factor F in EO, responsible for adjusting the balance between global exploration and local exploitation, is calculated according to the following formula.

$$F = e^{-\lambda(t-t_0)} \quad (4)$$

where t is calculated according to the following equation.

$$t = \left(1 - \frac{Iter}{Max_iter}\right)^{\left(\frac{a_2 \cdot Iter}{Max_iter}\right)} \quad (5)$$

where $Iter$ and Max_iter denote the current iteration number and the maximum iteration number, respectively, and a_2 is a constant responsible for regulating the local search behavior. The parameter t_0 in the exponential term is calculated according to the following equation:

$$t_0 = \frac{1}{\lambda} \ln(-a_1 \cdot sign(r - 0.5)(1 - e^{\lambda})) + t \quad (6)$$

where a_1 is a constant responsible for adjusting global exploration, and r is a random vector between 0 and 1. Substituting the expression of t_0 into Eq. (4), the exponential factor can be revised as follows:

$$F = a_1 \cdot sign(r - 0.5) [e^{-\lambda} - 1] \quad (7)$$

The EO algorithm defines the concept of generation rate (G), which is calculated as follows:

$$G = G_0 e^{-\lambda(t-t_0)} = G_0 F \quad (8)$$

where

$$G_0 = GCP(C_{eq} - \lambda C) \quad (9)$$

$$GCP = \begin{cases} 0.5r_1 & r_2 \geq GP \\ 0 & r_2 < GP \end{cases} \quad (10)$$

where r_1 and r_2 are random numbers in $[0, 1]$, GP is mainly responsible for controlling whether the particles use GCP to update the concentration or not.

4. MRPP BASED ON THE OEO

4.1. Opposition-Based learning Mechanism Enhanced EO

The opposition-based learning (OBL) technique, proposed by Tizhoosh in 2005 [38], is an optimization learning strategy that involves seeking the opposite solution of a feasible problem solution. It evaluates both the original feasible solution and its corresponding inverse solution to determine the superior solution, which then serves as the optimization learning strategy for the next generation of individuals. Within the

framework of OBL, two key components are defined - the reverse point and OBL optimization. This approach explores an effective strategy by identifying the better-performing solution through evaluating the original and inverse solutions, thus guiding the optimization process for subsequent generations of individuals. In OBL, reverse point and OBL optimization are defined as follows:

Define 1 Suppose $x = (x_1; x_2; \dots; x_D)$ is any point in D -dimensional space, where $x_1; x_2; \dots; x_D$ belong to the real numbers R , and x_i is in the interval $[a_i, b_i]$. Then, the corresponding global reverse point of x is defined as $ox = (ox_1; ox_2; \dots; ox_D)$, where $ox_i = a_i + b_i - x_i$.

Define 2 Suppose $x = (x_1; x_2; \dots; x_D)$ is any point in D -dimensional space. The global reverse point of x is defined as $ox = (ox_1; ox_2; \dots; ox_D)$. For problems focused on minimization, if $f(ox) < f(x)$, then $x = ox$, and this is referred to as OBL optimization.

After updating the particle concentration according to Eq. (2), the OBL mechanism is executed, and the particle leaps to become an OBL individual. The subsequent optimization process retains the superior

individual between the current particle and the OBL individual. Through the application of the OBL mechanism, the ability of the basic EO algorithm to escape local optima is effectively enhanced. This improvement contributes to the overall optimization performance of the algorithm. The pseudo-code of the OBL algorithm is shown in Algorithm 1.

4.2. Fitness Function Construction

The problem of MRPP is often transformed as an optimization problem, and the objective is to minimize the path length while avoiding obstacles or threatening areas. We use artificial intelligence methods to solve this optimization problem by mathematically modelling the objective and proposing an objective function. The meta-heuristic algorithm evaluates the generated solution by scrutinizing the objective function to plan a suitable path for the AMR. The first mission is to find the shortest path, and the second task is to avoid the threatening areas. Based on these two goals, the objective function is designed as follows:

$$fit = L(1 + \rho \cdot Dis) \quad (11)$$

Algorithm 1 Pseudo-code of OEO

Input:Maximum iteration Max_iter , upper bound C_{max} and lower bound C_{min} , randomly generated initial population $C_i^{initial}$, $i=1,2,\dots,N$ and the dimension D

Output:The concentration of equilibrium candidate solution $C_{eq(1)}$

- 1: Initialize the population $C_i^{initial}$ ($i = 1, 2, \dots, N$) by considering C_{max} and C_{min}
 - 2: Evaluate the fitness value of each particle
 - 3: **while** $Iter < Max_iter$ **do**
 - 4: Establish the equilibrium pool
 - 5: Implement memory saving (if $Iter > 1$)
 - 6: **for** $i = 1$ to N **do**
 - 7: Randomly select a particle from the equilibrium pool
 - 8: Calculate the exponential term F according to Eq. (7)
 - 9: Calculate the generation rate G according to Eq. (8)
 - 10: Update the concentration of the i th particle according to Eq. (2)
 - 11: Execute the OBL mechanism and generate the OBL individual
 - 12: Select the better individual among the current and OBL individuals to proceed to subsequent iterations
 - 13: **end for**
 - 14: $Iter = Iter + 1$
 - 15: **end while**
 - 16: Return the equilibrium candidate $C_{eq(1)}$
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Table 2: The Minimum Path Length Comparison of OEO-Based MRPP Approach and Competitors under Three Terrains

Terrain	PSO	FA	ABC	GWO	SSA	OEO
	Path length	Path length	Path length	Path length	Path length	Path length
Map 1	14.3332	14.3200	14.3167	14.3299	14.5421	14.3049
Map 2	15.7275	15.8280	16.2014	16.0306	16.6224	15.7206
Map 3	22.1193	21.6858	25.2991	21.8575	22.2995	21.5458

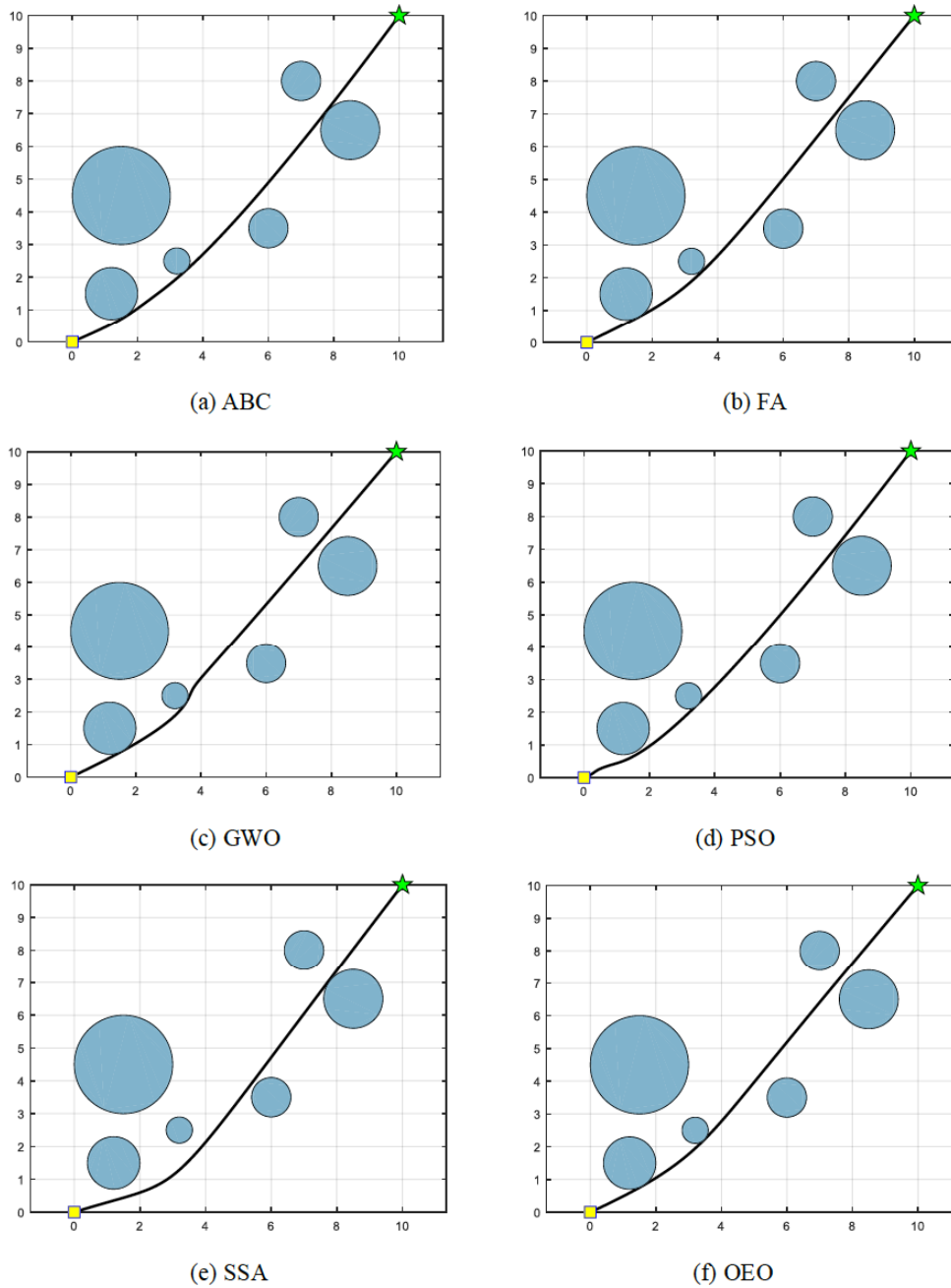


Figure 1: Map 1 (a) PSO, (b) FA, (c) ABC, (d) GWO, (e) SSA and (f) OEO.

to ascertain the stability and efficacy of the OEO approach, with the best-performing outcomes being

duly recorded.

Table 2 presents the optimal path lengths for three distinct terrains generated by all methods, with the shortest obstacle-free paths in each environmental map highlighted in bold. The table distinctly demonstrates that in the first two environmental maps, the conventional SSA method consistently chooses the longest obstacle-free path from the starting point to the target destination, while our proposed approach consistently opts for the shortest safe path. Even in the case of Terrain 3, our developed method continues to

prioritize the shortest safe path from the initial point to the endpoint, with ABC exhibiting the poorest performance in this regard.

Figure 1 (a-f) illustrates the optimal collision-free paths generated by six methods in the first environmental setting. It is evident from Figure 3 that all six approaches exhibit robust performance, successfully identifying collision-free paths. However, GWO, PSO, and SSA tend to get trapped in local

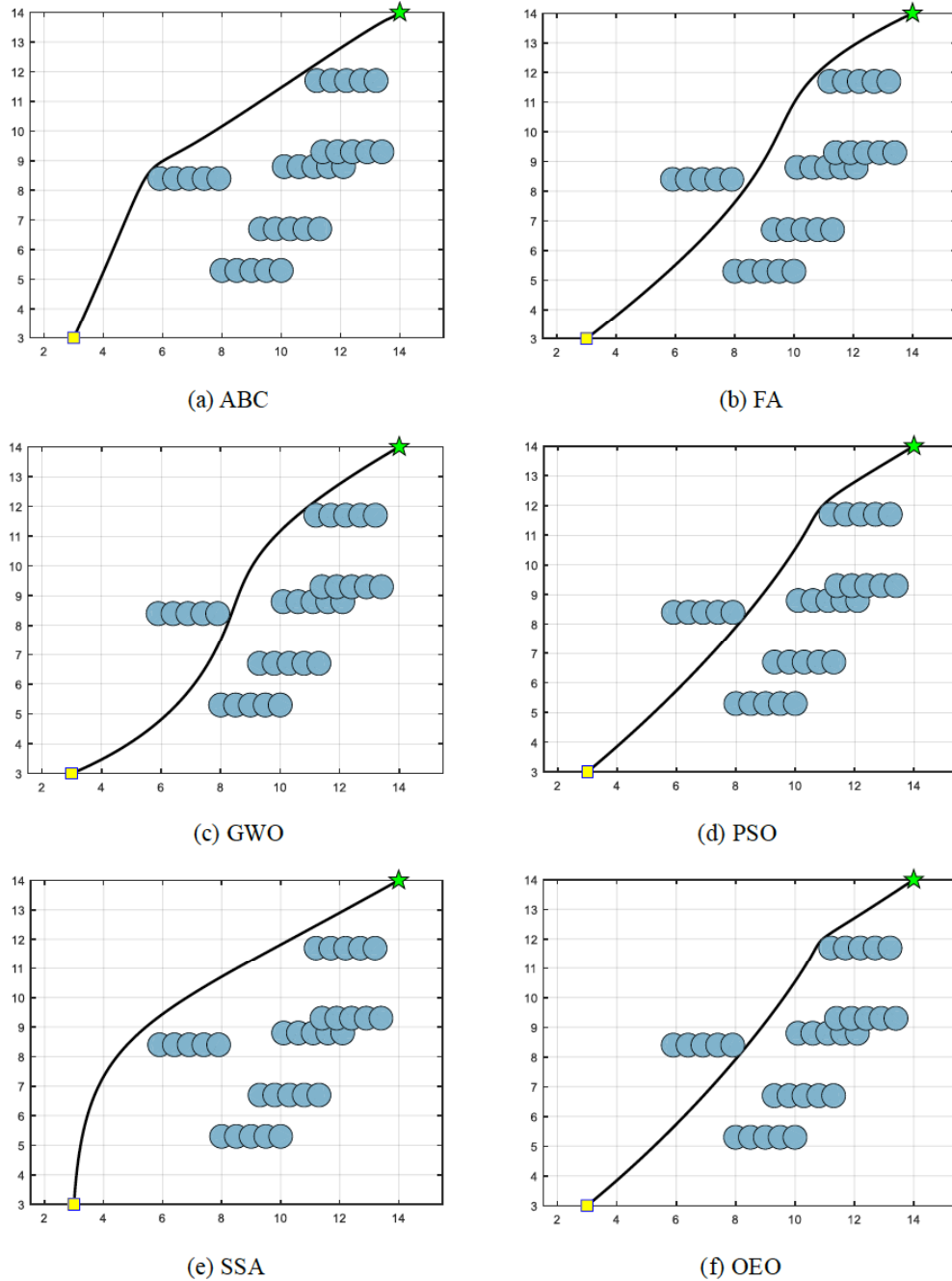


Figure 2: Map 2 (a) PSO, (b) FA, (c) ABC, (d) GWO, (e) SSA and (f) OEO.

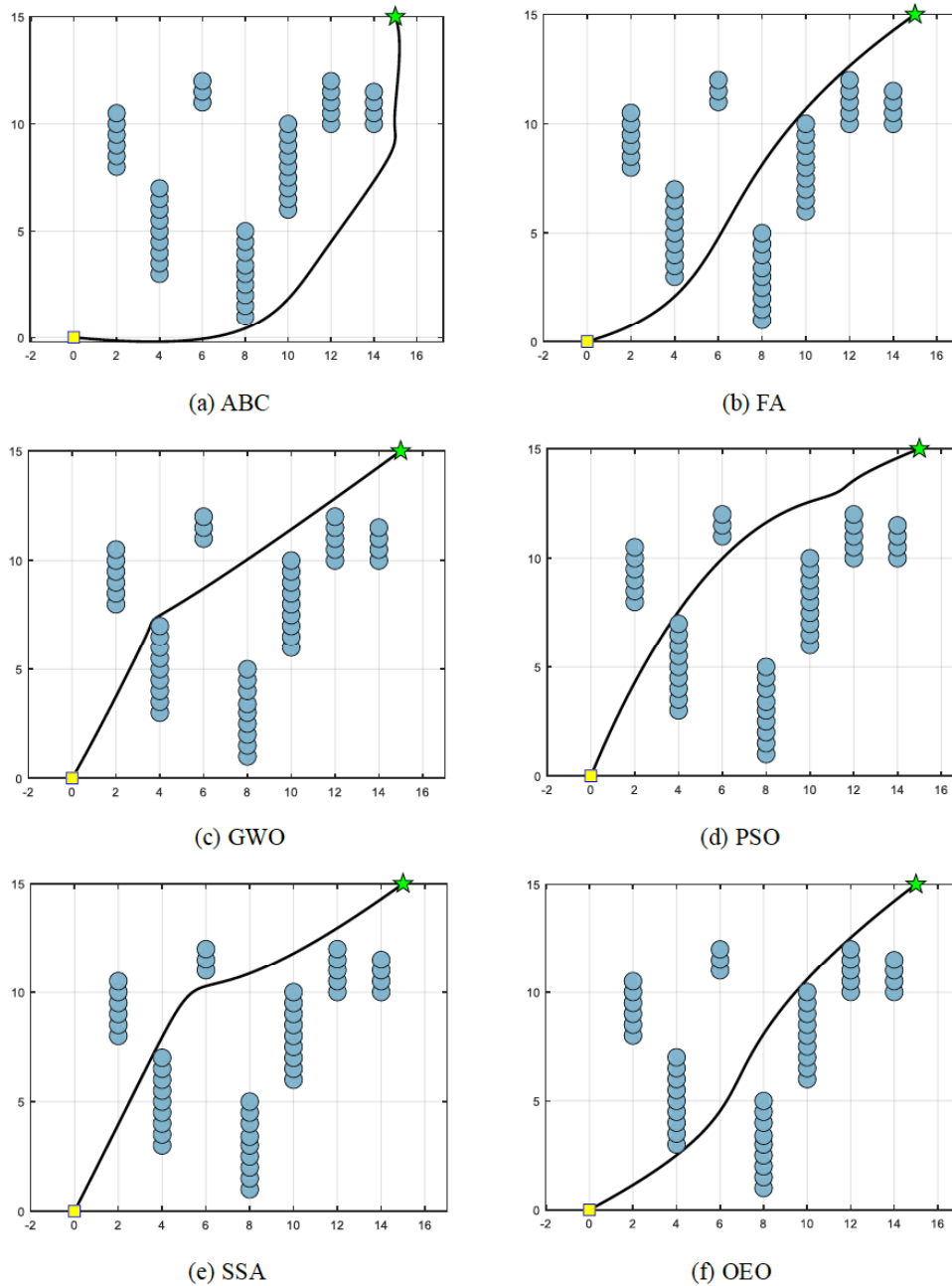


Figure 3: Map 3 (a) PSO, (b) FA, (c) ABC, (d) GWO, (e) SSA and (f) OEO.

optima, while ABC, FA, and OEO chart straightforward trajectories. Furthermore, in comparison to ABC and FA, paths generated by OEO are notably shorter.

Figure 2 (a-f) showcases the optimal performance of all methods in the second terrain with 30 threat zones. The plots reveal that both ABC and SSA chart identical trajectories, while FA, GWO, PSO and OEO generate different types of paths. OEO generates more direct paths compared to its competitors. In summary, the proposed method performs well in the second setting.

Figure 3 (a-f) compares the optimal collision-free routes developed by all methods for the third terrain configuration. Experimental results indicate that in this terrain, OEO outperforms its counterparts, boasting the shortest path length at 21.5458 units, followed by FA, GWO, PSO, SSA, and ABC. In conclusion, OEO demonstrates superior performance compared to rival algorithms.

6. CONCLUSIONS

As the complexity of optimization problems continues to grow, an increasing number of

computational intelligence algorithms have been developed to tackle these challenges. These algorithms have consistently delivered satisfactory results when applied to global optimization problems and have garnered a solid reputation. With the proven success of evolutionary algorithms in various engineering and scientific domains, they have also gained significant attention in the field of robotics. Specifically, these algorithms find extensive use in solving MRPP problems for AMRs.

In this study, we introduce a novel method called OEO, based on the principles of the basic EO. OEO incorporates the OBL mechanism, complementing the memory storage strategy inherent to basic EO. This innovation aids the population in escaping local optima and enhances the algorithm's ability to strike a balance between exploitation and exploration. We apply the developed OEO algorithm to the MRPP problem. To assess the performance of the OEO-based MRPP method, we conduct rigorous testing in three distinct environmental maps. We compare the results against those obtained using three classical metaheuristic algorithms, including PSO, GWO, ABC, SSA, and FA. The outcomes of these experiments reliably demonstrate that our proposed method consistently charts the shortest collision-free and smoothed path from the starting point to the endpoint across different environmental maps, outperforming the comparison algorithms. This promising performance positions OEO as a valuable tool for addressing challenges in path planning for AMRs within the domain of mobile robotics. In future research endeavors, we plan to explore multi-robot path planning within wireless sensor networks built upon the foundation of OEO.

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CONFLICT OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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