MRAC with SMC Applied to Lateral Control of a Fixed-Wing MAV

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Abstract: This paper presents a PD control law with adaptive gains with the MIT (Massachusetts Institute Technology) rule with different sliding modes; that is, the MIT rule has been designed with is known in the literature with first order sliding mode, second order sliding mode and high order sliding mode (HOSM) to obtain a better gain scheduling taking advantage the sliding modes techniques-the PD control law with adaptive gains that is designed for the lateral dynamics of a fixed-wing MAV. To apply the methodology of the model reference adaptive control (MRAC), sometimes called model reference adaptive system (MRAS), to the adaptive gains of the PD control, a sliding manifold is proposed considering the output of the lateral dynamics and with the output of the reference model.

Keywords: MIT Rule, Sliding mode, Fixed-wing MAV, Lateral control, MRAC.

1. INTRODUCTION

Some dynamic systems in control theory have constant uncertain parameters or slow variation [2]. For example, an MAV (Mini Aerial Vehicle) that has added sensors or batteries that have done some variations in its weight in quantities minor or significant will have unknown inertia values. Another example is when an MAV is flying in bad weather, and the MAV is exposed during the flight to a change in the temperature, which affects the performance of the MAV due to the density of air, which is usually considered or calculated as an approximation with a constant value.

Thus, to solve the problems mentioned above, an adaptive controller could be an option for realizing a stable flight with an MAV (Mini Aerial Vehicle) [1]. We can see the adaptive theory in different areas, such as robotics, aircraft, embedded systems, and remotely operated vehicles or underwater robots [2]. We can find research about the MIT rule, and it should be mentioned that the MIT rule name is because such methodology was developed in such institute in 1961. The model reference adaptive control or MRAC, sometimes defined as MRAS (model reference adaptive system), is a scheme of control where it is necessary to have a model reference and the real model or system to control. In such cases, the real model should follow the signal reference generated by the model reference (see Figure **2**).

For example, in [3], the theory based on the MIT rule is applied for a second-order system, and in [3], the methodology to obtain the adaptation of a unique controller is presented. Also, in [4], we can see the application of the MIT rule in a linearized model, and after considering such a model, we can see that it is designed as an adaptive control law. In other works, [5] has presented the two classical methodologies used in a MRAS: MIT rule and Lyapunov, and the adaptive controller shown in [5] is applied in a spherical tank. The results in [3-5] use Matlab software.

Thus, the contributions of this work are:

- The design of a robust gain scheduling, with the use of MIT rule and sliding mode theory.
- The reduction or almost elimination of the chattering effect with the use of high-order sliding mode.
- The adaptation of all the gains that make up the law of control.
- Change the adaptive gain with values small and big and achieve the control objective.

The organization of this work is as follows: Section 2 presents the equations for lateral dynamics—the controller methodology is presented in Section 3. Section 4 presented the simulation results. Finally, in section 5, the conclusions are presented.

2. LATERAL DYNAMICS

To obtain the model equations, the fixed-wing MAV is considered a rigid body by omitting any flexible structure of the MAV. Also, we do not consider the earth's curvature; it is regarded as a plane because we assume that fixed-wing MAVs will only fly short distances. With the previous considerations, we obtain the model by applying Newton's laws of motion. The

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E-ISSN: 2409-9694/23

following equations describe the dynamics of the roll angle:

$$\dot{\phi} = p \qquad (1)$$

$$\dot{p} = L_p p + L_{\delta a}\delta_a \qquad (2)$$

where p denotes the roll rate and ϕ describes the roll angle, see Figure **1**. It is observed that δ_a is the ailerons deflection. The lateral stability derivatives in roll are L_p and $L_{\delta a}$ [14], and are given by [10]:

$$L_p = \frac{\rho S V \bar{c}^2}{4 I_{xx}} C_{lp}$$

$$L_{\delta e} = \frac{\rho V^2 S \bar{c}}{2 I_{xx}} C_{l\delta a}$$

with:

V: Fixed-wing MAV velocity.

ρ: Air density (1.05 kg/m^3).

S: Wing area (0.09 m^2).

b: Wingspan (0.914 m).

c: Middle chord line (0.010 m)

I_{xx} : Roll angle moment of inertia (0.16 kg · m^2).

C_{lp} : Dimensionless coefficient for roll angle, obtained experimentally (-0.15).

$C_{l\delta a}$: Dimensionless coefficient for ailerons movement, obtained experimentally

(0.005).

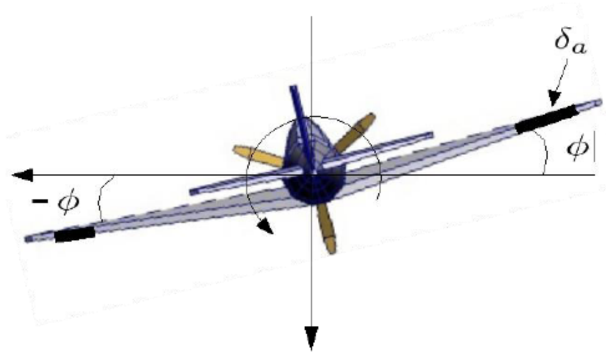

Figure 1: Pure rolling motion.

3. MRAC WITH SMC

The MIT rule is the original approach to model-reference adaptive control. The name is derived from the fact that it was developed at the Instrumentation Laboratory at MIT. To present the MIT rule, we will consider a closed-loop system in which the controller has one adjustable parameter θ. The desired closed-loop response is specified by a model whose output is y_m. Let e be the error between the output y of the closed-loop system and the output y_m, of the model. One possibility is to adjust parameters in such a way that the loss function (3) is minimized, the loss function is given by:

$$J(\theta) = \frac{1}{2} e^2 \qquad (3)$$

To make J small, we have to change the parameters in the direction of the negative gradient, that is,

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} = -\gamma e \frac{\partial e}{\partial \theta} \qquad (4)$$

The equation (4) is the MIT rule. The partial derivative $\partial e / \partial \theta$, which is called the sensitivity derivative of the system. It tells how the adjustable parameter influences the error. If it is assumed that the parameter changes are slower than the other variables in the system, then the derivative $\partial e / \partial \theta$ can be evaluated under the assumption that θ is constant. There are many alternatives to the loss function given by the equation (3). If it is chosen to be:

$$J(\theta) = |e| \qquad (5)$$

the gradient method gives:

$$\frac{d\theta}{dt} = -\gamma \frac{\partial e}{\partial \theta} \operatorname{sgn}(e) \qquad (6)$$

The first MRAS that was implemented was based on this formula. There are, however, many other possibilities, for example:

$$\frac{d\theta}{dt} = -\gamma \operatorname{sgn}\left(\frac{\partial e}{\partial \theta}\right) \operatorname{sgn}(e) \qquad (7)$$

The equation (7) is called the sign-sign algorithm.

DESIGN OF THE ADAPTIVE CONTROL

The controller to design is a PD controller with adaptation in the k_p and k_v gains and it is based on the model reference adaptive system or MRAS. The problem to work with MRAS is to determine the adjustment mechanism to stabilize the system. Thus, the adjustment mechanism for the gains has been designed with the MIT rule. To this rule, we added the sliding mode theory to design a robust controller to keep the desired roll angle in the presence of disturbances by wind gusts. The block diagram representing the adaptive system reference model is shown in Figure **2**, where the Plant defines the lateral dynamics, and the Model has equations that describe a stable performance.

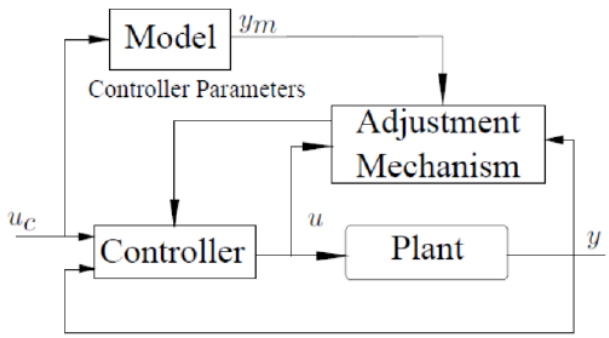

Figure 2: Block diagram.

To design the adaptive PD controller with the MIT rule and the technique of sliding mode for the roll angle of the MAV fixed-wing, we have considered. the equations (1)-(2), where δa defines the control input, and by the integral of the equation (1) has been obtained the roll angle. Thus, the adaptive control law is given by:

$$\delta_a = \hat{k}_{pl} e_\phi + \hat{k}_{vl} \dot{e}_\phi \tag{8}$$

where $\hat{k}_p$ and $\hat{k}_v$ are the called position gains and velocity, respectively; thus, these are the adaptive gains. The error of the roll angle has been defined as $e_\phi = \phi - \phi_d$. The adaptive gains of the PD control have a subscript to know the adjustment mechanism, which is tested. That is, $l \in a_1, a_2, a_3, a_4$ where a_1 correspond to the MIT rule, a_2 correspond to the MIT rule with sliding mode, a_3 MIT rule with 2-sliding modes and finally a_4 is for the MIT rule with HOSM. To design the MIT rule is introduced an error given by:

$$e = \phi_m - \phi \tag{9}$$

where ϕ is the roll angle from the MAV fixed-wing, and ϕ_m is the roll angle from the equations of the aerodynamic model. And then, to follow the methodology that have been shown in [6] for the MIT rule, the aerodynamic model is transform to transference function to develop the sensitivity derivatives are designed with partial derivatives considering $\hat{k}_{pl}$ and $\hat{k}_{vl}$. Then, the closed loop transfer function is given by:

$$\phi = \frac{L_{\delta a}(\hat{k}_{pl} + \hat{k}_{vl} s)}{s^2 + (L_p + L_{\delta a}\hat{k}_{vl})s + L_{\delta a}\hat{k}_{pl}}\phi_d \tag{10}$$

And the model of reference of the roll angle has been defined as:

$$\phi_m = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\phi_d \tag{11}$$

with ζ = 3,17 and ω = 3,16, remember that the equation (11) it should be designed as a stable system and it is the reference model and the parameters of (11) are computed or obtained with the root locus location on the left of the complex plain to obtain a stable system. Thus considering (10)-(11) are calculated the partial derivatives with respect to $\hat{k}_{pl}$ and $\hat{k}_{vl}$:

$$\frac{\partial e_{\phi m}}{\partial \hat{k}_{pl}} = \frac{L_{\delta a}}{s^2 + (L_p + L_{\delta a}\hat{k}_{vl})s + L_{\delta e}\hat{k}_{pl}}(\phi - \phi_d) \tag{12}$$

$$\frac{\partial e_{\phi m}}{\partial \hat{k}_{vl}} = \frac{L_{\delta a}}{s^2 + (L_p + L_{\delta a}\hat{k}_{vl})s + L_{\delta a}\hat{k}_{pl}}(\phi) \tag{13}$$

In general, the expressions (12) and (13) cannot be used because the parameters $\hat{k}_{pl}$ and $\hat{k}_{vl}$ are unknown, so an optimum case is assumed and is defined as:

$$s^2 + (L_p + L_{\delta a}\hat{k}_{vl})s + L_{\delta a}\hat{k}_{pl} = s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{14}$$

Thus, the terms L_p y $L_{\delta a}$ are included in the adaptation gain γ. Thus, the differential equations are:

$$\dot{\hat{k}}_{pa1} = -\gamma_1\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi - \phi_d)\right)e_{\phi m} \tag{15}$$

$$\dot{\hat{k}}_{va1} = -\gamma_2\left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi)\right)e_{\phi m} \tag{16}$$

Then, we have proposed a different formulation than the presented in (6) based on MIT rule with first order sliding modes (MIT-2SM), that is, sliding manifold is defined as $s_1 = \dot{\phi}_m - p + k_1 e$ to obtain a robust adjustment mechanism and to achieve a better performance in roll angle, with k_1 as a positive gain. Thus, the differential equations with MIT-2SM are:

$$\dot{\hat{k}}_{pa2} = -\gamma_1 \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi - \phi_d) \right)(\beta_p \operatorname{sgn}(s_1)) \quad (17)$$

$$\dot{\hat{k}}_{va2} = -\gamma_2 \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi) \right)(\beta_p \operatorname{sgn}(s_1)) \quad (18)$$

where $\beta_{p1}, \beta_{v1} \succ 0$. The first-order sliding mode has presented the chattering effect; we will design an adjustment mechanism with a sliding mode of second-order (MIT-2SM) to decrease the chattering impact, which is necessary for a robust first-order differentiator due to the derivatives in real-time are sensitive to noise at the time of computing the derivative [7]. This differentiator is given by:

$$\dot{x}_0 = v_0 = -\lambda_0 |x_0 - s_1|^{1/2} \operatorname{sgn}(x_0 - s_1) + x_1$$
$$\dot{x}_1 = -\lambda_1 \operatorname{sgn}(x_1 - v_0)$$

where $x_0 = s_1$ and $x_1 = \dot{s}_1$, $\lambda_1, \lambda_2 \succ 0$. Thus, the differential equations of the MIT-2SM are:

$$\dot{\hat{k}}_{pa3} = -\gamma_1 \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi - \phi_d) \right) \times (\beta_{p1} \operatorname{sgn}(s_1) + \beta_{v2} \operatorname{sgn}(\dot{s}_1)) \quad (19)$$

$$\dot{\hat{k}}_{va3} = -\gamma_1 \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi) \right) \times (\beta_{p1} \operatorname{sgn}(s_1) + \beta_{v2} \operatorname{sgn}(\dot{s}_1)) \quad (20)$$

where β_{p1}, β_{p2}, β_{v1}, β_{v2} $\succ 0$. Then, with the objective of reduce or eliminated the chattering effect, it is designed the MIT rule with HOSM (MIT-HOSM), and is necessary a robust differentiator of second order [7]. This differentiator is given by:

$$\dot{x}_0 = v_0 = -\lambda_0 |x_0 - s_1|^{2/3} \operatorname{sgn}(x_0 - s_1) + x_1$$
$$\dot{x}_1 = v_1 = -\lambda_1 |x_1 - v_0|^{1/2} \operatorname{sgn}(x_1 - v_0) + x_2$$
$$\dot{x}_2 = -\lambda_2 \operatorname{sgn}(x_2 - v_1)$$

where $x_0 = s_1$, $x_1 = \dot{s}_1$ and $x_2 = \ddot{s}_1$ are real-time estimations of s_1, $\dot{s}_1$ and $\ddot{s}_1$. The values of λ_0, λ_1 and λ_2 are constants defined positive. Finally, the differential equations of the adaptive PD controller with HOSM are defined by:

$$\dot{\hat{k}}_{pa4} = -\gamma_1 \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi - \phi_d) \right) \times (\alpha_p (\ddot{s}_1 + 2(|\dot{s}_1|^3 + |s_1|^2)^{1/6} \operatorname{sgn}(\dot{s}_1 + |s_1|^{2/3} \operatorname{sgn}(s_1)))) \quad (21)$$

$$\dot{\hat{k}}_{va4} = -\gamma_2 \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}(\phi) \right) \times (\alpha_v (\ddot{s}_1 + 2(|\dot{s}_1|^3 + |s_1|^2)^{1/6} \operatorname{sgn}(\dot{s}_1 + |s_1|^{2/3} \operatorname{sgn}(s_1)))) \quad (22)$$

where α_p and α_v are positive gains.

4. SIMULATION RESULTS

Figure **3** presents the results obtained by the MIT rule in roll angle. The red solid line is the reference that the model reference has generated, and the blue solid line is the response obtained from the roll angle and has to converge to the dashed line. Figure **4** presents the error between the model reference and the actual roll angle. In Figure **5**, we can appreciate the control response of the adaptive PD control, and the control law signal has been saturated to $\pm 40^{\circ}$. The deflection value of the ailerons that have been allowed by the fixed-wing MAV are $\pm 20^{\circ}$. We have tried to reduce saturation to achieve the allowed values, but obtaining a good response to the control law was impossible. The MIT rule has presented some noise in its control signal when it has been saturated the control law in Figure **15** shows a zoom of the MIT rule to appreciate the noise mentioned.

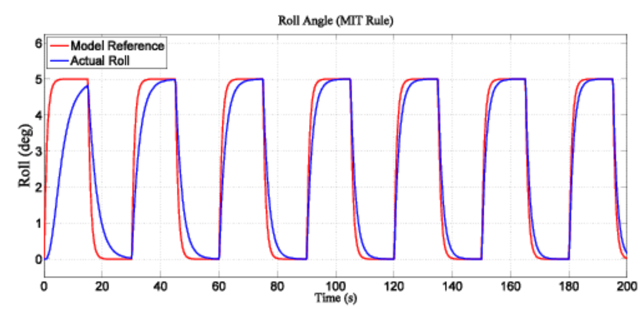

Figure 3: Response of the adaptive PD control with the MIT rule.

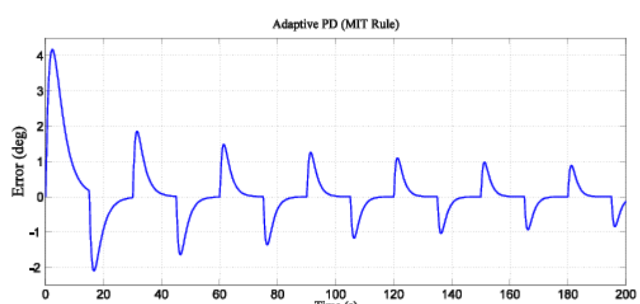

Figure 4: Error signal with the MIT rule.

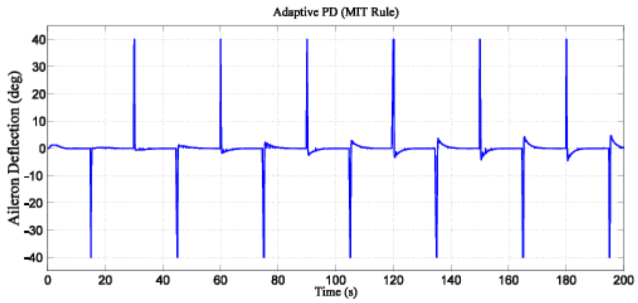

Figure 5: Control response with the MIT rule.

Figure **6** shows the results obtained by the MIT rule with sliding mode. Figure **6** presented a considerable improvement compared to the results obtained with the MIT rule without the sliding mode; it is appreciated that the response obtained in roll angle achieves the desired signal (red solid line) in a time not longer than 60 seconds. Figure **7** and Figure **8** presented the error and the control effort applying the MIT rule with sliding mode, respectively.

The error is less of $\pm 1^{\circ}$ after of the 20 seconds, see Figure **8**. Even in Figure **7**, it is appreciated that the control response is saturated in $\pm 20^{\circ}$; this angle deflection is allowed by the control surface of the fixed-wing MAV. The inconvenience in this algorithm-like adjustment mechanism is the effect chattering in Figure **15**, shown as a zoom of the MIT rule with second-order sliding mode to appreciate the effect mentioned.

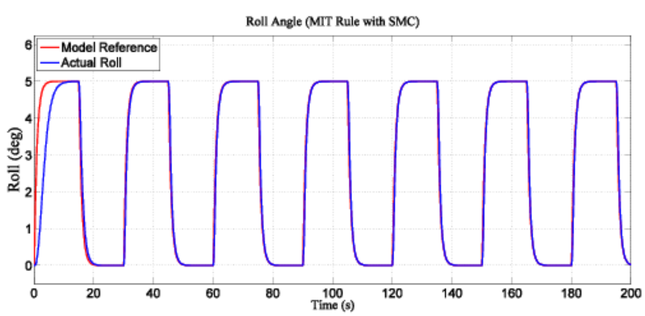

Figure 6: Response of the adaptive PD control with the MIT rule and first order sliding mode.

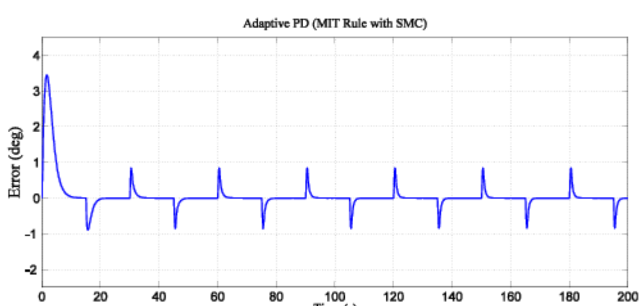

Figure 7: Error signal with the MIT rule and first order sliding mode.

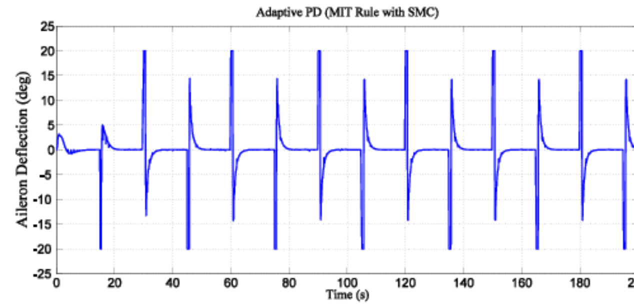

Figure 8: Control response with the MIT rule and first order sliding mode.

Figure **9** presented the response of the MIT rule with second-order sliding mode; it has been appreciated that the response obtained from the roll angle (blue solid line) achieves the desired roll angle (red solid line) after 60 seconds, and it presented a stationary state error of $\pm 1^{\circ}$.

The error signal is shown in Figure **10**. The signal control has been shown in Figure **11**; there is a reduction of the chattering effect, and the control signal is saturated to ± 20 we have achieved a good response to the control law, but the control signal still presents the chattering effect. To appreciate this effect better, see Figure **15**.

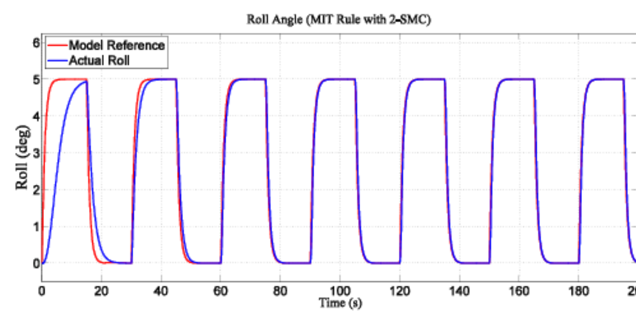

Figure 9: Response of the adaptive PD control with the MIT rule and second order sliding mode.

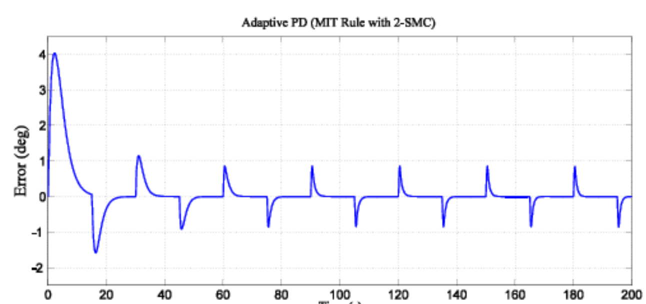

Figure 10: Error signal with the MIT rule and second order sliding mode.

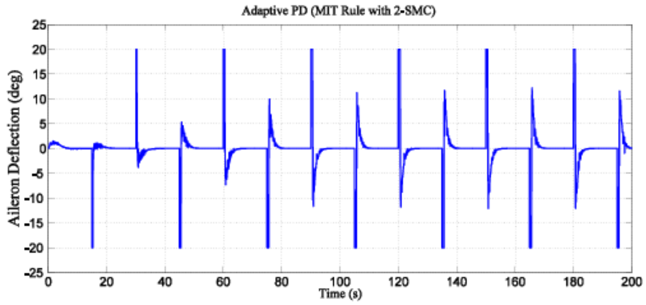

Figure 11: Control response with the MIT rule and second order sliding mode.

Figure **12** shows the results obtained by the MIT rule and high order sliding mode (HOSM); it is appreciated that the response obtained from the roll angle (blue solid line) achieved the model reference signal after 80 seconds.

Figure **14** shows that the error is less than $\pm 1^{\circ}$ after 120 seconds, but the chattering has been eliminated with the MIT rule, and the high-order sliding mode see Figure **15**, and even the control signal is in the values $\pm 20^{\circ}$ for the ailerons of the fixed-wing MAV, see the Figure **13**.

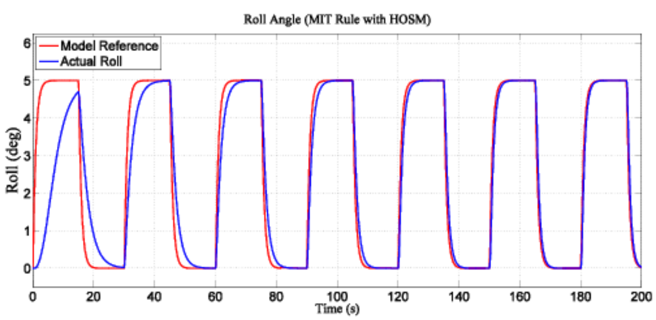

Figure 12: Response of the adaptive PD control with the MIT rule and high order sliding mode.

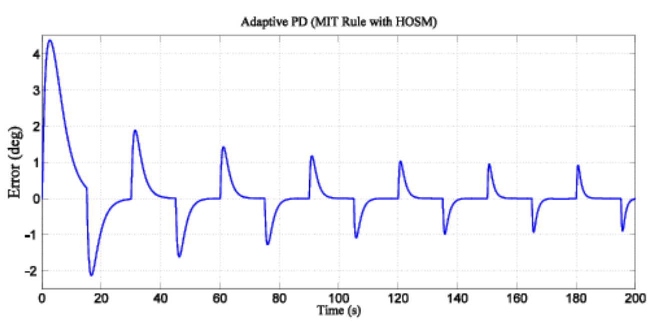

Figure 13: Error signal with the MIT rule and high order sliding mode.

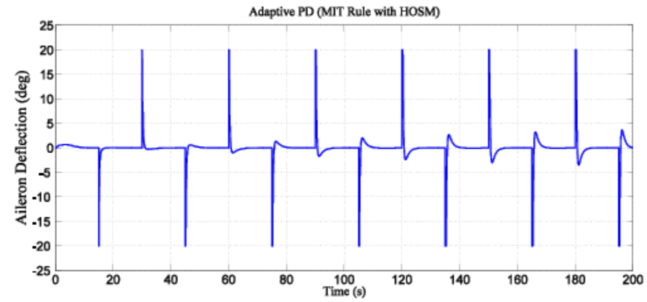

Figure 14: Control response with the MIT rule and high order sliding mode.

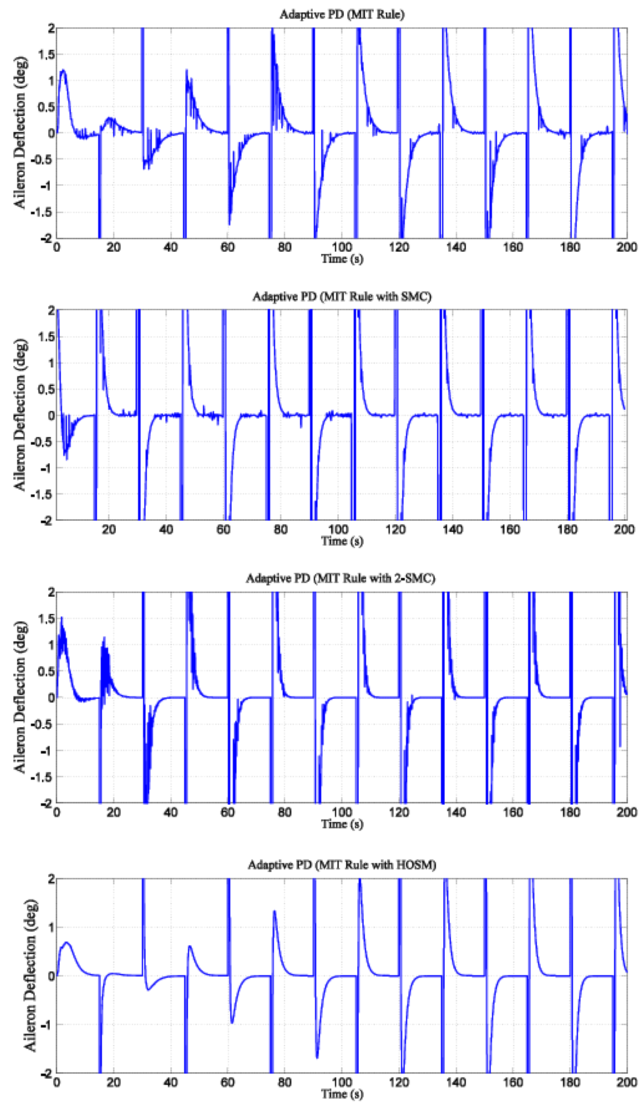

Figure 15: Zoom of the reduction of the chattering effect of the adaptive PD controller with MIT rule and sliding mode theory.

5. CONCLUSIONS

This work used the MIT rule with the sliding mode technique to design a robust adjustment mechanism for an adaptive PD control.

Then, the adaptive control law is applied to achieve the desired values of the roll angle of a fixed-wing MAV. The adaptive PD control with the MIT rule as an adjustment mechanism was only possible saturate to $\pm 40^{\circ}$ and these values are not allowed by the fixed-wing MAV, and even the control signal has presented noise; also, the tuning of the adaptive gains is complex because the system tends to be unstable with some decimal changes in the adaptation gain.

On the other side, with the MIT rule with sliding mode, it is possible to obtain the desired roll angle with the saturation in $\pm 20^{\circ}$, but in the control signal, it has presented the undesired chattering effect. Thus, the chattering effect in the control signal is reduced by the MIT rule with second-order sliding mode. Still, the response obtained from the roll angle (blue solid line) converges to the reference model in a more significant time than the MIT rule with sliding mode.

Finally, the MIT rule with high-order sliding mode presented a better response in the control signal with the chattering effect eliminated, and even more, the control signal is inside of the desired values by the ailerons of the MAV fixed-wing, that is, in $\pm 20^{\circ}$ of ailerons deflection. We appreciate that tuning the adaptive gain with the MIT rule with a high-order sliding mode is more accessible. The unique inconvenience is that fewer errors $\pm 1^{\circ}$ are achieved after 120 seconds.

FUNDING

This work was not financially supported by any grant.

CONFLICT OF INTEREST

The authors declare that a conflict of interest does not exist.

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Received on 07-11-2023 Accepted on 15-12-2023 Published on 18-12-2023

DOI: https://doi.org/10.31875/2409-9694.2023.10.11