Finite Horizon Memory Control of Networked Systems Using Chain-like Lyapunov Function

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Abstract: This paper proposes a novel finite horizon memory control (FHMC) design framework for networked systems by using input delay approach. A chain-like input delay model is established to characterize the networked control system (NCS) under memory control architecture in order to make full use of historic sampled-data. Based on the proposed chain-like delay model, the corresponding chain-like Lyapunov-Krasovskii function, which records the historic sampling information of NCS, is well constructed for facilitating further analysis and synthesis of the proposed FHMC scheme. Both state feedback controllers and static output feedback controllers are derived by solving LMIs (Linear matrix inequalities). The proposed FHMC scheme is skilled in improving control performance of networked systems. Simulations show the effectiveness of the presented FHMC scheme.

Keywords: Networked control systems, Input delay, Finite horizon memory control, Sampled-data control.

1. INTRODUCTION

Networked control system (NCS) has been widely applied in many promising areas such as smart grids, Internet of Things and intelligent manufacture [1]. Although fruit results on analysis and synthesis of NCS have been arrived at in the past decades, some new and interesting challenges also arise [2].

In fact, sampling and time delay are the two key features that play an important difference in the stability and stabilization of the NCS [3]. Therefore, a fundamental design of the NCS should take such two factors into account. By considering continuous-time dynamics under discrete-time sampled-data control fashion, the input delay approach [4] is well developed to cope with both time delay and discrete-time sampling. On the one hand, such an input delay approach is beneficial to characterize the sampling characterization and time delay under a unified framework. On the other hand, it is convenient to use the well-studied Lyapunov-Krasovskii functional method to conduct analysis and synthesis for the NCS [5]. It is well known that Lyapunov-Krasovskii functional method exploiting the length of time delay can reach a delay-dependent condition for stabilization of the NCS. In addition, one can also relax the conservative of the time delay system by constructing some novel Lyapunov-Krasovskii functional. Although time delay can be neglected for modern advanced communication and computation technologies, it makes no difference in the fact that the input delay approach becomes a primary design tool for the analysis and design of the NCS. At present, the input delay approach has been widely founded in networked analysis and design [3, 6].

However, the historic sampled-data are rarely considered in the presented networked control design [7,8]. It is clear that the performance under a memoryless controller can not be better than a memory one although a memoryless control scheme has the advantage of easy implementation. Thus, one can pursue a memory-based control scheme, which includes both current state and past information, to improve the robustness and performance of the NCS [9, 10]. In fact, only current state measurement and delayed measurement are used to design memory controllers in most existing works [7,10]. In order to make full use of the historic sampled-data, the finite horizon memory control (FHMC) scheme, where its idea originated from finite impulse response (FIR) filters, is developed [11]. In essence, finite horizon memory control is similar to the FIR filter which can smooth the system state and reject disturbances or noises by measurements. This is also the reason why the FHMC scheme is able to mitigate the influence of abnormal signal from outside and improve the performance of control systems.

Unfortunately, FHMC scheme is not easily implemented under networked control framework. Actually, the latest sampled-data is often used when one transforms the discrete-time sampling control to a time delay fashion of continuous-time style. However, the successive historic sampled-data are rarely

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considered in the traditional memory control models under such hybrid systems which contain both continuous and discrete dynamics. On the one hand, memory control schemes are often founded in discrete-time systems rather than the referred hybrid systems [12]. On the other hand, the presented construction of Lyapunov-Krasovskii function just only pays attention to derive a less conservative criterion for a time-delayed NCS by using the delay interval division method. To our best knowledge, there is no general result on the FHMC scheme for the NCS under sampled-data control with time delay.

Based on the above observations, the main contributions of this paper can be summarized as follows

• A FHMC model for the NCS is established by using input delay approach. Different from the previous results which only the latest sampling information is used [7] or some probability distribution is needed [12, 13], the proposed FHMC model makes full use of finite available historic sampled-data and characterizes it as a chain-like input delay model for the NCS.

• A novel chain-like Lyapunov-Krasovskii candidate is well constructed based on the propose chain-like delay model. Different from the tradition delay interval divisions [14], we will develop a chain-like Lyapunov-Krasovskii candidate to characterize the chain-like delays step-by-step. Thus, each delayed sampling information is included in the proposed Lyapunov-Krasovskii candidate.

• Static output feedback controllers are derived by solving matrix pseudo-inverse. By comparing with the existing dynamic output feedback controller design method [15] or static output feedback method [16], the propose method will significantly simplify the static output feedback controller design by using direct matrix analysis and operations.

The reminder of this paper is organized as follows. Section 2 establishes the FHMC model for the NCS. Section 3 conducts the stability analysis and controllers design of the NCS based on the FHMC scheme. Section 4 verifies the proposed theory results through simulations. Section 5 concludes this paper.

2. MODELING OF FHMC UNDER NETWORKED ENVIRONMENT

The dynamics of the interested NCS to be controlled is described as follows

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t) \\ y = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$, $y(t) \in R^m$, $u(t) \in R^p$ are the state, output and control vectors, respectively. A, B, C, D are the constant matrices with appropriate dimensions. $w(t)$ is the external disturbance.

Under networked control environment, suppose the sensor is time-driven with sampling period h and the controller and actuator are event-driven. In order to exploit the historical sampled-data, the FHMC scheme is designed as

$$u(t) = \begin{cases} \sum_{i=0}^{N-1} K_{Si} x(kh - ih) & \text{State feedback} \\ \sum_{i=0}^{N-1} K_{Oi} Cx(kh - ih) & \text{Output feedback} \end{cases} \tag{2}$$

for $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$. Here, K_{Si} and K_{Oi} are the state feedback controller gain and output feedback controller gain, respectively. τ_k is the transmission delay due to communication network, N is the memory length of sampled-data.

Then substituting (2) into dynamics (1) yields

$$\dot{x}(t) = Ax(t) + B_i x(kh - ih) + Dw(t) \tag{3}$$

for $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$. Here, $B_i = BK_{Si}$ (state feedback case) or $B_i = BK_{Oi}C$ (output feedback case).

Remark 1: Traditionally, only $x(kh)$ is used for the memoryless state or output feedback controller design, namely $u(t) = Kx(kh)$ [17]. However, the most recently N sampled-data, *i.e.* $x(kh), x((k-1)h), \cdots x((k-N+1)h)$ are used for the proposed FHMC scheme. Under FHMC scheme, it is clear that both current measurement and historic samplings are feeded back in the closed-loop NCS. Thus, the improvement of control performance is expected.

Define $\tau(t) = t - kh$, thus one can arrive at that $\tau(t)$ is a linear piecewise function with $0 < \tau(t) < \tau$ and historical sampling instants satisfy

$$\begin{cases} kh = t - \tau(t) \\ (k-1)h = t - \tau(t) - h \\ (k-2)h = t - \tau(t) - 2h \\ \text{L} \\ (k-N+1)h = t - \tau(t) - (N-1)h \end{cases} \tag{4}$$

Then the closed system (3) can be further described by

$$\dot{x}(t) = Ax(t) + \sum_{i=0}^{N-1} B_i x(t - \tau(t) - ih) + Dw(t) \tag{5}$$

for $t \in [kh + \tau_k, (k+1)h + \tau_{k+1})$. The initial condition is given by $x(0) = x_0$ and $x(-ih) = 0$ for $i = \{1, 2, \cdots, N-1\}$.

Remark 2: It is clear that the time delay for each sampling instant can be represented by $(k-i)h = t-\tau(t)-ih$. In fact, the time delay can be divided into two parts which includes fixed part and time varying part. The time varying part $\tau(t)$ characterizes by the network transmission delay and the fixed part h characterizes the history sampling instants. Due to the fact that the fixed part is caused by every two adjacent sampling instants with a fixed sampling period h, we call this chain-like delay.

The control objectives of this paper is to pursue the finite horizon memory controllers of the NCS (5) while the ISS property is guaranteed. The definition of ISS is given by

Definition 1: [18] The FHMC of NCS (5) is said to be ISS if there exist a KL function $\mu(\cdot)$ and a K function $\nu(\cdot)$ such that

$$\| x(t,t_0) \| \leq \mu(\| x(t,t_0) \|) + \nu(\| w(t) \|_\infty) \tag{6}$$

where $w(t)$ is a bounded external disturbance.

3. STABILITY ANALYSIS AND CONTROLLERS DESIGN

In this section, the stability criterion and controller design for the NCS under FHMC scheme are presented.

3.1. Stability analysis

Theorem 1: Consider the FHMC of NCS (5). For some positive scalars h, τ, α and ς, the NCS (5) is ISS if there exists some positive definite matrices P, Q_i and R_i, $i \in \{0,1,\cdots,N-1\}$ satisfying

$$\begin{bmatrix} \Sigma_{11}^{(N-1)} & \Sigma_{12}^{(N-1)} \\ * & \Sigma_{22}^{(N-1)} \end{bmatrix} < 0 \tag{7}$$

where

$$\Sigma_{11}^{(N-1)} =$$

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} & \cdots & \varphi_{1(N-1)} & PD \\ * & \varphi_{22} & 0 & \cdots & 0 & 0 \\ * & * & \varphi_{33} & \cdots & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \varphi_{(N-1)(N-1)} & 0 \\ * & * & * & * & * & -\varsigma^2 I \end{bmatrix}$$

with

$$\varphi_{11} = A^T P + PA + 2\alpha P + Q_0 - \sum\nolimits_{i=0}^{N-1} e^{-2\alpha(\tau+ih)} R_i \,,$$

$$\varphi_{12} = PB_{K_0} + e^{-2\alpha\tau} R_0 \,,$$

$$\varphi_{13} = PB_{K_1} + e^{-2\alpha(\tau+h)} R_1 \,,$$

$$\cdots,$$

$$\varphi_{1(N-1)} = PB_{K_{N-1}} + e^{-2\alpha(\tau+(N-1)h)} R_{N-1} \,,$$

$$\varphi_{22} = -e^{-2\alpha\tau}(Q_0 + R_0) + e^{-2\alpha(\tau+h)} R_1 \,,$$

$$\varphi_{33} = -e^{-2\alpha(\tau+h)}(Q_1 + R_1) + e^{-2\alpha(\tau+2h)} R_2 \,,$$

$$\cdots,$$

$$\varphi_{(N-1)(N-1)} = -e^{-2\alpha(\tau+(N-1)h)}(Q_{N-1} + R_{N-1}) \,,$$

$$\Sigma_{12}^{(N-1)} =$$

$$\begin{bmatrix} \tau(R\Gamma)^T & (\tau+h)(R\Gamma)^T & \cdots & (\tau+(N-1)h)(R\Gamma)^T \end{bmatrix}$$

with

$$\Gamma = \begin{bmatrix} A & B_{K_0} & B_{K_1} & \cdots & B_{K_{N-1}} & D \end{bmatrix},$$

$$\Sigma_{22}^{(N-1)} = diag\begin{Bmatrix} -R_0 & -R_1 & \cdots & -R_{N-1} \end{Bmatrix}.$$

Proof: We choose the following chain-like Lyapunov-Krasovskii candidates as

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$
(8)

where

$$\begin{aligned} V_1(t) &= x^T(t) P x(t) \\ V_2(t) &= \sum_{i=0}^{N-1} \int_{t-\tau-ih}^{t-ih} e^{2\alpha(s-t)} \dot{x}^T(s) Q_i \dot{x}(s) ds \\ V_3(t) &= \sum_{i=0}^{N-1} (\tau+ih) \int_{-(\tau+ih)}^{0} \int_{t+\theta}^{t} e^{2\alpha(s-t)} \dot{x}^T(s) R_i \dot{x}(s) ds d\theta \end{aligned} \tag{9}$$

Differentiating (8) with respect to t along trajectory (3) yields

$$\begin{aligned} \dot{V}_1(t) &= x^T(t)(A^T P + PA)x(t) + 2x^T(t) PDw(t) \\ &\quad + 2x^T(t) P \sum_{i=0}^{N-1} B_i x(t-\tau(t)-ih) \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{V}_2(t) &= 2\alpha V_2(t) \\ &\quad + x^T(t) Q_0 x(t) - e^{-2\alpha\tau} x^T(t-\tau) Q_0 x(t-\tau) \\ &\quad + \sum_{i=1}^{N-1} [x^T(t-\tau-ih) Q_i x(t-\tau-ih) \\ &\quad - x^T(t-\tau-(i+1)h) Q_i x(t-\tau-(i+1)h)] \end{aligned} \tag{11}$$

$$\begin{aligned} \dot{V}_3(t) &= 2\alpha V_3(t) + \sum_{i=0}^{N-1} [(\tau+ih)^2 \dot{x}^T(t) R \dot{x}(t) \\ &\quad - (\tau+ih) \int_{t-(\tau+ih)}^{t} e^{2\alpha(s-t)} \dot{x}^T(s) R \dot{x}(s) ds] \end{aligned} \tag{12}$$

Using Jessen inequality [19], there exists

$$-(\tau+ih)\int_{t-\tau-ih}^{t-ih} e^{2\alpha(s-t)}\dot{x}^T(s)R_i\dot{x}(s)ds \leq$$

$$-e^{-2\alpha\tau}\zeta_i(t)^T \begin{bmatrix} R_i & -R_i \\ * & R_i \end{bmatrix} \zeta_i(t) \tag{13}$$

where $\zeta_i(t) = \begin{bmatrix} x^T(t-ih) & x^T(t-\tau(t)-ih) \end{bmatrix}^T$.

Combining with (10)~(13), we can arrive at

$$\dot{V}(t) + 2\alpha V(t) - \varsigma w^T(t)w(t)$$

$$\leq \xi^T(t)\Xi_{11}\xi(t) + \tau(t)^2\dot{x}^T(t)R_i\dot{x}(t) \tag{14}$$

where
$\xi(t) = col\{x(t), x(t-\tau(t)), \cdots, x(t-\tau(t)-(N-1)h), w(t)\}$
and Σ_{11} is given by (7).

Applying Schur complement lemma to the right side of (14), we obtain that

$$\dot{V}(t) + 2\alpha V(t) - \varsigma w^T(t)w(t) < 0 \tag{15}$$

if (7) holds.

This yields

$$\dot{V}(t) + 2\alpha V(t) < \varsigma w^T(t)w(t). \tag{16}$$

Multiplying both sides of (16) with $e^{2\alpha t}$, we have

$$e^{\alpha t}\dot{V}(t) + 2\alpha e^{2\alpha t}V(t) < \varsigma e^{2\alpha t}w^T(t)w(t). \tag{17}$$

From (17), one can obtain

$$\frac{d(e^{2\alpha t}V(t))}{dt} < \varsigma e^{2\alpha t}w^T(t)w(t). \tag{18}$$

Integrating both sides of (24) from t to $+\infty$, then

$$e^{2\alpha t}V(t)) - V(0) < \varsigma(e^{2\alpha t}w^T(t)w(t) - w^T(0)w(0)), \tag{19}$$

which implies

$$V(t) < e^{-2\alpha t}V(0) + \varsigma\int_0^t e^{-2\alpha t}w^T(s)w(s)ds \tag{20}$$

by multiplying both sides of (20).

Let $a = \lambda_{min}(P)$, $b = \lambda_{max}(P)$, $\|\overline{w}\| = \max(\|w(t)\|)$, it gives

$$\|x(t)\| \leq \sqrt{\frac{b}{a}}e^{-2\alpha t}\|x(0)\| + \varsigma\sqrt{\frac{1}{a}}\|\overline{w}\| \tag{21}$$

which completes proof.

Remark 3: From the constructed chain-like Lyapunov-Krasovskii function (8), it is clear that the single integral parts, $\int_{t-\tau}^{t} f(\cdot)ds, \int_{t-\tau-h}^{t-\tau} f(\cdot)ds, \cdots, \int_{t-\tau-ih}^{t-\tau-(i+1)h} f(\cdot)ds$, describe a chain-like connection for historic sampled-data $x(kh), x((k-1)h), \cdots, x((k-i)h)$. Then, double integral parts, $\tau\int_{-\tau}^{0}\int_{t+\theta}^{t} f(\cdot)dsd\theta, (\tau+h)\int_{-\tau-h}^{0}\int_{t+\theta}^{t} f(\cdot)dsd\theta, \cdots, (\tau+ih)\int_{-\tau-ih}^{0}\int_{t+\theta}^{t} f(\cdot)dsd\theta a$, characterize the delay-dependent information for historic sampled-data $x(kh), x((k-1)h), \cdots, x((k-i)h)$. Thus, each collected sampled-data information are fully taken into account for the chain-like Lyapunov-Krasovskii function (8).

Remark 4: Traditional "interval division method", which divides a large delay interval into servals small delay intervals, pays more attentions on finding a more larger time delay bound by designing more positive matrices in Lyapunov-Krasovskii function. However, the chain-like Lyapunov-Krasovskii functions will devote themselves into exploiting the historic sampled-data.

Corollary 1: Consider the memoryless control of NCS (5). For some positive scalars α, τ, if there exists positive definite matrix P_0, Q_0, R_0 satisfying

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0 \tag{22}$$

where

$$\Sigma_{11} = [(1,1) = A^T P + PA + 2\alpha_0 P + Q_0 - e^{-\alpha\tau}R_0,$$

$$(1,2) = PBK + e^{-\alpha\tau}R_0, (1,3) = PD,$$

$$(2,2) = -e^{-2\alpha\tau}Q_0 - e^{-2\alpha\tau}R_0, (3,3) = -\varsigma]$$

$\Sigma_{22} = -R_0$,

$\Sigma_{12} = \tau R_0\Gamma$ with $\Gamma^T = [A, BK, D]$.

Then, the NCS (5) is ISS.

Proof: Let $i = 0$ in Theorem 3.1, one can easily arrive at the desired result. The similarly results can also be founded in [20].

3.2. State and Output Feedback Controllers Design

Theorem 2: Consider the FHMC of NCS (5). For some positive scalars h, τ, α γ, if there exists positive definite matrices X, $\widetilde{Q}_i$ and $\widetilde{R}_i$, $i \in \{0,1,\cdots,N-1\}$ satisfying

$$\begin{bmatrix} \overset{(N-1)}{\tilde{\Sigma}_{11}} & \overset{(N-1)}{\tilde{\Sigma}_{12}} \\ * & \overset{(N-1)}{\tilde{\Sigma}_{22}} \end{bmatrix} < 0 \tag{23}$$

where

$\tilde{\Sigma}_{11} =$

$$\begin{bmatrix} \tilde{\varphi}_{11} & \tilde{\varphi}_{12} & \tilde{\varphi}_{13} & \cdots & \tilde{\varphi}_{1(N-1)} & D \\ * & \tilde{\varphi}_{22} & 0 & \cdots & 0 & 0 \\ * & * & \tilde{\varphi}_{33} & \cdots & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & \tilde{\varphi}_{(N-1)(N-1)} & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix}$$

with

$\tilde{\varphi}_{11} = A^T X + AX + 2\alpha X + \tilde{Q}_0 - \sum_{i=0}^{N-1} e^{-2\alpha(\tau+ih)}\tilde{R}_i$,

$\tilde{\varphi}_{12} = B\tilde{Y}_0 + e^{-2\alpha\tau}\tilde{R}_0$,

$\tilde{\varphi}_{13} = B\tilde{Y}_1 + e^{-2\alpha(\tau+h)}\tilde{R}_1$,

$\cdots$,

$\tilde{\varphi}_{1i} = B\tilde{Y}_i + e^{-2\alpha(\tau+ih)}\tilde{R}_i$,

$\tilde{\varphi}_{1i} = D$,

$\tilde{\varphi}_{22} = -e^{-2\alpha\tau}(\tilde{Q}_0 + \tilde{R}_0) + e^{-2\alpha(\tau+h)}\tilde{R}_1$,

$\tilde{\varphi}_{33} = -e^{-2\alpha(\tau+h)}(\tilde{Q}_1 + \tilde{R}_1) + e^{-2\alpha(\tau+2h)}\tilde{R}_2$,

$\cdots$,

$\tilde{\varphi}_{(N-1)(N-1)} = -e^{-2\alpha(\tau+(N-1)h)}(\tilde{Q}_{N-1} + \tilde{R}_{N-1})$,

$\tilde{\Sigma}_{12} =$

$$\begin{bmatrix} \tau\tilde{\Gamma}^T & (\tau+h)\tilde{\Gamma}^T & \cdots & (\tau+(N-1)h)\tilde{\Gamma}^T \end{bmatrix}$$

with

$$\Gamma = \begin{bmatrix} AX & B\tilde{Y}_0 & B\tilde{Y}_1 & \cdots & B\tilde{Y}_{N-1} & D \end{bmatrix},$$

$$\Sigma_{22} = diag\left\{ \tilde{R}_0 - 2X \quad \tilde{R}_1 - 2X \quad \cdots \quad \tilde{R}_{N-1} - 2X \right\}.$$

Then state feedback controllers of FHMC for the NCS can be designed as

$$K_{Si} = Y_i X^{-1} \quad i \in \{0,1,2,\ldots,N-1\} \tag{24}$$

the output feedback controllers can be designed as

$$K_{Oi} = Y_i X^{-1} C^T (CC^T)^{-1} \quad i \in \{0,1,2,\ldots,N-1\}. \tag{25}$$

Proof: Define $X = P^{-1}$, $Y_i = K_{Si}X$, $Y_i = K_{Oi}CX$, $\tilde{Q}_i = XQ_iX$, $\tilde{R}_i = XR_iX$. We first pre-and-post multiply both sides of the first matrix inequality of (7) with a diagonal matrix $diag[\underbrace{X \quad X \quad \cdots \quad X}_{N} \quad I]$. Then we use the fact that $-X\tilde{Q}_i X \le \tilde{Q}_i - 2X$, $-X\tilde{R}_i X \le \tilde{R}_i - 2X - X\tilde{Q}_i X \le \tilde{Q}_i - 2X$ to deal with nonlinear items $\tilde{Q}_i = XQ_iX$, $\tilde{R}_i = XR_iX$, respectively. Then $K_{Si} = Y_i X^{-1}$ is reached. Further, $C^{-1} = C^T(CC^T)^{-1}$ is exploited to derive the output feedback controller $K_{Oi} = Y_i X^{-1} C^T (CC^T)^{-1}$. Then the desired results are reached.

Remark 5: In fact, it is not easy to derive a static output feedback controller for LMI (23) because of the existing of nonlinear item KCX . A common method to the static output feedback controller design one can refer to [21, 22]. However, a directly method using matrix pseudo-inverse is exploiting to reach the static output feedback controllers. Thus, state feedback controller and output feedback controller are designed in a unified framework [23].

Corollary 2: Consider the memoryless control of NCS (5). For some positive scalars α , τ , ς , if there exists positive definite matrix X_0 , $\tilde{Q}_0$, $\tilde{R}_0$ satisfy

$$\begin{bmatrix} \tilde{\Sigma}_{11} & \tilde{\Sigma}_{12} \\ * & \tilde{\Sigma}_{22} \end{bmatrix} < 0 \tag{26}$$

where

$\tilde{\Sigma}_{11} = [(1,2) = X_0 A^T + AX_0 - 2\alpha X_0 + \tilde{Q}_0 - e^{-2\alpha_0\tau}\tilde{R}_0,$

$(1,2) = BY_0 + e^{-2\alpha\tau}\tilde{R}_0, (1,3) = D,$

$((2,2) = -e^{-2\alpha\tau}\tilde{Q}_0 - e^{-2\alpha\tau}\tilde{R},(3,3) = -\varsigma]$

$\bar{\Sigma}_{22} = \tilde{R}_0 - 2X_0$,

$\bar{\Sigma}_{12} = \tau[AX_0, BY_0, D]^T$.

Then the static state feedback controller can be designed as

$$K_{S0} = Y_0 X^{-1}, \tag{27}$$

the static output feedback controller can be designed as

$$K_{Oi} = Y_0 X^{-1} C^T (CC^T)^{-1}. \tag{28}$$

Proof: Let $i=0$ in Theorem 3.2, one can easily arrive at the desired result. The similarly results can also be founded in [20].

4. EXAMPLE AND DISCUSSIONS

In this section, the simulation experiments on speed tracking of PMSM are give to illustrate the proposed memory-based control scheme.

Consider the $d-q$ model of permanent-magnet synchronous motor given as follows

$$\begin{cases} \dfrac{di_d(t)}{dt} = -\dfrac{R_m}{L_d} i_d(t) + n_p w(t) i_q(t) + \dfrac{1}{L_d} u_d(t) \\ \dfrac{di_q(t)}{dt} = -\dfrac{R_m}{L_q} i_q(t) + n_p w(t) i_d(t) + \dfrac{1}{L_d} u_q(t) - \dfrac{n_p w \varphi_f}{L_q} \\ \dfrac{dw(t)}{dt} = -\dfrac{T_L}{J} + \dfrac{3n_p\varphi_f}{J} i_q(t) - \dfrac{B_v w(t)}{J} \end{cases} \tag{29}$$

where R_m is the resistance, L is the inductance, n_p is the number of pole pairs, φ_f is the permanent magnet flux, J is the moment of inertia, J is the load torque, $i_d(t)$ is the d-axis current, $w(t)$ is the rotor speed, $i_q(t)$ is the q-axis current, $u_q(t)$ is the q-axis voltage, $u_d(t)$ is the d-axis voltage.

Denote $w^*(t)$ the reference rotor speed. By defining the tracking error $e(t) = w^*(t) - w(t)$, $e(t) = w^*(t) - w(t)$ and $x_2(t) = e(t)$, we can arrive at

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ b \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) \\ y(t) = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix} x(t) \end{cases} \tag{30}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$, $a = \dfrac{B_v}{J}$, $b = \dfrac{3n_p\varphi_f}{2J}$ and $d(t) = \dfrac{\dot{T}_L}{J} + a\dot{w}^*(t) + \ddot{w}^*(t)$.

The detailed parameters of PMSM are given as following Table **1**.

Then, set the simulation time $T = 30s$, sampling period $h = 0.01s$, $\alpha = 0.2$, $\alpha = 0.2$. Based on the above derived speed control model (30) , we use YALMIP [24] to solve LMIs in Theorem 3.2. It is easy to see that $\dot{w}^*(t) = \ddot{w}^*(t) = 0$ for a constant speed tracking.

• State feedback case

Based on Theorem 3.2, the state feedback controllers of FHMC scheme are solved and given as follows

$$\begin{aligned} K_{S0} &= \begin{bmatrix} -0.0021 & -0.0035 \end{bmatrix} \\ K_{S1} &= \begin{bmatrix} -0.0014 & -0.0031 \end{bmatrix} \\ K_{S2} &= \begin{bmatrix} -0.0021 & -0.0027 \end{bmatrix} \end{aligned} \tag{31}$$

We first apply only K_{S0} and then the memory controllers K_{S0}, K_{S1}, K_{S2} to the controlled system (30). The simulations on state response and control inputs are given as below Figures **1** and **2**.

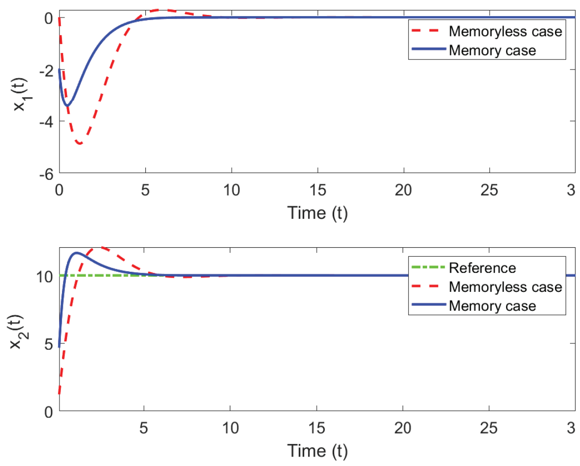

Figure 1: Responses on state feedback case.

Table 1: Parameters of PMSM

$w^*(r/s)$	$J(kg \cdot m)$	n_p	$\varphi_f(Wb)$	$B_v(N \cdot m \cdot s / rad)$
10	2.77×10^{-3}	4	0.175	5.6×10^{-6}

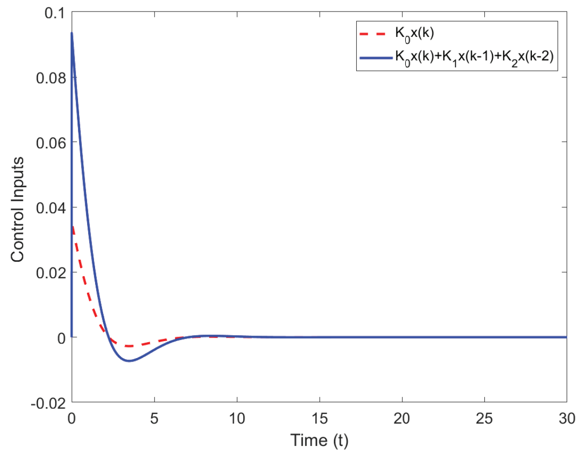

Figure 2: Control inputs under state feedback case.

The above Figure **1** shows that the performance of speed tracking under memory controllers K_{S0}, K_{S1}, K_{S2} is obviously improved by comparing the memoryless case with only K_{S0}. However, one can see that more control effort is needed while improving such tracking performance from Figure **2**.

• Output feedback case

Similar to state feedback case, the output feedback controllers of FHMC scheme are solved and given as follows

$$\begin{aligned} K_{O0} &= \left[-0.0061\right] \\ K_{O1} &= \left[-0.0045\right] \\ K_{O2} &= \left[-0.0054\right] \end{aligned} \tag{32}$$

by solving LMI (23). Here, matrices X, $\tilde{Q}_0$, $\tilde{Q}_1$, $\tilde{Q}_2$, $\tilde{R}_0$, $\tilde{R}_0$, $\tilde{R}_2$ are the same as in (31). By applying K_{O0} and K_{O1}, K_{O1}, K_{O2} to the controlled system (30) respectively, we obtained the following state response and control input results. The simulations given by the following Figure **3** and **4**.

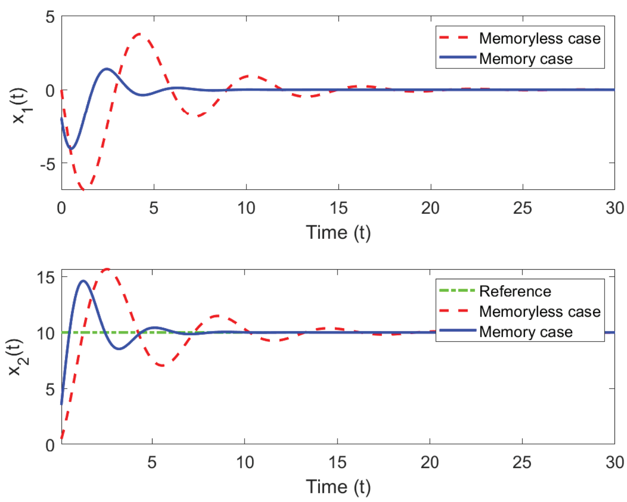

Figure 3: Responses on output feedback case.

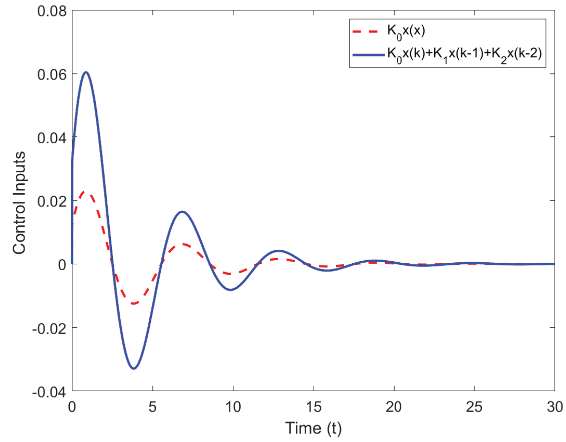

Figure 4: Control inputs under output feedback case.

From Figure **3**, it is obvious that the performance degradation is shown under output feedback case by comparing with the state feedback case which indicated by Figure **1**. However, the improvement of speed tracking performance under memory control case is obvious for the output feedback case.

Based on Figure **1** and Figure **3**, one can see that the memory control scheme is benefit to improve the control performance for both state and output feedback cases. It is well known that the tracking control performance is worse that the feedback case due to the fact that less information is obtained under output feedback case. Such fact is also confirmed by the above simulation results.

5. CONCLUSIONS

In this paper, FHMC scheme has been developed for the NCS by using the historic sampled-data. First, a chain-like delay model has been established to make full use the historic sampled-data. In order to cope with such step-by-step input delays, the corresponding chain-like Lyapunov-Krasovski function has been constructed such one can exploit the historic sampling knowledge. Then, LMIs have been derived which is readily to arrive at the memory controllers design. At last, the effectiveness of the proposed memory control scheme is confirmed via some simulation results.

In fact, the proposed FHMC scheme shows some advantages on improvement of the control performance of the NCS by exploiting the historic sampling-data under networked environment. This extended the results on input delay approach with applications to the NCSs. In the future, the event-triggered control and security issues should be further considered based on the proposed FHMC scheme.

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