The SPH-EL Method for the Simulation of the Interaction between Water Drops and Solid Wall

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Abstract: In raining days, the water drops can enter the compressor and cause security problems. However, there is still no appropriate numerical method for simulating the interaction between water drops and solid wall. The Euler-Lagrange (EL) method is usually regarded as the best solution for this problem, but it considers each water drop as a fluid particle and cannot allow the simulation of deformation. We propose to combine the Smoothed Particle Hydrodynamics (SPH) method with the EL method, which is named as SPH-EL method. In this contribution we introduce the idea of SPH-EL method as well as two key techniques: tracking the particle deformation and regrouping the distorted particles. Finally we present a 2D simulation with SPH method and will apply the SPH-EL method to improve the simulation in the future.

Keywords: Euler-Lagrange method, smoothed particle hydrodynamics, particle deformation, particle regrouping.

1. INTRODUCTION

In raining days, the water drops can enter the compressor and cause security problems, and accurate prediction is therefore necessary. In general, water drops can reduce the efficiency of compressor, and sometimes can even lead to flameout [1-5]. However there are also experiments showing that in some cases the water drops may increase the efficiency [6]. Currently experiments are always needed to evaluate the effect of water drops, but the related cost is quite expensive. In this case, an accurate numerical simulation of the interaction between water drops and solid wall would be important.

There are already many studies on this topic of numerical method. The group of Murthy has been working on simulating the water-wall interaction for over 30 years with the program WINCOF-I and simplified models [7-9]. In this century, researchers have become interested on the flow details, which cannot be predicted using these models. Recently Nikolaidis and Plidis performed a RANS (Reynolds Averaged Navier-Stokes) simulation under the Euler-Lagrange (EL) framework by using a commercial software CFX [5]. The air flow was calculated by RANS in the Eulerian framework, and every water drop was considered as a fluid particle, which is tracked under the Lagrangian view. However, the water drop and water film are treated separately by using different models, which may lead to inconsistency. Indeed, we would like to remark on the problems of present numerical methods in the following.

(1) For the tracking technique of water drops, the present EL method considers each drop as a fluid particle. As commented in Ref. [5], this technique cannot consider the deformation, split or merge of water drops. In addition, in the present method the effect from water drops to the air flow is not considered, thus if the process of evaporation is calculated, the conservation of mass and momentum cannot be guaranteed.

(2) For the calculation method of water film, the present studies often use simplified assumptions. For instance, in Ref. [5] the governing equation of film height was derived as the basic equation. Indeed, this equation is not consistent with the calculation of water drops. In particular, the conservation of momentum is difficult to be guaranteed. In addition, the phenomenon of "water rivulets" [6] is also difficult to be simulated.

(3) For the conclusions, there are also conflictions. For instance, Multhy and Mullican concluded that even a little water (about 1%) can affect much the efficiency of compressors [8], but Williams and Freeman argued that the effect is negligible [6]. In brief, more precise numerical simulation is required.

In this case, it would be necessary to develop new numerical methods to simulate the problem of water-wall interaction. In the following, we will introduce our

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idea of the SPH-EL method, as well as two key techniques: tracking the particle deformation and regrouping the distorted particles. Finally we present a 2D simulation with SPH (Smoothed Particle Hydrodynamics) method to show the future objective of the SPH-EL development.

2. IDEA OF THE SPH-EL METHOD

As stated above, the present EL method cannot precisely simulate the interaction between water drops and solid wall. Instead, we propose to combine the SPH method together with the EL method. This concept of SPH-EL can be described as follows:

(1) The EL method is employed to combine the calculation of air and water. An Eulerian view is used for calculating the air flow, while a Lagrangian view is applied for the evaluation of liquid phase.

(2) Distinct from the traditional EL method, the SPH method is applied to simulate the behaviour of water drops and water film. It means that, every water drop should be calculated with many fluid particles instead of one particle. The interaction of these particles can then be considered appropriately. In addition, the water drops and water film can be simulated consistently.

This concept of SPH-EL method is proposed above, however, currently we have not implemented all the details in practice. Indeed, it is found that the traditional SPH method has great problem in considering the problem of particle deformation. Therefore in the following two sections we will focus on solving this problem, which would be important in the future SPH-EL implementation.

3. METHOD FOR TRACKING THE DEFORMATION OF FLUID PARTICLES

There are already several studies on tracking the deformation of fluid particles [10, 11], but they are not simple to be applied in practical problems. In this contribution we propose an alternative method to tracking the deformation of fluid particles, and present two applications in numerical practice.

A Cartesian coordinate system is defined in 2D plane as *A* with its basis $\vec{a}_1, \vec{a}_2$. Likewise, we can define in the same plane a new Cartesian coordinate system B, which can be considered as the sequence of rotation of the “old” reference frame A. Column matrix $[X]_A = \left([\alpha_1 \quad \beta_1]_A\right)^T$ or $[X]_B = \left([\alpha_2 \quad \beta_2]_B\right)^T$ (here *T* means transpose) can be used as the symbol for the same vector $\alpha_1\vec{a}_1 + \beta_1\vec{a}_2 = \alpha_2\vec{b}_1 + \beta_2\vec{b}_2$. The subscript of a column matrix identifies that the elements in this matrix are the coordinates in a certain reference frame. We use an oriented notation $(\vec{*}, \vec{*})$ to define the angle of rotation θ from the “old” coordinate *A* to “new” one $B: \theta = (\vec{a}_1, \vec{b}_1) = (\vec{a}_2, \vec{b}_2)$. The conversion of the two coordinates is written in matrix form:

$$\begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}_B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}_B^A \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}_A = M_B^A(r_\theta) \begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix}_A . \quad (1)$$

The elliptic fluid particle is a suitable model with a favorable criterion for the deformation: the ratio of lengths between the major axis and the minor axis. The ellipse can be represented in a quadratic form. For example, we establish a reference frame *E* using the major axis $\vec{e}_1$ and the minor axis $\vec{e}_2$ of an ellipse (see Figure **1**). We suppose $A = 1/a^2, B = 1/b^2$ and the coordinates of ellipse is $[X_1] = \left([x_1 \quad y_1]_E\right)^T$. Then the equation of ellipse in quadratic form is:

$$\left([X_1]_E\right)^T \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}_E^E [X_1]_E = 1 \quad (2)$$

I is the inertial coordinate system and the velocity of flow is $u\vec{i} + v\vec{j}$. According to the polar decomposition theorem, the velocity gradient tensor can be decomposed into a symmetric tensor s_{ij} and an anti-symmetric tensor a_{ij}. The tensor s_{ij} is diagonalizable and its two eigenvectors s_1, s_2 are the principal axes of strain:

$$s_{ij} = \left(U_I^I\right)^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}_I^I U_I^I \quad (3)$$

where U_I^I is an orthogonal matrix. Along the two axes of the orthonormal coordinate system *S*, the strain tensor s_{ij} becomes a pure stretch with no shear component (see Figure **1a**). The frame *E* rotates to the frame *S* with an angle $\psi = (\vec{e}_1, \vec{s}_1) = (\vec{e}_2, \vec{s}_2)$.

From the anti-symmetric tensor a_{ij} the angular velocity is obtained:

$$\omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \quad (4)$$

After a time step of *dt*, the ellipse is stretched by the tensor s_{ij} (see Figure **1b**) and then rotates like a rigid

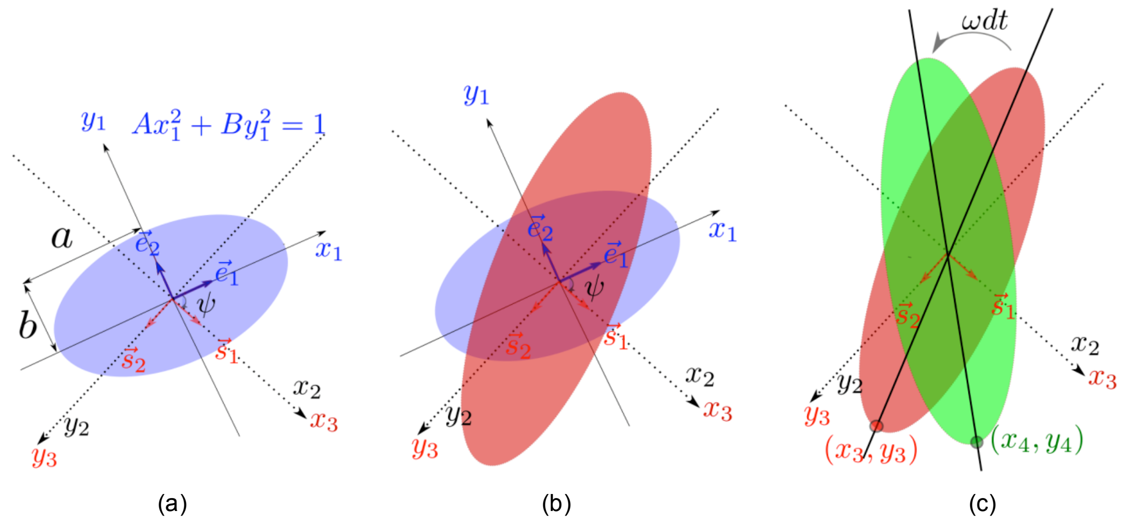

Figure 1: (**a**) a process of stretch with no shear component $[X_2]_s = M_S^E(r_\psi)[X_1]_E$; (**b**) sketch of $[X_3]_s M_S^S(s)[X_2]_s$;(**c**) sketch of $[X_4]_s = M_S^S(r_{-\omega dt})[X_3]_s$.

body under the influence of the tensor a_{ij} (see Figure **1c**). This stretch effect corresponds to a linear mapping s in 2D plane which can be expressed as a diagonal matrix $M_S^S(s)$. Then the ellipse rotates $-\omega dt$ in 2D plane, regardless of reference frame. We note that this rotation can be realized by relative motion: the frame (here we suppose the frame *S*) rotates $-\omega dt$ while the ellipse is fixed.

We concentrate on the evolution of a certain vector. The relation between its old coordinates $[X_1]_E$ on the initial ellipse under the frame *E* and its new coordinates $[X_4]_s$ under frame:

$$[X_4]_s = M_S^S(r_{-\omega dt}) M_S^S(s) M_S^E(r_\psi)[X_1]_E. \quad (5)$$

Substituting Eq.(5) into Eq.(2), we can obtain the equation of new ellipse after a time step of *dt*.

$$\left([X_4]_s\right)^T M_S^S(r_{-\omega dt}) M_S^S(s) M_S^E(r_\psi) \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}_E^E M_E^S(r_{-\psi}) \left(M_S^S(s)\right)^{-1} M_S^S(r_{\omega dt})[X_4]_s = 1. \quad (6)$$

In fact, the matrix between $\left([X_4]_s\right)^T$ and $[X_4]_s$ in Eq.(6) is a symmetric matrix. Consequently, we can simplify the equation of new ellipse Eq.(6) if we diagonalize this matrix. After solving this eigenvalue problem, we find

$$M_S^S(r_{-\omega dt}) M_S^S(s) M_S^E(r_\psi) \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}_E^E M_E^S(r_{-\psi}) \left(M_S^S(s)\right)^{-1} M_S^S(r_{\omega dt}) = \left(V_S^S\right)^{-1} \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix}_S^S V_S^S,$$

where V_S^S is an orthogonal matrix. The eigenvector $\vec{n}_1$ is associated with the eigenvalue μ_1, and the eigenvector $\vec{n}_2$ is associated with the eigenvalue μ_2. After calculating Eq.(3) and (4), we are able to solve the eigenvalue problem in Eq.(7). Then the new equation of new ellipse (green ellipse) in reference frame *N* is obtained as $\mu_1 x_5^2 + \mu_2 y_5^2 = 1$. Finally the ratio of lengths between the major axis and the minor axis can be calculated.

4. METHOD FOR REGROUPING THE DISTORTED FLUID PARTICLES

Although this method has great advantages on the surface simulation, the particle distortion is always an obvious problem and has never been fully solved [12]. For instance, the Taylor-Green Vortex will lead to inhomogeneous particle distribution and divergence (see Figure **2b**). Traditional method usually uses the remeshing method [13], but this involves too strong numerical dissipation. Moreover, it is difficult to apply the remeshing method on the free-surface problems. Another method to deal with the particle distortion is generating randomly-distributed particles instead of the homogeneous ones, and using the periodic redistribution technique [14], but this still does not fully solve the distortion problem. In this contribution we introduce a local regrouping method for avoiding the particle distortion in the 2D situation.

We first develop the regrouping method in 2D, and the advantage of this choice is that the geometry properties of the particles are easily represented. Each particle is defined by its position (*x, y*), its short axis λ_1, its long axis λ_2 (the method of calculating λ_1 and λ_2

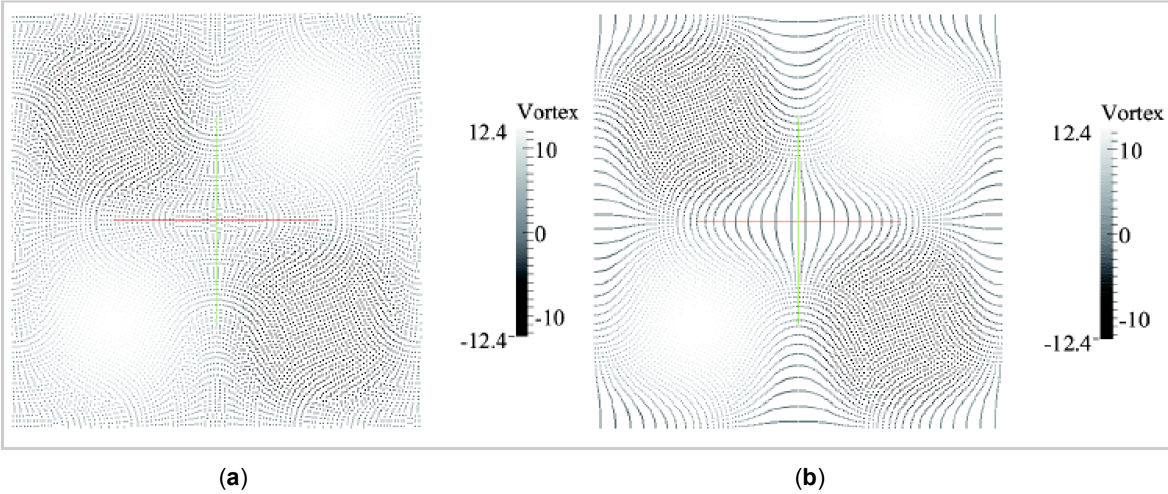

Figure 2: Distribution of the particles for the 2D Taylor Green vortex simulation at step 2000. (**a**) with regrouping and (**b**) without regrouping.

can be found in the last section), its unit direction vector of short axis $a=(a_x,a_y)$, and its unit direction vector of long axis $b=(b_x,b_y)$. We may also define its rate of deformation γ as λ_2/λ_1. For simplicity, the particle is considered as a rectangle. In order to handle the problem of deformation efficiently, the particles are classified into two types, the ordinarily deformed particles whose rate of deformation is greater than c_1, and the extraordinarily deformed particles of which the rate of deformation is greater than c_2. In this paper, we choose $c_1=2.3$ and $c_2=4.6$.

This method is only adapted to the ordinarily deformed particles. Two ordinarily deformed particles marked (1) and (2) are taken into account, and after the regrouping, two new particles marked (3) and (4) are obtained eventually. This transformation is illustrated in Figure **3**.

A new open-source program "SPHturb" of 2D/3D SPH simulation for homogeneous isotropic turbulence, inspired by the formulation of Gingold and Monaghan, is developed as a platform for the present work. Here we focus on the Taylor-Green vortex in two dimensions, and the velocity $V=(u,w)$ is specified by

$$\begin{aligned} u &= A\sin\left(2\pi\left(\frac{x}{L}+\phi\right)\right)\cos\left(2\pi\left(\frac{z}{L}+\phi\right)\right)F(t),\\ w &= C\cos\left(2\pi\left(\frac{x}{L}+\phi\right)\right)\sin\left(2\pi\left(\frac{z}{L}+\phi\right)\right)F(t), \end{aligned} \tag{8}$$

where $F(t)=e^{-2\nu t}$. At time $t=0, F(t)=1$. Boundary conditions are considered as periodic with a period *L*. In this case, we choose *A* = 1, *C* = -1, and the kinematic viscosity of the fluid ν = 0. In the flowing simulation, the problem domain is a square with side length *L* = 1*m,* and the number of the particles is 100×100. The time step is 10^{-4}s.

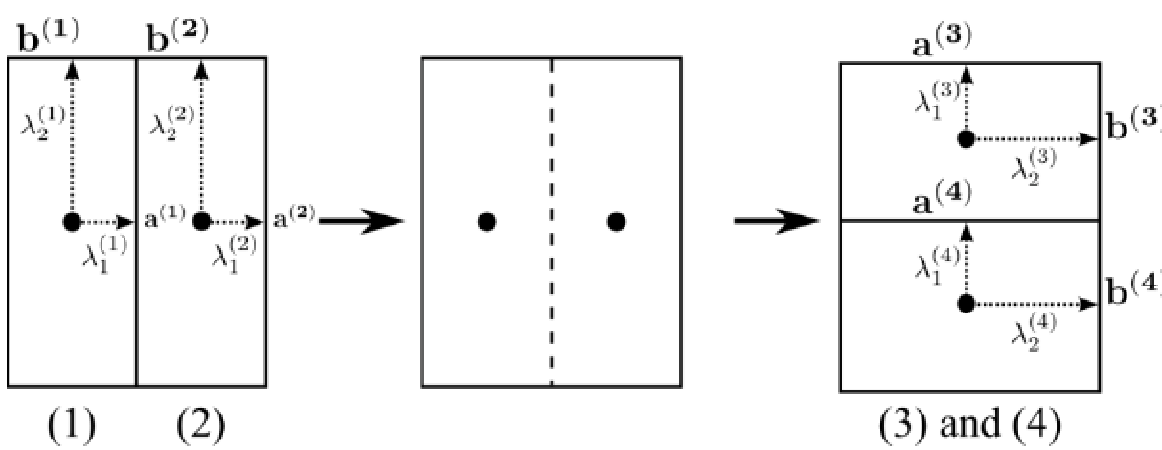

Figure 3: Illustration of two particles' regrouping.

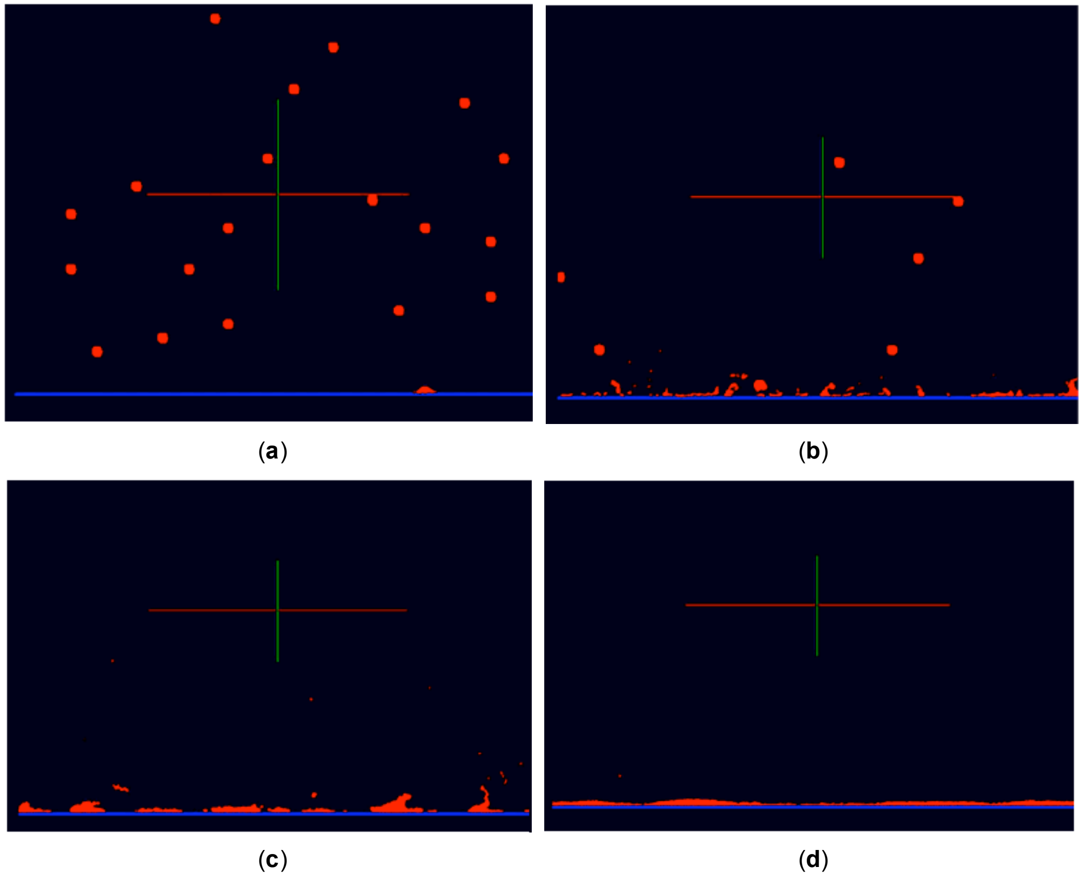

Figure 4: Two-dimensional SPH simulation of water drops. From (**a**) to (**d**): time advance.

To demonstrate the effectiveness of these methods, we compare the simulation of the Taylor-Green vortex with regrouping to the one without it. It is shown in Figure **2** that, at the same time, the distribution of particles with regrouping is much more ordered than the one without it. This fact results in the numerical stability of the test case with local regrouping.

5. TWO-DIMENSIONAL SPH SIMULATION OF WATER DROPS AND SOLID WALL

In order to illustrate the idea of SPH-EL, here we perform a two-dimensional SPH simulation on the interaction between water drops and solid wall. The open-source program SPHysics [15] is employed for this calculation. Figure **4** shows the process from water drops to water film. Although the process is already simulated, the details are not analyzed yet. In particular, the techniques of tracking deformation and regrouping, which are introduced in the last two sections, have not been implemented in this case. This implementation will be performed in the near future, illustrating the possibility of employing the SPH concept to improve the EL method.

6. CONCLUSION

In this contribution, the concept of SPH-EL method is introduced as a possible improvement of the simulation of the interaction between water drops and solid wall. Two key techniques, that is, tracking the particle deformation and regrouping the distorted particles, are then developed. A test case of the Taylor-Green vortex shows the performance of the new techniques. In addition, we present a 2D simulation with SPH method and will apply the SPH-EL method to improve the simulation in the future.

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REFERENCES

[1] Kissel G. Rain and Hail Extremes at Altitude. J Aircr 1980; 17(7): 464-67. http://dx.doi.org/10.2514/3.57926

[2] Luers J and Haines P. Heavy Rain Influence on Airplane Accidents. J Aircr 1983; 20(2): 187-91. http://dx.doi.org/10.2514/3.44850

[3] Santa I. The Effect of Water Ingestion on the Operation of the Gas Turbine Engine. Proceedings to the 22nd ICAS Congress 2000; 524.1-524.8.

[4] Kennedy JB and Roberts J. Rain Ingestion in a Gas Turbine Engine. Proceedings to the 4th ILASS Meeting. Institute of Liquid Atomization and Spray Systems 1990; pp 154.

[5] Nikolaidis T and Pilidis P. The Effect of Water Ingestion on an Axial Flow Compressor Performance. In: Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 2014; 228(3): 411-23. http://dx.doi.org/10.1177/0954410012474421

[6] Williams J, Freeman C and Day I. Rain Ingestion in Axial Flow Compressors at Part Speed. Proceedings to the ASME Turbo Expo 2005: power for land, sea, and air. http://dx.doi.org/10.1115/1.2366511

[7] Tsuchiya T and Murthy SNB. Water Ingestion into Axial Flow Compressors, Part I: Analysis and Predictions. WFWAL-TR-80-2090, Wright-Patterson Air Force Base 1976.

[8] Murthy SNB and Mullican A. WINCOF-I Code for Prediction of Fan-compressor Unit with Water Ingestion. Report No. DOT/FAA/CT-TN 89/63, and NASA Contractor Report CR 185157, 1990.

[9] Murthy SNB. Effect of atmospheric water ingestion on the performance and operability of flight gas turbines. In: Proceedings of 32nd AIAA/ASME/SAE/ASEE joint propulsion conference and exhibit 1996. http://dx.doi.org/10.2514/6.1996-3059

[10] McKenzie D. Finite Deformation During Fluid Flow. Geophys J R astr Soc 1979; 58(3): 689-715. http://dx.doi.org/10.1111/j.1365-246x.1979.tb04803.x

[11] Subramanian N, Kellogg LH and Turcotte DL. Statistics of Advective Stretching in Three-dimensional Incompressible Flows. J Stat Phys 2009; 136: 926-944. http://dx.doi.org/10.1007/s10955-009-9807-0

[12] Koumoutsakos P. Multiscale Flow Simulations Using Particles. Annu Rev Fluid Mech 2005; 37: 457-87. http://dx.doi.org/10.1146/annurev.fluid.37.061903.175753

[13] Cleveland W and Loader C. Smoothing by Local Regression: Principles and Methods, In: W. Hardle and M.G. Schimek. Proceedings to the Statistical Theory and Computational Aspects of Smoothing, Physica-Verlag HD 1996; 10-49. http://dx.doi.org/10.1007/978-3-642-48425-4_2

[14] De Leffe M and Le Touze D. SPH modeling of shallow-water coastal flows. J Hydraul Res 2010; 48: 118-25. http://dx.doi.org/10.1080/00221686.2010.9641252

[15] Gómez-Gesteira M, Rogers BD, Dalrymple RA, *et al.* User guide for the SPHysics code v2. 0. [updated 2012]: Available from: http://www.sphysics.org.

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