Steady Free Convection Boundary Layer Flows at a Vertical Plate with Variable Fluid Properties

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Abstract: This paper investigates the similarity solutions of the steady two-dimensional flow of a stream of viscous fluid with far field viscosity past a vertical plate. The variable viscosity, thermal conductivity and heat sink in momentum and energy equations are incorporated. The governing system of equations are transformed into dimensionless equations and solved numerically by using Maple-13 software for different boundary conditions and for various values of parameters. The effects of different values of physical parameters on the velocity and temperature profiles as well as on the skin-friction coefficient and Nusselt number are discussed.

Keywords: Free convection, Similarity solutions, variable properties and Heat source.

1. INTRODUCTION

The flow and heat transfer of a thin film determining the coating process, chemical processing equipment's and heat exchangers design. It is having other applications include food stuff processing, wire and fibre coating and transpiration cooling etc. The plasma optical emission from various distance from graphite surface moving with temperature varied from the plasma plume have been explained by Diamant *et al*. [1].

For the case of flow without heat transfer the non-dimensionalized thermal equation depends on the viscosity parameter, dimensionless temperature and the equation depends on the thermal conductivity, Prandtl number and dimensionless temperature Reynolds Number and hence all physical realizations of the related experiment will have the same value of non-dimensionalized variables for the same Reynolds Number. Arunachalam and Rajappa [2] studied forced convection in liquid metal with variable thermal conductivity and obtained explicit analytical solutions in closed form. Carey and Mollendorf [3] investigated heat transfer in fluid flow of low Prandtl number with variable thermal conductivity. Fluid flow and heat transfer characteristics of a stretching sheet with variable temperature condition was investigated by Grubka and Bobba [4]. Effect of variable viscosity and the thermal diffusivity on mixed convection flow along vertical isothermal plate have been reported by Seddeek and Salem [4].

Mahanti and Gaur [6] studied the effects of linearly varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. Mohammad Rashidi *et al.* [7] applied one parameter continuous group method to investigate magneto-hydrodynamics (MHD) heat and mass transfer flow of a viscous incompressible fluid over a flat plate.

The aim of the present work is twofold: firstly to derive systematically the similarity transformation under similarity requirement for the governing equations, secondly the highly nonlinear PDE's governing the particular fluid flow of boundary layer theory is transformed into an ODE by searching the group of transformation subject to the similarity requirement. The reduced non-linear ODE-BVP is numerically solved by Runge-Kutta shooting method using MAPLE 13 computational algorithm.

2. PROBLEM FORMULATION

We consider steady two dimensional flow of a thin layer (boundary layer) of incompressible fluid past a vertical plate along the x-axis. We incorporate heat sink in the energy equation. The x-axis is taken along the plate and y-axis is normal to the plate. The physical model and the coordinate system are shown in Figure **1**.

Under the Buossinesq approximation, the governing continuity, momentum and energy equations are written as,

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Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(u*\frac{\partial u}{\partial y})+u_e\frac{du_e}{dx}+g\beta(T|-T_\infty) \tag{2}$$

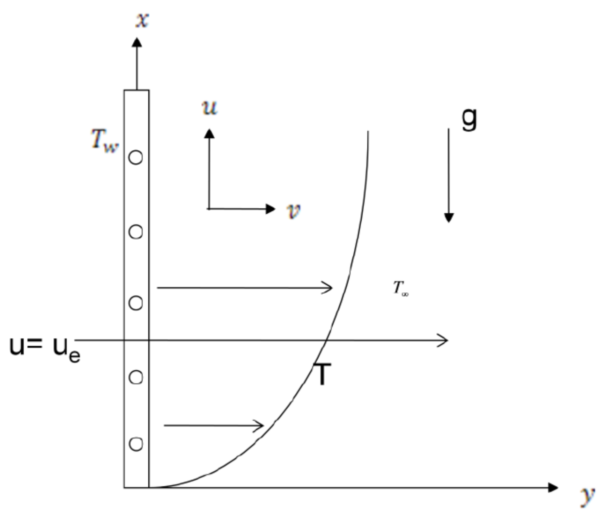

Figure 1: Physical and Coordinate system.

Energy Equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k*\frac{\partial T}{\partial y}\right)+Q \tag{3}$$

Where u and v are the velocity components in the x and y directions respectively. T is the fluid temperature, v is the kinematic viscosity, ρ is the fluid density. $u_e(x)$ is the velocity at the edge of boundary layer, g is the acceleration due to gravity, β is the co efficient of thermal expansion, T_w is the wall temperature, T_∞ is the ambient temperature, c_p is the specific heat, $k*$ is the variable thermal conductivity.

Subjected to the boundary conditions are,

$$\left.\begin{array}{l} u=0, v=0, w=0,\ T=T_w \quad at \quad y=0 \\ u\to u_e(x)=u_\infty x, \quad T\to T_\infty\, as \quad y\to\infty \end{array}\right\} \tag{4}$$

we may take the following suitable similarity variables as discussed by Darji and Timol [8].

$$x^*=\frac{x}{L}, y^*=\frac{y}{L}\sqrt{\mathrm{Re}}, u^*=\frac{u}{u_\infty}, v^*=\frac{v\sqrt{\mathrm{Re}}}{u_\infty}, \mathrm{Re}=\frac{u_\infty L}{\upsilon}$$

$$u_e^*=\frac{u_e}{u_\infty},$$

$$\theta-\frac{T-T_\infty}{T_w-T_\infty} \tag{5}$$

$$K*=k[1+\in\theta]$$

$$\mu*=\mu[1+\lambda(\theta-\frac{1}{2})]$$

$$Q=S\{k(T_w-T_\infty)/x^2(Gr/4)^{1/2}(T-T_\infty)\}$$

where L is the reference Length, $u_\infty(x)$ is the velocity of main stream, v is the kinematic viscosity, R_e is the Reynolds number.

The boundary layer equations (1-3) reduces to,

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{6}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=\frac{\partial}{\partial y^*}\left[\left\{1+\gamma\left(\theta-\frac{1}{2}\right)\right\}\frac{\partial u^*}{\partial y^*}\right]+x\frac{u_\infty^2}{u_0^2}L+ \\ g\beta\theta L\,(T-T_\infty) \tag{7}$$

$$\frac{u_0}{L}u^*\theta'(T_w-T_\infty)+\frac{u_0}{L}v^*\theta'(T_w-T_\infty)=\frac{L}{u_0}k\varepsilon\left(\frac{\partial\theta}{\partial y}\right)^2 \\ +k(1+\varepsilon\theta)\frac{L}{u_0}+\frac{QL}{u_0(T_w-T_\infty)} \tag{8}$$

The above equations can be reduced to a system of ordinary differential equations by defining new variables solved by Rashidi *et al*. [7].

$$\eta=y$$

$$\Psi=xf(\eta)$$

$$u=\frac{\partial\Psi}{\partial y}=xf'(\eta)$$

$$v=-\frac{\partial\Psi}{\partial x}=-\frac{\partial}{\partial x}(xf(\eta)) \\ =-xf'(\eta).0-f(\eta)=-f(\eta)$$

Using mathematical manipulations, we can write the transformed momentum and energy equations for two dimensional incompressible thin layer flow as,

$$\left[1+\gamma\left(\theta-\frac{1}{2}\right)\right]f'''+(\gamma\theta'+f)f''-f'^2+u_\infty^2+Gr\theta=0 \tag{9}$$

$$(1+\varepsilon\theta)\theta''+\varepsilon\theta'^2+pr\theta'f+S\theta=0 \tag{10}$$

Equations (4) are subject to the following boundary conditions,

$$\left.\begin{array}{l} f(0)=0, f'(0)=0, \theta(0)=1 \\ f'(\infty)=u_\infty, \theta(\infty)=0 \end{array}\right\}$$

In the above prime denote differentiation with respect to η. Where Grashof number is defined as $Gr=\frac{g\beta L(T_w-T_\infty)}{u_\infty^2}$ and Heat sink parameter is defined as $S=\frac{QL}{\rho c_p(T_w-T_\infty)}$

From the engineering point of view the important characteristics of the flow and the skin friction co efficient and the Nusselt number respectively.

Skin friction co efficient:

$\tau_{yx}=\mu\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)$ where $\frac{\partial v}{\partial x}\to 0$

Since $(v<<u)$, therefore the skin friction coefficient as the non dimensional wall shear stress given by

$$c_f=\frac{\tau_{wall}}{0.5\rho u_\infty^2}=\frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{0.5\rho u_\infty^2}=\frac{\mu\left[1+\gamma(\theta-0.5)\right]}{0.5\rho u_\infty^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$=\frac{\mu\left[1+\gamma(\theta-0.5)\right]}{0.5\rho u_\infty^2}f''(0)$$

The rate of heat transfer in terms of the Nusselt number at plate may be written as:

$$N_u=\frac{xq_w}{k(T_w-T_\infty)};\ where\ q_w=-k\frac{\partial T}{\partial y}$$
$$=\frac{x\left(-k\frac{\partial T}{\partial y}\right)_{wall}}{k(T_w-T_\infty)}$$
$$=\frac{x[-k\theta'(T_w-T_\infty)]_{wall}}{k(T_w-T_\infty)}$$
$$=-\,x\theta'(0)$$

3. NUMERICAL RESULTS AND DISCUSSION

The coupled system of equations (9) - (10) subject to boundary conditions in (11) were solved numerically by RungeKutta-Fhelberg method along with shooting technique using Maple-13 software, see Aziz [9]. The asymptotic boundary conditions given by (11) were replaced by using a value of 6 for the similarity variable η_{max} as follows:

$$\eta_{max}=6, f'(6)=u_\infty=0.5, \theta(6)=0$$

The choice of η_{max} = 6 confirms that the far field boundary conditions satisfy correctly. It is worth mentioning that a uniform grid of $\Delta\eta=0.01$ was found to be satisfactory for convergence criterion of 10^{-6} is all most all the cases. Solutions for a range of $\gamma, Gr, \in, Pr$ and S are however useful since they illustrate the main features of the response boundary layers.

We can demonstrate the variation of velocity, temp profile shapes and the missing slopes ($f''(0), \theta'(0)$) near the plate.

The values of $f''(0), \theta'(0)$ for different values of γ, Pr, S and $\in$ are tabulated in Table **1** as these are used for the evaluation of skin friction and Nusselt number. It is observed from this table that the velocity and the temperature of the fluid decrease with an increase in Prandtl number.

Figure **2** shows the corresponding velocity profiles for several values of the Prandlt number Pr =1,7,10 with Gr = 1, u∞ = 0.5, © = -0.4, S = 0, ε = 0.1 when Pr>>1 (heavy, high viscosity oils) the velocity boundary layer is very much thicker.

The temperature profiles in Figure **3** are for a vertical plate with fluids of different Prandtl number where u∞ = 0.5, Gr = 1, © = -0.4, S = 0, ε = 0. The profile thickness is greatly affected by the Prandtl number, but the effect on profile shape is rather smaller at higher values.

Figures **4-5** shows the Grashof number (Gr = 1,1.5,2) results obtained with u∞ = 0.5, Pr =1, S = 0.0, © = -0.4, ε = 0.3. It is seen that the velocity profile increases with increasing Gr and the temperature profile decreases with increasing Gr.

Figures **6-7** depict the effects of © = -0.4,0, 0.4 with u∞ = 0.5, Pr = 1, S = 0.0, Gr = 1, ε = 0.3 as might be expected the higher values of γ with corresponding decrease of velocity fluid flow and increase of temperature fluid flow.

Figures **8-9** present the influence of S = 0, 0.2, 0.5 with u∞ = 0.5, Gr = 1, © = -0.4, Pr = 1, ε = 0.1 on the velocity and temperature profiles. It is seen that the velocity flow profile is accelerating with the increase of S as it moves away from the vertex. On the temperature of fluid first accelerating and then decelerating due to increasing S.

Table 1: Missing Slopes of $f''(0)$ and $-\theta'(0)$ for Different Values of γ, Pr, S and $\in$

	$\gamma=-0.4$		$\gamma=0.0$		$\gamma=0.4$	
S=0.0 **P_r = 1**	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
$\in=0.0$	0.875777	0.384717	0.753160	0.377660	0.665743	0.371659
$\in=0.1$	0.884052	0.363770	0.761043	0.357162	0.673253	0.351534
$\in=0.3$	0.898691	0.330512	0.774964	0.324631	0.686502	0.319610
S=0.0 **P_r = 7**						
$\in=0.0$	0.656103	0.772291	0.555777	0.753062	0.485388	0.737263
$\in=0.1$	0.664478	0.732947	0.563347	0.714670	0.492547	0.699635
$\in=0.3$	0.679631	0.670694	0.577383	0.653971	0.505460	0.640186
S=0.1 **P_r = 7**						
$\in=0.0$	0.664463	0.724870	0.563862	0.704375	0.493274	0.687457
$\in=0.1$	0.672656	0.688873	0.571397	0.669429	0.500233	0.653362
$\in=0.3$	0.687488	0.631849	0.585007	0.614115	0.512286	0.599434
S=0.2 **P_r = 7**						
$\in=0.0$	0.673074	0.676073	0.572244	0.654188	0.501395	0.636031
$\in=0.1$	0.681074	0.643543	0.579597	0.622820	0.508230	0.605614
$\in=0.3$	0.695564	0.591932	0.592886	0.573095	0.520560	0.557430
S=0.2 **P_r = 10**						
$\in=0.0$	0.618393	0.765630	0.533015	0.755851	0.461646	0.723136
$\in=0.1$	0.636858	0.742959	0.540222	0.719448	0.468622	0.687394
$\in=0.3$	0.651166	0.683288	0.553279	0.661829	0.480168	0.633012

S=0.0 $\in=0.3$ **P_r = 1**	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$	$f''(0)$	$-\theta'(0)$
Gr = 1	1.227487	0.437548	1.046749	0.426792	0.918317	0.417720
Gr = 1.5	1.554203	0.469785	1.325511	0.458246	1.163067	0.448538
Gr = 2	1.859452	0.496465	1.585957	0.484278	1.391731	0.474042

Figures **10-11** show the corresponding velocity and temperature profiles for various values of ε = 0, 0.1, 0.3 with u_∞ = 0.5, Pr = 1, G r = 1, © = -0.4, S = 0.0.

The profiles in Figure **11** show that ε has some effect on the temperature profiles but much less than on the velocity profiles (Figure **10**). Of course this effect is exerted via changes in the velocity profile, ε as such does not appear in the momentum equation for incompressible flow, equation (10) with the increase in the value of ε, the temperature of fluid increases. The temperature profile thickness is greatly affected by ε.

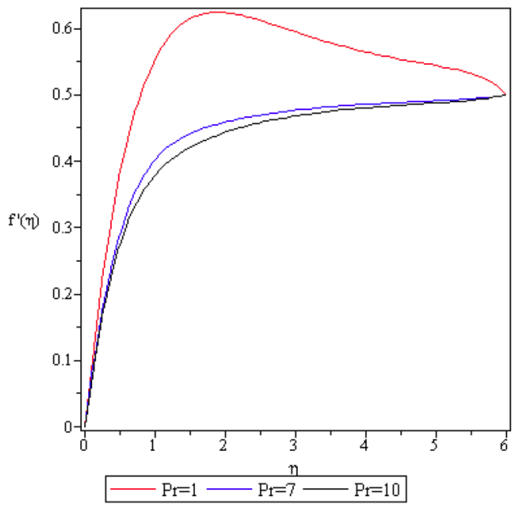

Figure 2: The dimensionless velocity f' as a factor of η for various values of Pr.

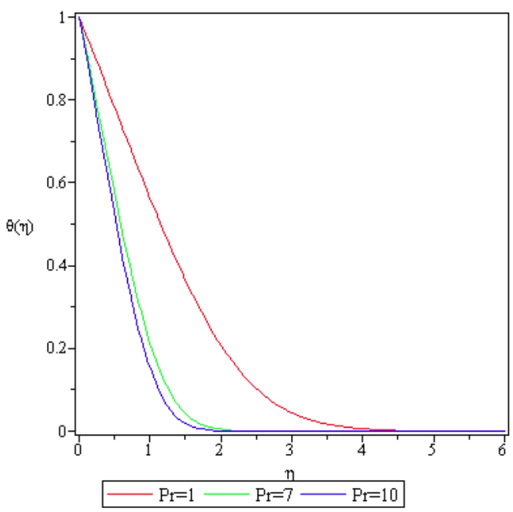

Figure 3: The dimensionless temperature θ as a factor of η for various values of Pr.

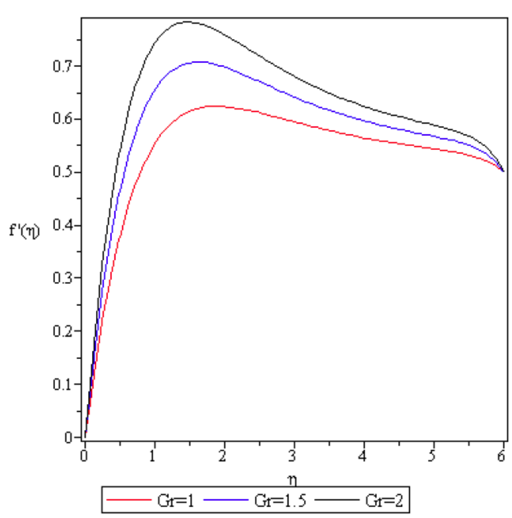

Figure 4: The dimensionless velocity f' as a factor of η for various values of Gr.

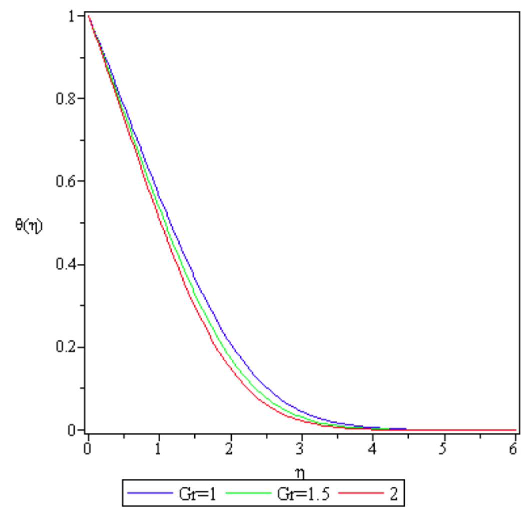

Figure 5: The dimensionless temperature θ as a factor of η for various values of Gr.

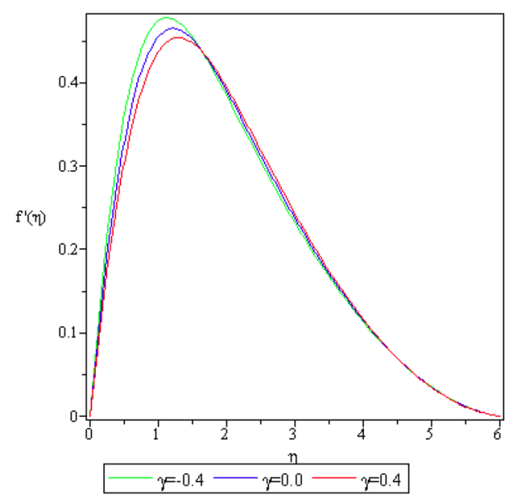

Figure 6: The dimensionless velocity f' as a factor of η for various values of γ.

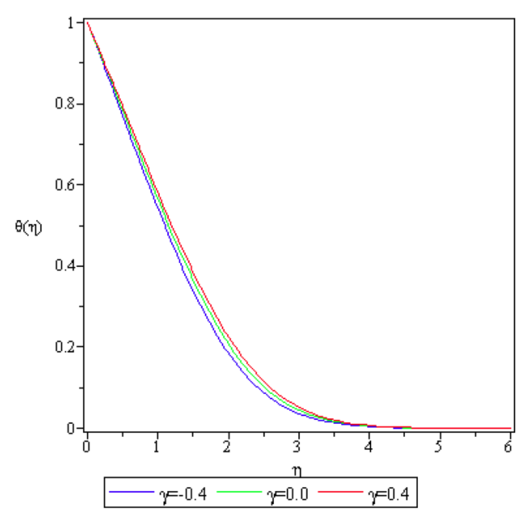

Figure 7: The dimensionless temperature θ as a factor of η for various values of γ.

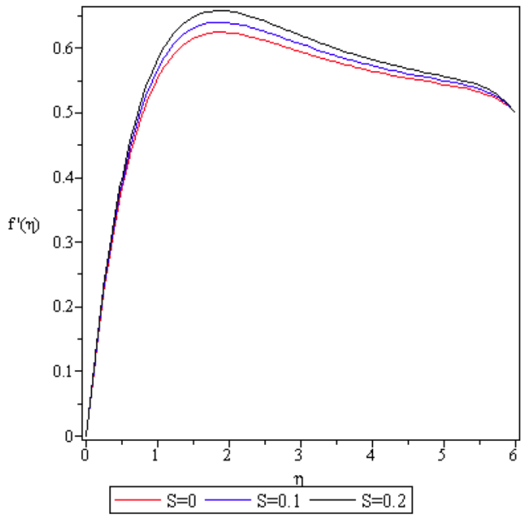

Figure 8: The dimensionless velocity f' as a factor of η for various values of S.

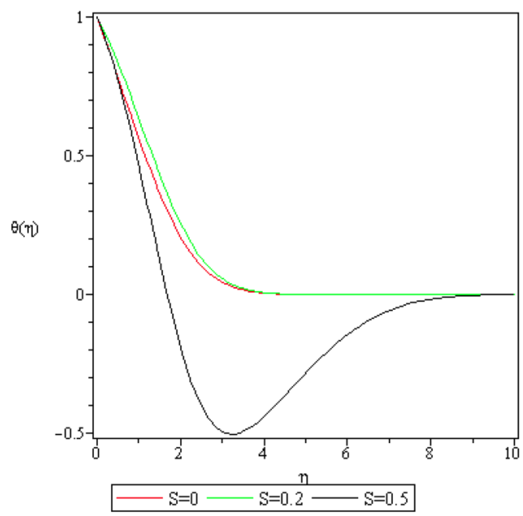

Figure 9: The dimensionless temperature θ as a factor of η for various values of S.

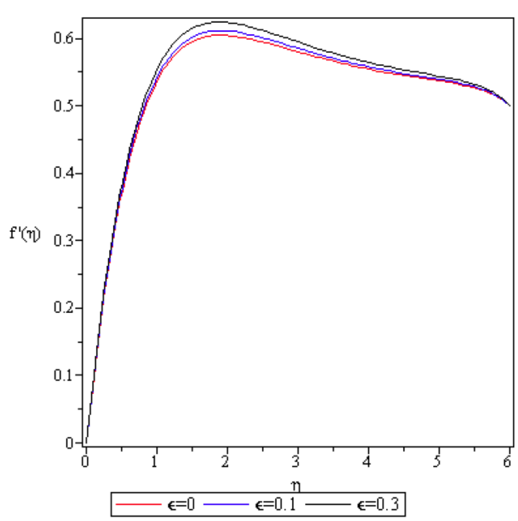

Figure 10: The dimensionless velocity f' as a factor of η for various values of ε.

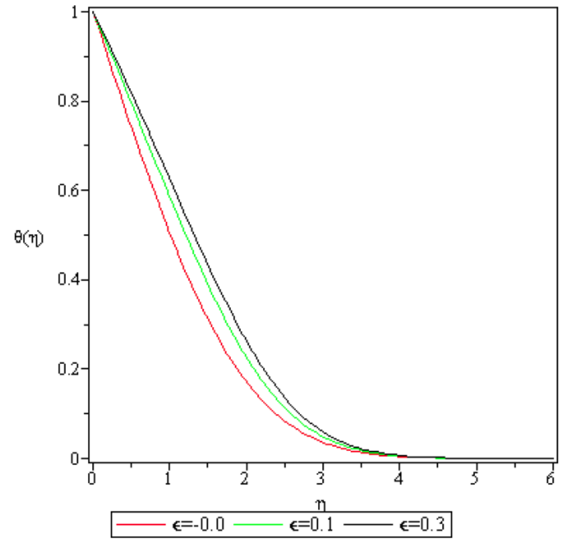

Figure 11: The dimensionless temperature θ as a factor of η for various values of ε.

4. CONCLUSION

In the present work we have transformed the highly nonlinear PDEs governing the particular fluid flow of boundary layer theory into an ODE by searching the group of transformation subject to the similarity requirement. The reduced non-linear ODE-BVP is numerically solved by Runge-Kutta shooting methodusing MAPLE 13 computational algorithm. It has been shown that:

1. The velocity and the temperature of the fluid decrease with the increase in Prandlt number.
2. As the Prandlt number increases, the velocity and the thermal boundary layers thickness decrease.
3. With the decrease in heat sink parameter S, the velocity and the temperature of the fluid decrease.
4. The velocity and the thermal boundary layers thickness decrease with decrease in heat sink parameter S.
5. The increase in thermal conductivity parameter ε increase the velocity and temperature of fluid irrespective of value of heat sink parameter S.
6. The increase in viscosity parameter γ decrease the velocity of fluid near the plate however the effect of viscosity parameter © is negligible on the temperature of the fluid.

7. Skin –friction co-efficient increase, while the rate of heat transfer increase with the increase in thermal parameter conductivity parameter.

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