A Note on Heat Transfer Unsteady Similar Boundary Layer Flow Over a Streatching Surface with Power Law Temperature

M. Ferdows[1,*], Zavid Iqbal Bangalee[1], Selina Parvin[2] and Don Liu[3]

[1]*Research Group of Fluid Flow Modeling and Simulation, Department of Applied Mathematics, University of Dhaka, Dhaka-1000, Bangladesh*

[2]*Department of Mathematics, University of Dhaka, Dhaka-1000, Bangladesh*

[3]*Mathematics and Statistics, and Mechanical Engineering, Louisiana Tech University, Ruston, USA*

Abstract: The aim of this paper is to investigate the effects of thermal radiation and viscous dissipation on unsteady two dimensional viscous and incompressible boundary layer flow and heat transfer from a stretching surface. The governing partial differential equations are transformed into a system of ordinary differential equations by using similarity transformation. The governing equations are solved through the use of Spectral relaxation method. The solution is found to be dependent on the governing parameters including the Prandtl number, unsteadiness parameter, thermal radiation parameter, Eckert number and power law temperature parameter. Flow profiles are presented graphically for various values of the problem parameters.

Keywords: boundary layer, temperate variation, Spectral method, similarity.

1. INTRODUCTION

Flow and heat transfer over a continuously moving surfaces constitutes numerous applications for engineering purposes. The basic pioneering work seems to be [1] and lots of work has been done based on this work. Elbashbeshy [2] investigated the effects of injection and suction on the heat transfer from stretching surface with variable surface heat flux. Chen [3] considered the combined effect on flow over a stretching surface. The unsteady boundary-layer flow over stretching sheet has been studied by [4]. However the unsteady laminar incompressible flow over a stretching sheet configurations was studied numerically [5, 6]. The objective of the present study is to find the similarity solution of flow over a linearly stretching sheet with power law surface temperature considering viscous dissipation and radiation effects. The nonlinear, coupled governing partial differential equations is solved by Spectral relaxation method.

2. BASIC EQUATIONS

Consider an unsteady, two-dimensional laminar boundary layer flow over a continuously stretching surface immersed in an incompressible viscous fluid. At time t = 0, the plate is impulsively stretched with the velocity u_w (x, t) along the x-axis, keeping the origin fixed in the fluid of ambient temperature T_∞. The stationary Cartesian coordinate system has its origin located at the leading edge of the plate with the positive x-axis extending along the plate, while the y-axis is measured normal to the surface of the plate. We focus on the simulation when the surface temperature T_w is a power function of distance along the stretching surface. Under the boundary layer approximation with the Boussinesq assumption, the governing equations for continuity, momentum, and energy conservation, incorporating viscous dissipation and radiation effects, may be shown to take the form:

2.1. Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

2.2. Momentum Equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}$$

2.3. Energy Equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}(\frac{\partial u}{\partial y})^2$$

where, u and v are the velocity components along the x and y axes, respectively, T is the fluid temperature in the boundary layer, t is time and ν, ρ and α are the kinematic viscosity, fluid density and thermal diffusivity, respectively. σ_1 is the Stefan Biltzmann constant, k_1 is the mean absorption coefficient, c_p is the specific heat at constant pressure.

*Address correspondence to this author at the Research Group of Fluid Flow Modeling and Simulation, Department of Applied Mathematics, University of Dhaka, Dhaka-1000, Bangladesh; Tel: +1 318 308 6157; E-mail: ferdowsltech@gmail.com

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We assume that the boundary conditions of the above equations are:

$$\left.\begin{array}{l} u\,|_{y=0}=u_w,\ v\,|_{y=0}=0,\ \ T\,|_{y=0}=T_{\mathrm{w}}=T_\infty+Ax^\lambda \\ u\,|_{y\to\infty}=0,\quad T\,|_{y\to\infty}\to T_\infty \end{array}\right\}$$

Assume the stretching velocity $u_w(x,t)=\dfrac{ax}{1-ct}$ and the surface temperature $T_w(x,t)=T_\infty+\dfrac{bx^\lambda}{1-ct}$.

Where a, band c are constants. It should be noticed that at t = 0 (initial motion), the governing equations describes the steady flow over a stretching surface. This particular form of u_w (x, t) and T_w (x, t) has been chosen in order to be able to devise a new similarity transformation, which transforms the governing partial differential into a set of ordinary differential equations, thereby facilitating the exploration of the effects of the controlling parameters.

We look for suitable similarity variables in the form;

$$\eta=y\left(\frac{u_w}{\upsilon x}\right)^{\frac{1}{2}},\qquad u=u_w f'(\eta),\ v=-\left(\frac{u_w\upsilon}{x}\right)^{\frac{1}{2}}\left(\frac{\eta f'}{2}-f\right)$$

and $\theta(\eta)=\dfrac{T-T_\infty}{T_w-T_\infty}$.

After some algebra, the transformed coupled, ordinary differential equation:

$$f'''+f\,f''-f'^2-A\left(f'+\frac{\eta}{2}f''\right)=0 \tag{1}$$

$$(1+R)\theta''+\Pr\left(f\theta'-\lambda f'\theta\right)-A\Pr\left(\theta+\frac{\eta}{2}\theta'\right)+\Pr E_c f''^2=0 \tag{2}$$

The corresponding boundary conditions are;

$$\left.\begin{array}{l} f\,|_{\eta=0}=0,\ f'\,|_{\eta=0}=1,\ \theta\,|_{\eta=0}=1 \\ \quad f'\,|_{\eta\to\infty}\to 0,\ \theta\,|_{\eta\to\infty}\to 0 \end{array}\right\} \tag{3}$$

A is the unsteadiness parameter, $\Pr$ is the Prandtl number, R is the radiation parameter and Ec is the Eckert number are usual symbols.

3. SPECTRAL RELAXATION METHOD AND RESULTS

For numerical investigations we used an efficient numerical technique called Spectral relaxation method for solving our transformed ordinary differential equations. The key concept of this technique is the use of trail function and test functions [7, 8]. For trial function we use Chebychev polynomials and for test function we used the following assumptions:

$$f(\eta)=1-e^{-\eta},\ \ F_0(\eta)=e^{-\eta},\ \theta(\eta)=e^{-\eta},$$

The discretization procedure is very similar to Gauss-Seidel discretization idea. In the presence of Joule heating and viscous dissipation Motsa and Makukula [9] investigates the steady von Karman flow using SRM method of a Reiner-Rivlin fluid. Over a stretching surface for Maxwell fluid Shateyi [10] used the SRM to solve the MHD flow and heat transfer. Using the similar technique Shateyi and Makinde [11] studied stagnation point flow of an incompressible viscous fluid. In presence of binary chemical reaction and Arrhenius activation energy complex nonlinear system of equations of incompressible flow are solved using SRM technique by Awad *et al*. [12] over a stretching surface. Now for our model we discretize the transformed Eq. (8) to Eq. (11) using the following SRM algorithm:

1. Reduced the order $f(\eta)$ from three to two by taking $f'(\eta)=F(\eta)$ and defined the transformed equation in forms of $F(\eta)$, and the functions $\theta(\eta)$ which are in second order need not required to reduce the order.

2. Rewrite the transformed equation involving iteration notation.

3. In equation for $F(\eta)$ the scheme is developed by considering that only linear terms in $F(\eta)$ are evaluated in current i.c r+1 iteration level and all the other terms (linear and non-linear),

4. In $f(\eta),\theta(\eta)$ are assumed to be known from previous iteration (noted as r). Non-linear terms in $F(\eta)$ are evaluated as previous iteration level.

5. In equation for $\theta(\eta)$, all the linear term in $\theta(\eta)$ and all the term in $f(\eta),F(\eta)$ are calculated in current iteration level (as the update is available for $f(\eta),F(\eta)$) and other terms will use the value of previous iteration.

6. In similar manner the value will be evaluated.

Using the above three steps, equations (1-2) become;

$$f'''_{r+1} + f_r\, f''_{r+1} - f_r'^2 - A\left(f'_{r+1} + \frac{\eta}{2} f'' \right) = 0$$

$$(1+R)\theta''_{r+1} + \Pr\left[f_{r+1} + A\frac{\eta}{2} \right]\theta'_{r+1}$$
$$-\Pr\lambda\, f'_{r+1}\,\theta_{r+1} - \Pr A\,\theta_{r+1} + \Pr Ecf'' = 0$$

subject to boundary conditions:

$$F_{r+1}(0) = 1; \qquad F_{r+1}(\infty) = 0$$

$$\theta_{r+1}(0) = 1; \qquad \theta_{r+1}(\infty) = 0$$

Applying the Chebychev spectral collocation method on the above equations, we obtain:

$$A_1 F_{r+1} = B_1,$$

$$A_2 F_{r+1} = B_2,$$

$$A_3 \theta_{r+1} = B_3,$$

Where,

$$A_1 = D^2 + diag\left[f_r + A\frac{\eta}{2} \right] D - A\,I,$$

$$B_1 = F_r^{\,2},$$

$$A_2 = D,$$

$$B_2 = F_{r+1},$$

$$A_3 = D^2 + diag\left(\Pr\left[f_{r+1} + A\frac{\eta}{2} \right] \right) D - diag\left(\Pr\lambda\, f'_{r+1} + \Pr A \right),$$

$$B_3 = -\Pr Ec\, F''^2 .$$

I is an identity matrix and diag [] is a diagonal matrix, all size $(N+1)\times(N+1)$, where N is the number of grid points, $f, F, h, \theta, \phi, \chi$ respectively, when evaluated at the grid points and the subscript r denotes the iteration number. In our present study we take N=80 collocation point. These values gave accurate result for all the quantities of physical interest. Starting from the initial approximation the SRM scheme is repeatedly solve until the following condition is satisfied,

$$\max\left(F_{r+1} - F_\infty,\ \theta_{r+1} - \theta_\infty, \phi_{r+1} - \phi_\infty, \chi_{r+1} - \chi_\infty\right) \le \varepsilon_r ,$$

Where, ε_r is a prescribed error tolerance, which in this study is taken to be 10^{-6}.

The solution to the system of transformed governing equations (1)-(2) and boundary conditions (3) is accomplished as described in [7, 8]. Note that the effect of A, R and Ec are found to be almost not significant for velocity and temperature profiles. The dimensions velocity and temp profiles are presented in Figures **1-2** for different values of steadiness parameter A. It can be seen both profiles decrease with the increase of A. That is decreases the velocity and temp boundary layer thickness.

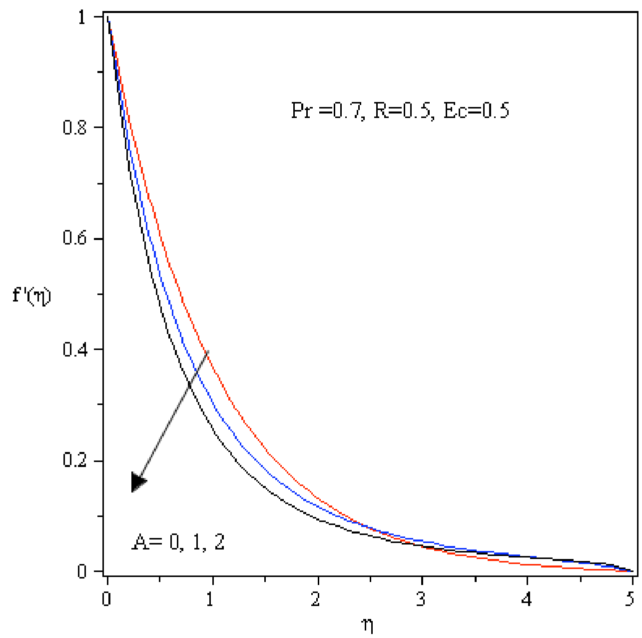

Figure 1: Velocity profiles f'(η) against η.

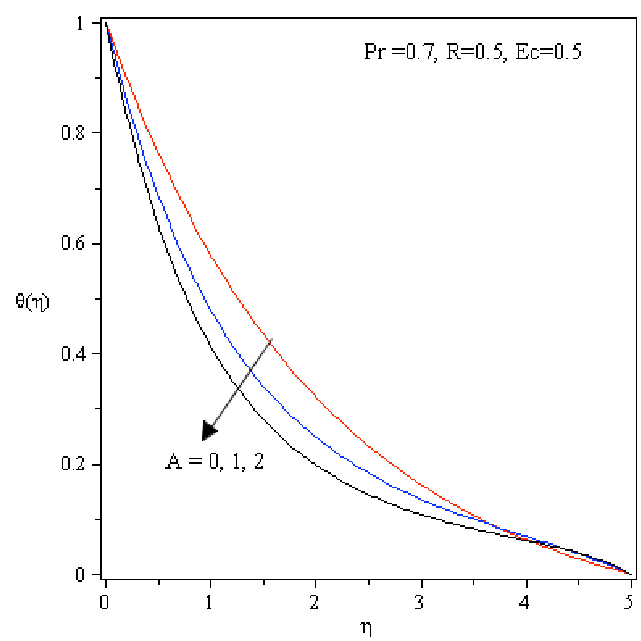

Figure 2: Temperature profiles $\theta(\eta)$ against η.

Figure **3** shows the dimensionless temp profiles for different values of the thermal radiation parameter. It can seen that increasingly thermal radiation increased the conductive fluid temp inside the boundary layer and decreased the heat transfer rates from the stretching surface.

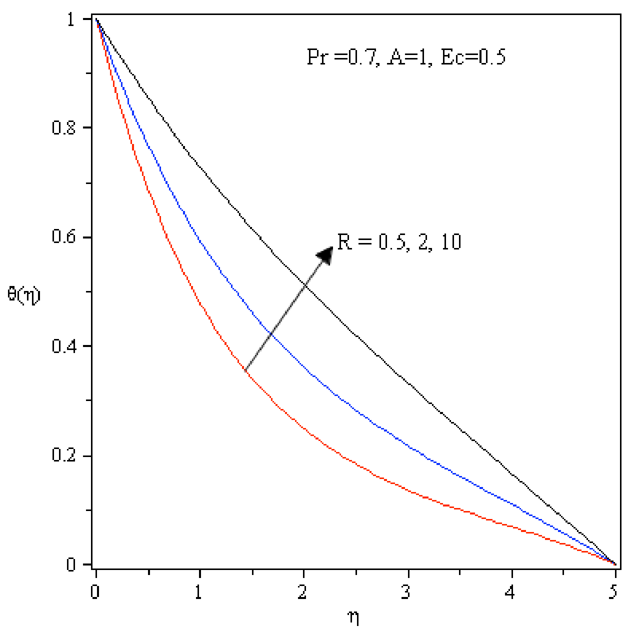

Figure 3: Temperature profiles $\theta(\eta)$ against (η).

The effect of Eckert number on flow profiles is displayed in figure **4**. From this graph we notice that the effect of Ec is to increase temp through out the boundary layer flow field and enhance temp significantly in the flow field.

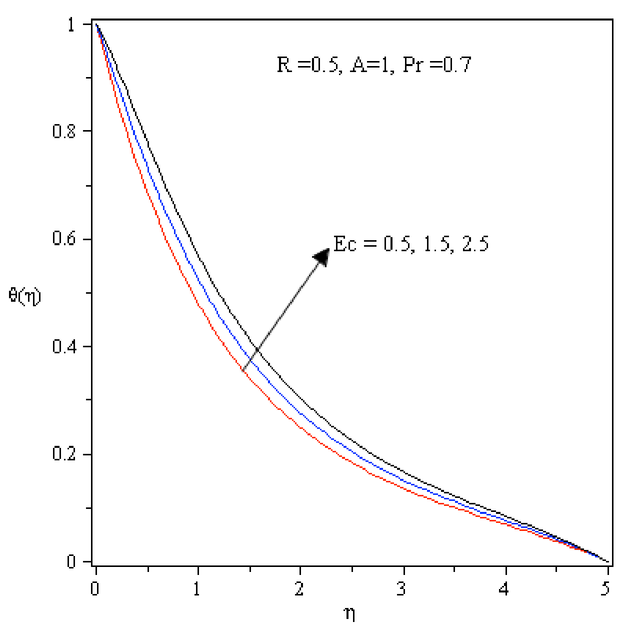

Figure 4: Temperature profiles $\theta(\eta)$ against η.

Figure **5** show a sample temperature profile against similarity variable for $\lambda = 0$ (isothermal surface), $\lambda = 1/3$ or (uniform heat flux), $\lambda = 1$ (non-isothermal surface). It is evident that for temperature variations the thermal boundary layer thickness decreases and a consequent decrease in the heat transfer.

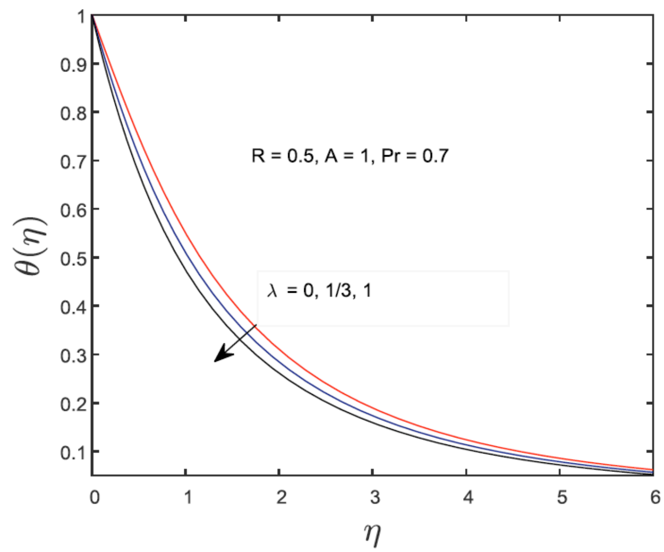

Figure 5: Temperature profiles $\theta(\eta)$ against η.

CONCLUSION

The specific conclusions derived from this study can be listed as follows:

1. Effect of increasing values of thermal radiation R and Eckert number Ec, is to increase temp significantly in the boundary layer flow field.

2. Effect of increasing values of A is to reduce the velocity temp boundary.

3. The effect of A, R and Ec are found to be almost not significant for velocity.

4. Power law temperature has a significant impact on heat transfer as expected.

It is expected that this model has an application of metallurgy and chemical engineering such as extrusion process, heat materials etc.

REFERENCES

[1] Sakiadis BC. Boundary-Layer Behaviour on Conti- nuous Solid Surfaces: I. Boundary-Layer Equations for Two-Dimensional and Axisymmetric Flow. AIChE Journal 1961; 7: 26. https://doi.org/10.1002/aic.690070108

[2] Elbashbeshy EMA and Bazid MAA. Heat transfer over an unsteady stretching surface. Heat and Mass transfer 2004; 41: 1. https://doi.org/10.1007/s00231-004-0520-x

[3] Chen CK and Char MI. Heat Transfer of a Continuous, Stretching Surface with Suction or Blowing. Journal of Mathematical Analysis and Applications 1988; 135: 568. https://doi.org/10.1016/0022-247X(88)90172-2

[4] Wubshet Ibrahim and BandariShanker. Unsteady Boundary Layer Flow and Heat Transfer Due to a Stretching Sheet by Quasilinearization. Technique, World Journal of Mechanics 2011; 1: 288. https://doi.org/10.4236/wjm.2011.16036

[5] Seripah Awang Kechil and Ishak Hashim. Series Solution for Unsteady Boundary-Layer Flows Due to Impulsively Stretching Plate. Chinese Physics Letters 2007; 24: 139. https://doi.org/10.1088/0256-307X/24/1/038

[6] Wenli Cai, Ning Su and Xiangdong Liu. Unsteady Convection Flow and Heat Transfer over a Vertical Stretching Surface. Plos one 2014; 9: 1. https://doi.org/10.1371/journal.pone.0107229

[7] Haroun N, Sibanda A, Mondal PS and Motsa SS. On unsteady MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection using the spectral relaxation method. Boundary value problems 2015; (1): 24. https://doi.org/10.1186/s13661-015-0289-5

[8] Canuto C, Hussaini MY, Quarteroni A and Zang TA. Spectral Methods in Fluid Dynamics,Springer-Verlag, Berlin 1988. https://doi.org/10.1007/978-3-642-84108-8

[9] Motsa SS and Makukula ZG. On spectral relaxation method approach for steady von Kármán flow of a Reiner-Rivlin fluid with Joule heating, viscous dissipation and suction/injection. Central European Journal of Physics 2013; 11(3): 363-374. https://doi.org/10.2478/s11534-013-0182-8

[10] Shateyi S. A new numerical approach to MHD ow of a maxwelluid past a vertical stretching sheet in the presence of thermophoresis and chemical reaction, Boundary Value Problems 2013; 196. https://doi.org/10.1186/1687-2770-2013-196

[11] Shateyi S and Makinde OD. Hydromagnetic stagnation-point flow towards a radially stretching convectively heated disk. Mathematical Problems in Engineering 2013. https://doi.org/10.1155/2013/616947

[12] Awad FG, Motsa S and Khumalo M. Heat and mass transfer in unsteady rotating fluid flow with binary chemical reaction and activation energy. Plos one 2014; 9(9): e107622. https://doi.org/10.1371/journal.pone.0107622

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