

# Investigation on the Influence of the Interfacial Slippage on the Whole Moving Surface in an Inclined Fixed Pad Thrust Slider Bearing

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**Abstract:** The influence of the interfacial slippage on the whole moving surface on the carried load and friction coefficient of an inclined fixed pad thrust slider bearing is analytically investigated. The calculation results show that the carried load of this mode of bearing is normally much lower than that of the conventional inclined fixed pad thrust slider bearing, while the friction coefficient of this mode of bearing is significantly higher than that of the conventional bearing, when the operating condition is the same. With the reduction of the contact-fluid interfacial shear strength on the moving surface, the performance of this mode of bearing is worsened. These results suggest the necessity of preventing the interfacial slippage on the moving surface in an inclined fixed pad thrust slider bearing.

**Keywords:** Thrust bearing, Interfacial slippage, Load, Friction coefficient.

## 1. INTRODUCTION

In fluid mechanics, the contact-fluid interfacial slippage was found long time ago [1-3]; It will change the flow through a channel not to be predictable from the classical flow theory. It was suggested that the boundary condition should be generally considered as the interfacial slippage in a fluid flow [4].

The hydrodynamic bearings have been designed according to conventional hydrodynamic lubrication theories [5], which ignore the contact-fluid interfacial slippage. The actual performances of those bearings were found to be often inferior to the classical theory predictions. A lot of factors have been considered for explaining this phenomenon, including the lubricant film viscous heating effect, the lubricant non-Newtonian effect and the surface roughness effect [6-8]; But no satisfactory explanations were obtained.

Rozeanu and Snarsky [9, 10] and Rozeanu and Tipei [11] experimentally found that the contact-fluid interfacial condition on the moving shaft surface has a pronounced influence on the load-carrying capacity of a hydrodynamic journal bearing. They suggested the contact-fluid interfacial slippage responsible for the great reduction of the carried load of the bearing. Jacobson and Hamrock [12] also analytically showed that the interfacial slippage can significantly reduce the carried load of a lubricated line contact at small slide-roll ratios. Zhang *et al.* [13] analytically showed the drastic reduction of the carried load of a lubricated line

contact at high slide-roll ratios, caused by the interfacial slippage. Yan *et al.* [14] analytically showed that the interfacial slippage can severely worsen the performance of a hydrodynamic lubricated step bearing with normal contact surfaces, including reducing the carried load of the bearing but increasing the friction coefficient of the bearing.

There have also been a lot of attempts to apply the interfacial slippage for improving the performance of a hydrodynamic bearing or for designing the novel bearings with abnormal geometrical configurations [15-17]. The design of these bearings relies on the occurrence of the interfacial slippage in specific lubricated areas.

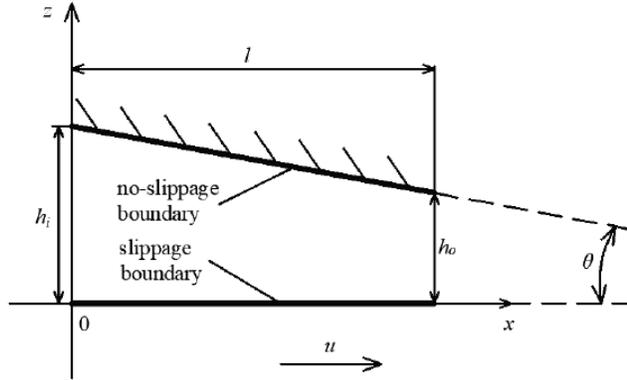
The present paper aims to study the influence of the interfacial slippage on the whole moving surface in a hydrodynamic lubricated inclined fixed pad thrust slider bearing. It can answer the questions that whether this interfacial slippage can be applied for improving the performance of the bearing or it should be prevented for avoiding the deterioration of the performance of the bearing.

## 2. WORKING CONDITION OF THE STUDIED BEARING

Figure 1 shows the working condition of the studied bearing where the interfacial slippage occurs on the whole moving (lower) surface but is absent on the stationary (upper) surface. The lubricating film thicknesses respectively on the entrance and exit of the bearing are  $h_i$  and  $h_o$ , the whole width of the bearing is  $l$ , the tilting angle of the bearing is  $\theta$ , and the sliding speed of the bearing is  $u$ . The used coordinates are also shown in Figure 1.

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The bearing in Figure 1 may occur in the condition of high sliding speeds and heavy loads when the moving surface has a weaker adsorption strength with the lubricating fluid than the stationary surface. It can also be artificially designed for a normal operating condition when we take the moving surface as a hydrophobic surface while take the stationary surface as a hydrophilic surface.



**Figure 1:** The studied hydrodynamic lubricated inclined fixed pad thrust slider bearing.

### 3. ANALYSIS

An analysis was made for the carried load and friction coefficient of the bearing in Figure 1 based on the interfacial limiting shear strength model [18]. It is based on the following assumptions:

1. Within the lubricating film, the fluid is Newtonian;
2. Across the lubricating film thickness, the film pressure is constant;
3. The compressibility of the fluid is negligible;
4. The fluid is isoviscous;
5. The fluid inertia is negligible;
6. The fluid is in laminar flow;
7. The operating condition is isothermal and steady-state.

#### 3.1. Film Pressure and Volume Flow Rate

Within the lubricating film, the rheological model of a Newtonian fluid reads:

$$\tau = \eta \frac{\partial v}{\partial z} \quad (1)$$

where  $\tau$  is the fluid film shear stress,  $\eta$  is the fluid dynamic viscosity, and  $v$  is the fluid film velocity in the  $x$  coordinate direction.

The well-known momentum equilibrium equation for an infinitesimal element of the fluid film is:

$$\frac{\partial p_{slip}}{\partial x} = \frac{\partial \tau}{\partial z} \quad (2)$$

where  $p_{slip}$  is the fluid film pressure.

Substituting Eq.(1) into Eq.(2) and rearranging gives that:

$$\frac{\partial p_{slip}}{\partial x} = \eta \frac{\partial^2 v}{\partial z^2} \quad (3)$$

Integrating Eq.(3) yields that:

$$\frac{\partial p_{slip}}{\partial x} z = \eta \frac{\partial v}{\partial z} + c_1 \quad (4)$$

Substituting Eq.(4) into Eq.(1) yields that:

$$\tau = \frac{\partial p_{slip}}{\partial x} z - c_1 \quad (5)$$

When the film slips on the moving surface, the magnitude of the shear stress on the moving surface will reach the fluid-moving surface interfacial shear strength  $\tau_{sb}$ ; Thus, according to  $\tau|_{z=0} = -\tau_{sb}$ , it is obtained that  $c_1 = \tau_{sb}$ .

Substituting  $c_1 = \tau_{sb}$  into Eq.(4) and integrating Eq.(4) gives that:

$$v = \frac{z^2}{2\eta} \frac{\partial p_{slip}}{\partial x} - \frac{\tau_{sb}}{\eta} z - c_2 \quad (6)$$

Since no film slippage occurs on the stationary surface, the film velocity on the stationary surface is equal to the speed of that surface i.e.  $v|_{z=h} = 0$ ;

Thus, it is obtained that:

$$c_2 = \frac{h^2}{2\eta} \frac{\partial p_{slip}}{\partial x} - \frac{\tau_{sb}}{\eta} h \quad (7)$$

where  $h$  is the lubricating film thickness.

Substituting Eq.(7) into Eq.(6) and rearranging gives the film velocity as:

$$v = \frac{z^2 - h^2}{2\eta} \frac{\partial p_{slip}}{\partial x} + (h - z) \frac{\tau_{sb}}{\eta} \quad (8)$$

The volume flow rate per unit contact length through the bearing is:

$$q_v = \int_0^h v dz = \frac{\tau_{sb} h^2}{2\eta} - \frac{h^3}{3\eta} \frac{\partial p_{slip}}{\partial x} \quad (9)$$

Rearranging Eq.(9) gives the following Reynolds equation for the whole lubricated area in the bearing [13]:

$$\frac{\partial p_{slip}}{\partial x} = \frac{3\tau_{sb}}{2h} - \frac{3\eta q_v}{h^3} \quad (10)$$

Integrating equation (10) gives that:

$$p_{slip} = -\frac{3}{2k} \left( \frac{\eta q_v}{h^2} + \tau_{sb} \ln h \right) + c_3 \quad (11)$$

where  $k = \tan(\theta)$ , and  $c_3$  is an integral constant. By using the boundary condition  $p|_{x=0} = 0$ , it is solved that  $c_3 = 3(\eta q_v / h_i^2 + \tau_{sb} \ln h_i) / (2k)$ . Then the fluid film pressure in the bearing is:

$$p_{slip} = \frac{3}{2k} \left[ \eta q_v \left( \frac{1}{h_i^2} - \frac{1}{h^2} \right) + \tau_{sb} \ln \frac{h_i}{h} \right] \quad (12)$$

By further using the boundary condition  $p|_{x=l} = 0$ , it is solved from Eq.(12) that:

$$q_v = \frac{\tau_{sb} \ln \frac{h_o}{h_i}}{\eta \left( \frac{1}{h_i^2} - \frac{1}{h_o^2} \right)} \quad (13)$$

### 3.2. Carried Load, Shear Stress, Friction Coefficient and Interfacial Slipping Velocity

The carried load per unit contact length of the bearing is:

$$\begin{aligned} w_{slip} &= \int_0^l p dx \\ &= -\frac{3}{2k} \left[ \tau_{sb} h_o \ln \frac{h_i}{h_o} + \tau_{sb} (h_o - h_i) + \eta q_v \left( \frac{h_o}{h_i^2} + \frac{1}{h_o} - \frac{2}{h_i} \right) \right] \end{aligned} \quad (14)$$

The shear stress on the upper contact surface is:

$$\tau_a = \frac{\partial p}{\partial x} h - \tau_{sb} = \frac{\tau_{sb}}{2} - \frac{3\eta q_v}{h^2} \quad (15)$$

The shear stress on the lower contact surface is:

$$\tau_b = -\tau_{sb} \quad (16)$$

The friction force per unit contact length on the upper contact surface in the bearing is:

$$F_{f,a,slip} = \int_0^l \tau_a dx = -\frac{1}{k} \left[ \frac{\tau_{sb}}{2} (h_o - h_i) + 3\eta q_v \left( \frac{1}{h_o} - \frac{1}{h_i} \right) \right] \quad (17)$$

The friction force per unit contact length on the lower contact surface in the bearing is:

$$F_{f,b,slip} = \int_0^l \tau_b dx = \frac{\tau_{sb}}{k} (h_o - h_i) \quad (18)$$

The friction coefficients on the upper and lower contact surfaces in the bearing are respectively:

$$f_{a,slip} = \frac{|F_{f,a,slip}|}{w_{slip}}, \quad f_{b,slip} = \frac{|F_{f,b,slip}|}{w_{slip}} \quad (19)$$

The fluid film slipping velocity on the lower contact surface is the film velocity on this surface minus the moving speed of this surface, i.e. [13]:

$$\Delta u_b = v|_{z=0} - u = \frac{3q_v}{2h} + \frac{\tau_{sb} h}{4\eta} - u \quad (20)$$

For ensuring the occurrence of the fluid film slippage, it should be satisfied that [13]:

$$\Delta u_b < 0 \quad (21)$$

### 3.3. Normalization

For generality, the following dimensionless parameters are defined:

$$\begin{aligned} W_{slip} &= \frac{w_{slip}}{u\eta}, \quad P_{slip} = \frac{p_{slip} h_o}{u\eta}, \quad Q_v = \frac{q_v}{uh_o}, \quad \bar{\tau}_{sb} = \frac{\tau_{sb} h_o}{u\eta} \\ \bar{F}_{f,a,slip} &= \frac{F_{f,a,slip}}{u\eta}, \quad \bar{F}_{f,b,slip} = \frac{F_{f,b,slip}}{u\eta}, \quad X = \frac{x}{l}, \\ H &= \frac{h}{h_o}, \quad H_i = \frac{h_i}{h_o}, \quad \alpha = \frac{h_o}{l} \end{aligned}$$

#### 3.3.1. For the Present Bearing

The dimensionless volume flow rate through the bearing is:

$$Q_v = \frac{\bar{\tau}_{sb} \ln \frac{1}{H_i}}{\frac{1}{H_i^2} - 1} \quad (22)$$

where  $H_i = 1 + k / \alpha$ .

The dimensionless pressure is:

$$P_{slip} = \frac{3}{2k} \left[ Q_v \left( \frac{1}{H_i^2} - \frac{1}{H^2} \right) + \bar{\tau}_{sb} \ln \frac{H_i}{H} \right] \quad (23)$$

where  $H = 1 + k(1 - X) / \alpha$ .

The dimensionless load carried by the bearing is:

$$W_{slip} = -\frac{3}{2k^2} \left[ \bar{\tau}_{sb} \ln H_i + \bar{\tau}_{sb} (1 - H_i) + Q_v \left( \frac{1}{H_i^2} + 1 - \frac{2}{H_i} \right) \right] \quad (24)$$

The dimensionless friction force per unit contact length on the upper contact surface in the bearing is:

$$\bar{F}_{f,a,slip} = \frac{1}{k} \left[ \frac{\bar{\tau}_{sb}}{2} (H_i - 1) + 3Q_v \left( \frac{1}{H_i} - 1 \right) \right] \quad (25)$$

The dimensionless friction force per unit contact length on the lower contact surface in the bearing is:

$$\bar{F}_{f,b,slip} = \frac{\bar{\tau}_{sb}}{k} (1 - H_i) \quad (25)$$

The friction coefficients on the upper and lower contact surfaces in the bearing are respectively:

$$f_{a,slip} = \frac{|\bar{F}_{f,a,slip}|}{W_{slip}}, f_{b,slip} = \frac{|\bar{F}_{f,b,slip}|}{W_{slip}} \quad (27)$$

According to Eq.(21), the condition for the present bearing is:

$$\bar{\tau}_{sb} < \frac{4 \left( \frac{1}{H_i^2} - 1 \right)}{\left( \frac{1}{H_i^2} - 1 \right) + 6 \ln \frac{1}{H_i}} \quad (28)$$

### 3.3.2. For the Conventional Bearing

Conventional hydrodynamic lubricated thrust slider bearings have been designed according to conventional hydrodynamic lubrication theory [5], which neglects the contact-fluid interfacial slippage. Those bearings may exist for the condition of light loads and low sliding speeds. However, for the condition of heavy loads and high sliding speeds, those bearings may actually become the present mode of the bearing, where the interfacial slippage occurs on specific bearing surfaces. If it occurs, conventional hydrodynamic lubrication theory may fail to describe the performance of such bearings because of ignoring the interfacial slippage effect.

For comparison, the results for the bearing in Figure 1 given by conventional hydrodynamic lubrication theory is also presented in this section. According to conventional hydrodynamic lubrication theory [5], for the conventional hydrodynamic lubricated inclined fixed pad thrust slider bearing, where no interfacial slippage occurs, the dimensionless volume flow rate per unit contact length through the bearing is:

$$Q_v = \frac{H_i}{1 + H_i} \quad (29)$$

The dimensionless pressure in the bearing is [5]:

$$P_{conv} = \frac{6}{k} \left[ Q_v \left( \frac{1}{H_i^2} - \frac{1}{H^2} \right) + \frac{1}{H} - \frac{1}{H_i} \right] \quad (30)$$

The dimensionless load carried by the bearing is [5]:

$$W_{conv} = -\frac{6}{k^2} \left[ Q_v \left( \frac{1}{H_i^2} - \frac{2}{H_i} + 1 \right) + \ln \frac{1}{H_i} - \frac{1}{H_i} + 1 \right] \quad (31)$$

The dimensionless friction force per unit contact length on the upper contact surface in the bearing is [5]:

$$\bar{F}_{f,a,conv} = -\frac{1}{k} \left[ 2 \ln \frac{1}{H_i} + 6Q_v \left( 1 - \frac{1}{H_i} \right) \right] \quad (32)$$

The dimensionless friction force per unit contact length on the lower contact surface in the bearing is [5]:

$$\bar{F}_{f,b,conv} = \frac{1}{k} \left[ 4 \ln \frac{1}{H_i} + 6Q_v \left( 1 - \frac{1}{H_i} \right) \right] \quad (33)$$

The friction coefficients on the upper and lower contact surfaces in the bearing are respectively:

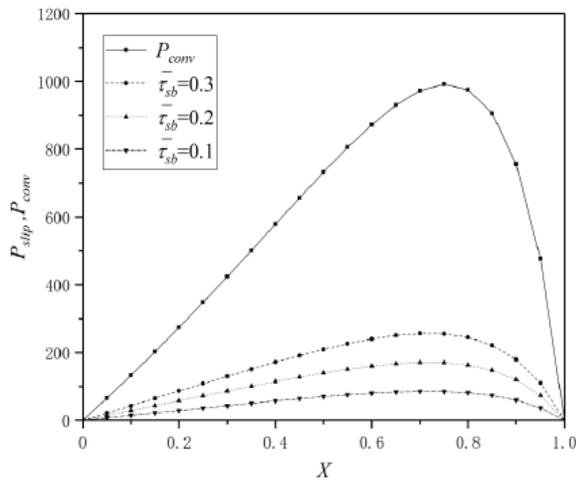
$$f_{a,conv} = \frac{|\bar{F}_{f,a,conv}|}{W_{conv}} \quad \square \quad f_{b,conv} = \frac{|\bar{F}_{f,b,conv}|}{W_{conv}} \quad (34)$$

## 4. RESULTS

### 4.1. Pressure Distribution

Figure 2 shows the dimensionless pressure distributions in the present bearing for different  $\bar{\tau}_{sb}$  and their comparisons with that ( $P_{conv}$ ) calculated from conventional hydrodynamic lubrication theory when  $\alpha = 2.5 \times 10^{-4}$  and  $\theta = 3.0 \times 10^{-2}$ . It is shown that the pressures in the present bearing are normally much lower than those in the conventional hydrodynamic inclined fixed pad thrust slider bearing for the same

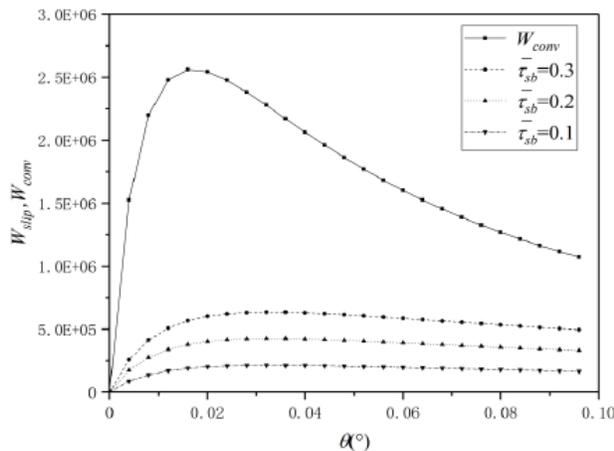
operating condition. With the reduction of the contact-fluid interfacial shear strength  $\bar{\tau}_{sb}$  on the moving surface, the pressures in the present bearing are significantly reduced.



**Figure 2:** Dimensionless pressure distributions in the present bearing for different  $\bar{\tau}_{sb}$  and their comparisons with that calculated from conventional lubrication theory,  $\alpha = 2.5 \times 10^{-4}$  and  $\theta = 3.0 \times 10^{-2}$ .

### 4.2. Carried Load

Figure 3 shows that the carried load of the present bearing is normally much smaller than that of the conventional hydrodynamic inclined fixed pad thrust slider bearing for the same operating condition. The carried load of the present bearing is significantly reduced with the reduction of the contact-fluid interfacial shear strength  $\bar{\tau}_{sb}$  on the moving surface. Figure 3 also shows that for a given operating

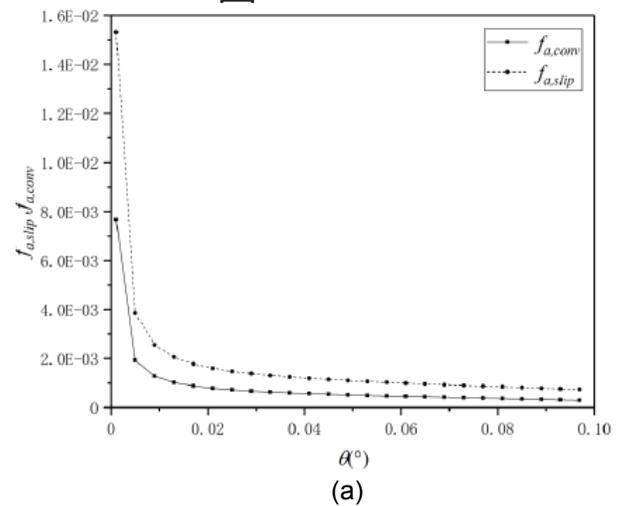


**Figure 3:** Dimensionless carried loads of the present bearing and their comparisons with those of the conventional hydrodynamic inclined fixed pad thrust slider bearing for the same operating conditions,  $\alpha = 2.5 \times 10^{-4}$ .

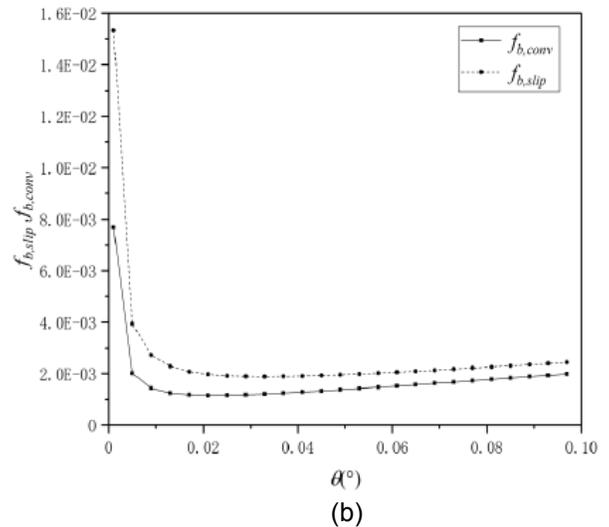
condition, there is an optimum tilting angle  $\theta$  which yields the highest load-carrying capacity of the bearing. This optimum  $\theta$  value for the present bearing is obviously greater than that for the conventional hydrodynamic inclined fixed pad thrust slider bearing for the same case.

### 4.3. Friction Coefficient

Figures 4(a) and (b) respectively compare the friction coefficients ( $f_{a,slip}$  and  $f_{b,slip}$ ) on the upper and lower contact surfaces in the present bearing with those ( $f_{a,conv}$  and  $f_{b,conv}$ ) in the conventional hydrodynamic inclined fixed pad thrust slider bearing for the same operating conditions when  $\alpha = 2.5 \times 10^{-4}$ .



(a)



(b)

**Figure 4:** Friction coefficients ( $f_{a,slip}$  and  $f_{b,slip}$ ) respectively on the upper and lower contact surfaces in the present bearing and their comparisons with those ( $f_{a,conv}$  and  $f_{b,conv}$ ) in the conventional hydrodynamic inclined fixed pad thrust slider bearing for the same operating conditions,  $\alpha = 2.5 \times 10^{-4}$ .

The friction coefficient in the present bearing is independent on the interfacial shear strength  $\bar{\tau}_{sb}$  on the moving surface as formulated above. It is shown that for the same operating condition, the friction coefficient in the present bearing is significantly greater than that in the conventional hydrodynamic inclined fixed pad thrust slider bearing especially when the tilting angle  $\theta$  of the bearing is small.

## CONCLUSION

The influence of the interfacial slippage on the whole moving surface is analytically investigated in a hydrodynamic inclined fixed pad thrust slider bearing based on the interfacial limiting shear strength model. In this bearing, there is no interfacial slippage on the stationary surface. The calculation results show that for the same operating condition, the load-carrying capacity of the present bearing is normally much smaller than that of the conventional hydrodynamic inclined fixed pad thrust slider bearing, but the friction coefficient of the present bearing is significantly higher than that of the conventional bearing. It is shown that the performance of the studied bearing is severely deteriorated because of the occurrence of the interfacial slippage on the whole moving surface. The bearing performance is further deteriorated with the reduction of the interfacial shear strength on the moving surface. The present study strongly suggests the necessity of preventing the interfacial slippage on the whole moving surface for avoiding the inferior performance of the bearing. The study supports the experiment by Rozeanu and Snarsky [10] on the interfacial slippage effect on the rotating shaft surface in a hydrodynamic journal bearing.

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