# Systems of Measurement for the Construction of Geometrical Models 

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#### Abstract

In the present work, the relevant components of the two basic system of measurement are defined through the analysis of existing systems of measurement for geometrical modeling. The major features of such systems related to the ways of formation of the two-dimensional complex drawing of a geometrical image on the basis of laws of projective connections are revealed. Interrelations of the elements of different systems of measurement with various projections of a geometrical image are defined. The relative location of projections of a geometrical image into the constructed twodimensional complex drawings for various systems of measurement is discussed. The rationale behind a particular arrangement of views in the projective drawing of a product in the analyzed systems of measurement is explained.


Keywords: Geometry, systems of measurement, image, projection, law, projective connections, drawing, interrelations, arrangement of views.

## 1. PROBLEM STATEMENT

Various systems of measurement for geometrical modelling are used in the different countries [1-54]. In Australia, England, Holland, India, Japan, the United States of America (USA), and some other countries the American system of an arrangement of projections in engineering drawing is used. With respect to the front view, the top view is located above, the bottom view is located below, the left-side view - to the left, the rightside view - to the right of the front view, and the rear view - to the right of the product right-side view [1, 2, 41, 43, 45].

In Belarus, Germany, Kazakhstan, Kirgizstan, Poland, Russia, Ukraine and other countries the European system of an arrangement of projections in engineering drawing is used. Relative the front view, the top view is located below, the bottom view - above, the left-side view - to the right, the right-side view - to the left of the front view, and the rear view - to the right of the product left-side view [1-4, 6, 7].

The Indian Standard Institution (ISI) and the British Standard Institution (BSI) recommend the use of First Angle Projection method (the European system) now in all the institutions too $[52,53]$.

As a rule, at various universities of the world one any system of an arrangement of projections in the drawing is taught as applied in design and technological works of the given country [8-13,15-27,40-51].

[^0]It may happen that an engineer developing a production process comes across a part drawing where the views' arrangement is different than he leans. As a result, he or she may not adequately understand the drawing, and thus part design including tolerances and requirements. As a consequence, an inadequate manufacturing process can be developed and/or a good part can be rejected on its inspection.

Realizable ways of formation of projections and views in the drawing are connected in consciousness of an engineer with a particular system of measurement for construction of geometrical models of a product [2, 4, 28-39].

The American system of measurement for the construction of a geometrical model of a product differs from the European system of measurement.

Thus, the essence of the problem (contradiction) consists in a necessity of adequate understanding the features of formation of projections in the drawing in used system of measurement (by everyone involved in design and product manufacture) and lack of the systematized knowledge of features of various ways of formation of projections in different systems of measurement in the geometrical modeling.

Therefore, the purpose of this work is to reveal the features of various systems of the measurement used in the geometrical modeling, and to provide justifications for the arrangement of projections in the product drawing.

## 2. WORK TASKS

1. To analyze the existing systems of measurement used in the geometrical modeling defining their relevant components.
2. To reveal the features of the known systems the measurements connected with the ways of formation of the two-dimensional complex drawing of a geometrical image.
3. To define interrelations of elements of the discussed systems of measurement with different projections of a geometrical image.
4. To define relative positioning of projections of a geometrical image in the two-dimensional complex drawing for various systems of measurement.
5. To carry out a logic substantiation of an arrangement of projections in the projective drawing of a product.

## 3. THE MAIN PART

The first mutual component of the systems of measurement used for geometrical modeling is a set of three mutually perpendicular planes (Fig. 1).

In relation to the subject (to the student, the engineer, the researcher), one of these three planes is located horizontally $\left(\mathrm{H}-\Pi_{1}\right)$ whereas other two planes ( $\mathrm{F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ ) - vertically.

Horizontally focused plane $\mathrm{H}-\Pi_{1}$ is called as the horizontal plane. Vertically located plane $\mathrm{F}-\Pi_{2}$ is called as the frontal plane. Vertically focused plane $\mathrm{P}-\Pi_{3}$ is called as the profile plane.

For geometrical modeling, the method of orthogonal projections [2,4, and 44] is used.

In the method of orthogonal projections, horizontal plane $\mathrm{H}-\Pi_{1}$ is called as the horizontal plane of projections, frontal plane $\mathrm{F}-\Pi_{2}$ is called as the frontal plane of projections, and profile plane $\mathrm{P}-\Pi_{3}$ is called as the profile plane of projections.

The second mutual component of systems of measurement used for geometrical modeling is the right three-dimensional system of coordinates OXYZ, developed by Rene Descartes (1596-1650). The beginning of system of coordinates (a point O-Latin, origo) coincides with the point of intersection of all three planes.

Thus, the system of measurement for geometrical modeling consists of three mutually perpendicular planes $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ and connected with them right [ 5,14 ] three-dimensional system of coordinates OXYZ.


Figure 1: Three mutually perpendicular planes of systems of measurement.

The allocation of a way of orientation of the right three-dimensional system of coordinates OXYZ relative to the planes of projections $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ can be considered as the first feature of systems of measurement for geometrical modeling.

1. In Australia, England, Holland, Japan, the USA and a number of other countries, axes OZ and OX coincide with horizontal plane $\mathrm{H}-\Pi_{1}$, axes OX and OY are located in frontal plane $\mathrm{F}-\Pi_{2}$, and axes OY and OZ belong to profile plane $\mathrm{P}-\Pi_{3}$ (Fig. 2). All axes of coordinates coincide with the lines of intersection of the discussed planes [43, 45].


Figure 2: Linking of the system of coordinates OXYZ with planes of projections $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ in the American system of measurement.

Because the positive direction of axis OZ relative to the origin O is directed to the subject (observer) for the right system of coordinates OXYZ, the positive coordinates of axis OX settle down to the right of the origin O and positive coordinates of axis OY are above the origin O .

Thus, axis OY is directed upwards concerning the origin O , axis OX is directed to the right concerning the origin O , and axis OZ is directed from the origin O towards the observer [45].

Accordingly, the portion of axis OX with negative coordinates is to the left of profile plane $\mathrm{P}-\Pi_{3}$, the portion of axis OY with negative coordinates settles down below horizontal plane $\mathrm{H}-\Pi_{1}$, the portion of axis OZ with negative coordinates is behind frontal plane F $\Pi_{2}[45]$.

The measurement system serves for placing and the description of geometrical images. Geometrical images are understood as abstract elements: a point, a line, a plane, a surface, a body [2-4, 6-27].

The second feature of systems of measurement for geometrical modeling is the way of space partition into semi spaces, quadrants, and octants.

For the description of the location of an object in a particular measurement, space is partitioned by a plane into two semi spaces, by two orthogonal planes into four quadrants (quarters), by three mutually perpendicular planes into eight octants (parts).
2. In the American system of measurement, space is partitioned into four quadrants by horizontal plane H $\Pi_{1}$ and profile plane $\mathrm{P}-\Pi_{3}$ (Fig. 3).

The third feature of systems of measurement for geometrical modeling is the way of numbering of the allocated parts of space (semi spaces, quadrants, and octants).
3. In the American system of measurement, numerical values of coordinates X and $Y$ are positive for both axes $-1:+X,+Y$. In the second quadrant, numerical values of coordinate $Y$ are positive, and coordinates $X$ are negative - 2: $-\mathrm{X},+\mathrm{Y}$. In the third quadrant, numerical values of coordinates $X$ and $Y$ are negative for both axes - 3: -X , -Y . In the fourth quadrant, numerical values of coordinate $Y$ are negative, and coordinates $X$ are positive -4 : $+\mathrm{X},-\mathrm{Y}[45]$.

Quadrant number increase at consecutive viewing the quadrants from the positive direction $+Z$ axes $O Z$
counter-clockwise (Fig. 3). Numbers 1, 2, 3, 4 quadrants are specified around the image.

The one eighth part of space, in which all numerical values of coordinates $X, Y, Z$ for three axes $O X, O Y, O Z$ are positive $+\mathrm{X},+\mathrm{Y},+\mathrm{Z}$ is chosen as the first octant.

Numbers of the first four octants correspond to four numbers quadrants (quarters). The fifth, the sixth, the seventh and the eighth octants are bred behind the frontal plane of projections $\mathrm{F}-\Pi_{2}$ from the positive direction $+Z$ axis $O Z$ accordingly in the first, in the second, in the third and the fourth quadrant (Fig. 3).


Figure 3: Quadrants and octants numbering in the American system of measurement.

The revealed features of the American system of measurement differ from similar characteristics of the European system of measurement although there are some similarities.

1. In the European system of measurement, the right $[5,14]$ the three-dimensional system of coordinates OXYZ is positioned differently relative to the planes of projections $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ (Fig. 1) as shown in Fig. (4).

Axes OZ and OX coincide with frontal plane $\mathrm{F}-\Pi_{2}$, axes $O X$ and $O Y$ settle down in horizontal plane $\mathrm{H}-\Pi_{1}$, and axes OY and OZ also belong to profile plane $\mathrm{P}-\Pi_{3}$ (Fig. 4). All the axes of coordinates are coincident with the intersection lines of the projection planes [4].

Because the positive direction of axis OZ with respect to the origin O is directed upwards for the right system of coordinates OXYZ, the positive coordinates of axis OX are to the left of the origin $O$ and positive coordinates of axis OY settle down before frontal plane $\mathrm{F}-\Pi_{2}$ closer to the observer to the origin O (Fig. 4).

Thus, axis $O Y$ is directed from the origin $O$ coordinates towards the observer, axis OX is directed to the left of the origin O , axis OZ is directed upwards relative to the origin O .

Accordingly, the portion of axis OX with negative coordinates is to the right of profile plane $\mathrm{P}-\Pi_{3}$, the portion of axis OY with negative coordinates settles down behind frontal plane $\mathrm{F}-\Pi_{2}$, the portion of axis OZ with negative coordinates is below horizontal plane H $\Pi_{1}[4]$.


Figure 4: Linking of system of coordinates OXYZ with planes of projections $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ in the European system of measurement.
2. In the European system of measurement, space is partitioned into four quadrants by horizontal plane H $\Pi_{1}$ and frontal plane $\mathrm{F}-\Pi_{2}$ (Fig. 5).
3. The way of numbering quadrants in the European system of measurement differs from the American system of measurement. In the first, quadrant numerical values of coordinates $Z$ and $Y$ are positive for both axes $-1:+Z,+Y$. In the second quadrant, numerical values of coordinate $Z$ are positive, and coordinates $Y$ are negative $-2:+Z,-Y$. In the third quadrant, numerical values of coordinates $Z$ and $Y$ are negative for both axes $-3:-Z,-Y$. In the fourth quadrant, numerical values of coordinate $Z$ are negative, and coordinates $Y$ are positive $-4:-Z,+Y$ [4]. The quadrant number increases at consecutive viewing quadrants from the positive direction $+X$ axes $O X$ counter-clockwise (Fig. 5).

In the European system of measurement, the same as in the American system of measurement, the first octant is chosen as one eighth part of space, in which
all the numerical values of coordinates $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ for three axes $O X, O Y, O Z$ are positive $+X,+Y,+Z$ is chosen.


Figure 5: Quadrants and octants numbering in the European system of measurement.

The difference for the first octant in the European system is that horizontal plane $\mathrm{H}-\Pi_{1}$ is located between axes OX and OY whereas in the American system frontal plane $\mathrm{F}-\Pi_{2}$ settles down, and frontal plane $\mathrm{F}-\Pi_{2}$ (Fig. 5) is located between axes OX and OZ, instead of horizontal plane $\mathrm{H}-\Pi_{1}$ (Figs. 2 and 3) in the American system.

Numbering of the first, the second, the third and the fourth octants, located to the left of profile plane $\mathrm{P}-\Pi_{3}$, coincides with numbering of the first, the second, the third and the fourth quadrants the European system (Fig. 5).

The fifth, the sixth, the seventh and the eighth octants in the European system settles down to the right of profile plane $\mathrm{P}-\Pi_{3}$ accordingly in the first, the second, the third and the fourth quadrants (Fig. 5).

Let's consider construction of model of a geometrical image in the American system of measurement.

The third octant in the third quadrant is used in the American system of measurement for construction of model of a geometrical image (Fig. 3).

Therefore, segment $A B$ of a straight line settles down in the third octant (Fig. 6).

The method of rectangular (orthogonal) projection is applied to construct the projections of segment $A B$ [44].

The results of projection of segment $A B$ into three mutually perpendicular planes $\mathrm{F}-\Pi_{2}, \mathrm{H}-\Pi_{1}, \mathrm{P}-\Pi_{3}$ are
frontal projection $\mathrm{A}_{\mathrm{f}} \mathrm{B}_{\mathrm{f}}$, horizontal projection $\mathrm{A}_{h} \mathrm{~B}_{h}$ and profile projection $A_{p} B_{p}$ [45]. The constructed threedimensional geometrical model of segment $A B$ is shown at the left site in Fig. (6).

For the development of a two-dimensional complex drawing from a three-dimensional geometrical model, segment $A B$ and projecting rays $A A_{f}, A A_{h}, A A_{p}, B B_{f}$, $B B_{h}, B_{p}$ are mentally removed. Horizontal plane $H-\Pi_{1}$ with projection $\mathrm{A}_{\mathrm{h}} \mathrm{B}_{\mathrm{h}}$ of the segment are rotated around the axis OX clockwise from the positive direction +X till its full coincidence with frontal plane $\mathrm{F}-\Pi_{2}$. Profile plane $\mathrm{P}-\Pi_{3}$ with projection $\mathrm{A}_{\mathrm{p}} \mathrm{B}_{\mathrm{p}}$ of the segment are rotated around the axis OY counter-clockwise from the positive direction $+Y$ till its full coincidence with frontal plane $F$ $\Pi_{2}$. The constructed two-dimensional geometrical model of segment $A B$ is shown on the right site in Fig. (6).

For the American system of measurement in the two-dimensional complex drawing of a geometrical image, the horizontal projection is located above the frontal projection, and the profile projection is located to the right of the frontal projection (Fig. 6).

Constructed frontal projection $\mathrm{A}_{\mathrm{t}} \mathrm{B}_{\mathrm{f}}$, horizontal projection $A_{h} B_{h}$ and profile projection $A_{p} B_{p}$ of segment AB satisfy the laws of projective connections 1-1, 2-2, 3-3 [4, 37].

Elements of the American system of measurement are unambiguously connected with the projections of a geometrical image.

1. Negative coinciding portions $-X \equiv-Z,-Y \equiv-Z$ of axes X and Y and axes of ordinates, applicates $\mathrm{OY}, \mathrm{OZ}$
limit frontal plane $\mathrm{F}-\Pi_{2}$ and the frontal projection of a geometrical image.
2. Coinciding negative portions $-X \equiv-Z$ of axes $O X$, OZ and coinciding positive portions $+Y \equiv+Z$ of axes OY , OZ limit horizontal plane $\mathrm{H}-\Pi_{1}$ and the horizontal projection of a geometrical image.
3. Coinciding negative portions $-Y \equiv-Z$ of axes $O Y$, OZ and coinciding positive portions $+\mathrm{X} \equiv+\mathrm{Z}$ of axes OX , OZ limit profile plane $\mathrm{P}-\Pi_{3}$ and the profile projection of a geometrical image.

For the realized way of construction of the complex drawing (Fig. 6) in the American system of measurement, the horizontal projection is the bottom view, and the profile projection is the left-side view.

Thus, if planes of projections are considered to be opaque in the projective drawing of a product then the bottom view settles down from above the front view and the left-side view settles down to the right of the front view. Conversely, in the countries using the American system of measurement, the bottom view settles down from below from the front view, and the left-side view settles down to the left of the front view. Conditions needed for realization of such accepted arrangement of views in the projective drawing of a product will be considered in the separate publication.

Let's consider the construction of model of a geometrical image in the European system of measurement. In the European system of measurement, the first octant in the first quadrant is used for construction of model of a geometrical image as shown in Fig. (5).


Figure 6: Geometrical models of segment $A B$ of a straight line in the American system of measurement.

That is why segment $A B$ of a straight line settles down in the first octant (Fig. 7).

The method of orthogonal projecting is also applied to develop projections of segment $A B$ [44].

The results of projections of segment $A B$ into three mutually perpendicular planes $\mathrm{F}-\Pi_{2}, \mathrm{H}-\Pi_{1}$, and $\mathrm{P}-\Pi_{3}$ are the frontal projection $\mathrm{A}_{2} \mathrm{~B}_{2}$, horizontal projection $A_{1} B_{1}$ and profile projection $A_{3} B_{3}$ [4]. The constructed three-dimensional geometrical model of segment $A B$ is shown to the left in Fig. (7).

For construction of the two-dimensional complex drawing from three-dimensional geometrical model, segment $A B$ and projecting rays $A A_{2}, A A_{1}, A A_{3}, B B_{2}$, $B B_{1}, B_{3}$ are mentally removed.

Horizontal plane $H-\Pi_{1}$ with projection $A_{1} B_{1}$ of segment $A B$ is rotated about the axis of abscissas $O X$ clockwise from a positive direction +X till its full coincidence with frontal plane $\mathrm{F}-\Pi_{2}$. Profile plane $\mathrm{P}-\Pi_{3}$ with projection $A_{3} B_{3}$ of segment $A B$ is rotated about the axis of applicate $O Z$ counter-clockwise from a positive direction $+Z$ till its full coincidence with frontal plane $F$ $\Pi_{2}$. The constructed two-dimensional geometrical model of segment $A B$ is shown to the right in Fig. (7).

A two-dimensional geometrical model is called as the complex drawing of a geometrical image [4, 6, 1922].

For the European system of measurement in the two-dimensional complex drawing of a geometrical image, the horizontal projection is located below the frontal projection, and the profile projection is located to the right of the frontal projection (Fig. 7).

Constructed frontal projection $\mathrm{A}_{2} \mathrm{~B}_{2}$, horizontal projection $A_{1} B_{1}$ and profile projection $A_{3} B_{3}$ of segment AB also satisfy the laws of projective connections 1-1, 2-2, 3-3 [4, 37].

Elements of the European system of measurement also correlated with different projections of a geometrical image as follows:

1. Coinciding ( $+\mathrm{X} \equiv-\mathrm{Y}$ ) the positive portion of the axis of abscissas OX and the negative portion of the axis of ordinates OY , and also coinciding ( $+\mathrm{Z} \equiv-\mathrm{Y}$ ) the positive portion of the axis OZ and the negative portion of the axis of ordinates OY limit frontal plane $\mathrm{F}-\Pi_{2}$ and the frontal projection of a geometrical image.
2. Coinciding ( $+\mathrm{X} \equiv-\mathrm{Y}$ ) the positive portion of the axis of abscissas OX and the negative portion of the axis of ordinates $O Y$, and also coinciding ( $+Y \equiv-Z$ ) the positive portion of the axis of ordinates OY and the negative portion of the axis OZ limit horizontal plane H $\Pi_{1}$ and the horizontal projection of a geometrical image.
3. Coinciding ( $+Z \equiv-Y$ ) the positive portion of the applicate axis OZ and the negative portion of the axis of ordinates OY , and also coinciding ( $+\mathrm{Y} \equiv-\mathrm{X}$ ) the positive portion of the axis of ordinates OY and the negative portion of the axis of abscissas OX limit profile plane $\mathrm{P}-\Pi_{3}$ and the profile projection of a geometrical image.

For the realized way of construction of the complex drawing (Fig. 7) in the European system of measurement, the horizontal projection is the top view, and the profile projection is the left-side view.


Figure 7: Geometrical models of segment AB of a straight line in the European system of measurement.

Hence, if planes of projections to consider opaque on the projective drawing of a product, the top view is located below the front view and the left-side view is located to the right of the front view. Such an arrangement of views corresponds to the accepted arrangement of views in the projective drawing of a product in the countries where the European system of measurement is used.

Interrelations of elements American (Fig. 6) and European (Fig. 7) systems of measurement with different projections of a geometrical image in the twodimensional complex drawing are not the same. The differences result in the various arrangements of views in the projective drawing of a product.

An attempt to reach conformity of projections of an image (Fig. 6) and proper visualization of product views under the American standard (ISO A of ISO 5456) by simple turn of planes of projections in such European direction leads to loss of clearness of the drawing of a geometrical image (Fig. 8).

## 4. CONCLUSIONS

1. Three mutually perpendicular planes $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}$, $\mathrm{P}-\Pi_{3}$ are considered as the components of system of measurement for geometrical modeling. The right three-dimensional system of coordinates OXYZ is allocated to determine the coordinates in these planes.
2. The way of orientation of the right threedimensional system of coordinates OXYZ with respect to the set planes of projections $\mathrm{H}-\Pi_{1}, \mathrm{~F}-\Pi_{2}, \mathrm{P}-\Pi_{3}$ is the
first feature of systems of measurement. In the American system of measurement, the axes OZ and OX coincide with horizontal plane $\mathrm{H}-\Pi_{1}$, the axes OX and OY settle down in frontal plane $\mathrm{F}-\Pi_{2}$, and axes OY and OZ belong to profile plane $\mathrm{P}-\Pi_{3}$ (Fig. 2). In the European system of measurement, the axis OZ and OX coincide with frontal plane $\mathrm{F}-\Pi_{2}$, axes OX and OY settle down in horizontal plane $\mathrm{H}-\Pi_{1}$, and axes OY and OZ also belong to profile plane $\mathrm{P}-\Pi_{3}$ (Fig. 4).
3. The second feature of systems of measurement for geometrical modeling is the way of conditional division of space into parts: semi spaces, quadrants, and octants. In the American system of measurement, the space is divided into four quadrants by horizontal plane $\mathrm{H}-\Pi_{1}$ and profile plane $\mathrm{P}-\Pi_{3}$ as shown in Fig. (3). In the European system of measurement, the space is divided into four quadrants by horizontal plane $\mathrm{H}-\Pi_{1}$ and frontal plane $\mathrm{F}-\Pi_{2}$ (Fig. 5).
4. The third feature of systems of measurement for geometrical modeling is the order of numbering of the allocated parts of space (semi spaces, quadrants, and octants). In the American system of measurement, this order shown in Fig. (3) differs from that used in the European system of measurement (Fig. 5).
5. Interrelations of elements American (Fig. 6) and European (Fig. 7) systems of measurement with different projections of a geometrical image in the twodimensional complex drawing are not the same. The differences result in the various arrangements of views in the projective drawing of a product.


Figure 8: The rotation of the plane H into the vertical position performed in the other direction (down). The same holds for the profile plane P (Left).

An attempt to reach conformity of projections of an image (Fig. 6) and proper visualization of product views under the American standard (ISO A of ISO 5456) by simple turn of planes of projections in such European direction leads to loss of clearness of the drawing of a geometrical image (Fig. 8).

A way of elimination of discrepancy of projections of an image and views of a product for the American system we will consider in a separate research.

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