

An Improved Multiple Damage Identification Method of Plate Structure Using Discrete Wavelet Transform

Bingrong Miao*, Ying Zhang, Xujuan Li, Yaoxiang Luo, Yangzhe Qiu

State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu 610031, China

Abstract: In order to improve the identification accuracy of multiple damage of the plate structures, a method based on a two-dimensional discrete wavelet transform is proposed. Firstly, the structural characteristic parameters of the undamaged and damaged plate structures are obtained through the finite element modal analysis. The discrete wavelet transform is used to decompose and evaluate the modal node displacement data of the rectangular thin plate, and the high frequency information and singular values of the structural damage location are calculated. Secondly, the quantitative analysis of damage is realized by the change rate of the maximum value of the wavelet coefficient. Finally, according to the proposed algorithm, the anti-noise performance of the algorithm is verified through different ratios of noise. The results are shown that the vibration-based damage identification method is effective to detect the location and severity of the rectangular thin plate structures, which can be used to solve the damage identification problems of complex structures.

Keywords: Damage Identification, Discrete Wavelet Transform, Wavelet Coefficients, Mode Shape, Finite analysis.

1. INTRODUCTION

Structural damage identification technology has been widely used in the structural health monitoring (SHM) of actual engineering objects, such as civil engineering, aerospace and mechanical engineering fields [1]. How to effectively carry out damage identification research to prevent economic losses caused by structural failure has attracted much more attention [2]. Even a small local damage may cause a decrease in the stiffness of the structure, an increase in damping, and a decrease in natural frequency, the Damage detection methods based on structural vibration can be easily applied to identify the existence of damage [3]. Compared with other vibration-based damage identification methods, the wavelet transform method can examine multi-scale signals in more details, and provide different levels of details and approximations, the use of wavelet transform to identify damage from the mode shape has gradually become the most extensive damage identification method [4]-[6].

Many structural components of railway vehicles are composed of basic elements such as beams and plates. Vibration-based damage identification technology can be applied to structural health monitoring of structural components to improve the safety of vehicles. Since minor structural damage is usually a local phenomenon, it may not significantly affect the overall vibration response parameters of the structure. This requires a method for extracting the required

detailed damage characteristic information from the vibration response data of the typical damaged structure. A common method is to use wavelet transform (WT) method to identify structural damage [7]. Since wavelet transform can be very sensitive to signals that show singularity in the presence of some tiny defects, signals decomposition using WT can effectively detect and locate the degree of structural damage. Currently, there are many types of wavelet transforms and different wavelet functions, such as Haar wavelet, Symlet, Mexican hat or Morlet, and other wavelet methods that are constantly evolving. It can be concluded from past experience that the most effective appearance is the fourth-order Daubechies wavelet with two vanishing moments. It can be achieved by using, for example, Lipschitz index [7]. However, sometimes when using continuous wavelet transform (CWT) or discrete wavelet transform (DWT) to process structural response signals in the process of plate structure damage identification, it is found that different wavelet functions may not be effective in identifying the types of structural damage identification defects. This requires people to propose improvements and improvements to the wavelet method. Miao *et al.* proposed an optimized damage identification method of beam combined wavelet with neural network in an attempt to improve the calculation iterative speed and accuracy damage identification [8]. A neuro-wavelet technique was proposed for damage identification of cantilever structure [9].

Palechor [10] and Gholizad [11] *et al.* performed structural identification using experimental modal analysis and CWT. Hansang Kim *et al.* [12] introduced

*Address correspondence to this author at the State Key Laboratory of Traction Power, Southwest Jiaotong University, Chengdu 610031, China; E-mail: brmiao@home.swjtu.edu.cn

the application of CWT and DWT in the SHM, and researched the damage identification problem of beam structures and mechanical gears. Moavenian *et al.* [13] studied the damage identification of 2D plate structures with different damage position and severity based on the finite method. Bagheri [14] and Amiri [15] *et al.* researched damage identification of plate structures using the discrete wavelet transform, and Bagheri verified the de-noising method and damage identification results through experiments. Thuan *et al.* [16] used the 2D wavelet transform method and the vibration modal data of the FGM plate to study the damage location of the plate structure. Masoumi *et al.* [17] researched the cases of different damage positions and restrictions of plate structures through the method of the 2D discrete wavelet transform. Masoumi and Ashory *et al.* [18] dealt with irregular boundaries and eliminated the noise of structural vibration modes by stationary wavelet and transform. The singularity of the vibration signal of the structural damage part can be effectively identified by the CWT [19].

The wavelet analysis method has multi-resolution characteristics that are used to identify damage by many researchers. Zhao *et al.* [20] used wavelet transform to transform the mode shape of reinforced concrete beams, and used the maximum curve of wavelet coefficient differences to conduct damage identification research. The positioning accuracy and quantization sensitivity are discussed. However, when the wavelet transform is used to analyze the mode shape data of the structure, the boundary effect due to the discontinuity of the data at the boundary will cause the damage identification result at the boundary to be wrong. Therefore, eliminating the influence of discontinuous boundary conditions is achieved through boundary expansion. Sivasubramanian and Umesha *et al.* [21] performed wavelet transformation on the static deflection response of continuous beam structures, and used the modulus maximum of wavelet coefficients to identify the damage location. The quantitative analysis of structural damage degree is realized by wavelet coefficients. Expanding the data boundary is achieved by the cubic spline extrapolation method, which can reduce the influence of boundary effects. Mardasi *et al.* [22] collected the profile data of aluminum beams with and without damage through a high-resolution laser profile sensor. Damage recognition is realized by amplifying the singularity of the contour signal of the damaged beam through wavelet transform analysis. In addition, while optimizing the scale factor and the number of repetitions of the wavelet transform, a window

function is introduced to reduce the edge effect of the wavelet transform, and better recognition results are obtained through experiments.

In addition, the damage location of the structure is determined by the singular point of a certain order of the modal shape. Since the damage at the node location may be hidden, the damage can be difficult to identify. Janeliukstis *et al.* [23] used wavelet transform technology to construct structural damage identification indicators, and reduced false alarms due to uncertain factors by a weighted average of multi-order modal data, and compared them with damage identification methods based on mode shape derivatives. Among them, the author uses the Fourier transform technique to simulate the undamaged structure by using a polynomial constructed from the mode shape data of the damaged structure, thereby avoiding the data collection of the undamaged structure. Serra *et al.* [24] used the cubic linear extrapolation method to expand the mode shape of the structure to eliminate the signal anomaly at the boundary. The continuous wavelet analysis is used to analyze the multi-order modes on the mode shape after stationary wavelet transformation. After the wavelet transform coefficients of the mode shapes are weighted and superposed, they are used for structural damage identification research. Yang and Oyadiji [25] obtained structural modals through modal testing, expanded the data by using double harmonic spline interpolation, then carried out wavelet transformation and extracted the detail coefficients of each mode, and then used different weighting coefficients for each mode. The detail factor is superimposed to amplify the cumulative effect of damage.

This paper makes use of the 2D discrete wavelet transform method to decompose modal nodes displacement data of the rectangular thin plate, and the high frequency signal information of the damage location is obtained, which is the wavelet coefficient of three directions, revealing singular points of mode, so the damage locations are determined. Meanwhile, the quantization of the damage severity is conducted for the different damage position and severity, which is realized by the change of maximum value of wavelet coefficients in many cases.

2. METHOD AND THEORETICAL BACKGROUND

The method proposed in this paper to identify the linear damage of the rectangular thin plate by using the two-dimensional discrete wavelet transform, which mainly uses the wavelet multi-resolution analysis princi-

ple to decompose the mode shape of the plate model. The vibration mode of the damaged rectangular plate is mainly derived and calculated by the finite element modal analysis method. The damage is represented by the assumed reduced thickness element, and then the modal shape is analysed by two-dimensional discrete wavelet transform.

2.1. Basic Concept of Wavelet Method

Wavelet transform is an effective time-frequency signal processing method. Wavelet transform has special analysis capabilities for non-stationary signals and singular signals. It can be widely used in structural damage identification by using the time-domain positioning characteristics of the detection of singular points. Wavelet analysis is better than other time-frequency analysis methods in characterizing high-frequency components of the signal, and it has great advantages in the detection of abrupt signals. The wavelet function can be expressed by Vanishing moments, where, N represented the number of vanishing moments of the wavelet [26]. The larger the vanishing moment, the longer the support length, and the flatter the response filter, but the wavelet function will become more oscillating. The basic function of wavelet analysis is defined by two parameters: scale and displacement. Wavelet analysis uses wavelet basic function (i.e. the mother wavelet) to carry out. For the N th wavelet, the basic function can be expressed as:

$$\psi(n) = \sum_{j=0}^{N-1} (-1)^j C_j (2n + j - N + 1) \quad (1)$$

where, C_j is the wavelet coefficients. The basic function must satisfied two conditions: (1) The integral of basis function is zero, i.e. $\int_{-\infty}^{+\infty} \psi(x) dx = 0$. (2) The energy value of the integral of the square of the basic function is limited, i.e. $\int_{-\infty}^{+\infty} |\psi(x)|^2 dx < \infty$.

The first condition indicates that the basic function oscillates. The second condition shows that most of the energy of basic function is limited in finite continuous time. The "Orthogonally" and "Bi-orthogonally" are the important features of basic function. These characteristics make the wavelet coefficient calculation is very valid. Using continuous wavelet transform (CWT) can detect the amplitude mutation point of the vibration signal, so as to realize the diagnosis of local defects. The definition of CWT is described as follows [27].

$$C(s, u) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x) \psi^* \left(\frac{x-u}{s} \right) dx = \int_{-\infty}^{+\infty} f(x) \psi_{s,u}^*(x) dx \quad (2)$$

where, $s(s>0)$ is the scale of wavelet, which reflects the width of specific basic function; u is the translation parameter, which indicates the position along the x -axis. ψ^* is the complex transpose of ψ . The concept of wavelet vanishing moments is related to the singularities of the wavelet functions. Wavelet $\psi(x)$ with n vanishing moments has expressions as $\int_{-\infty}^{+\infty} x^k \psi(x) dx = 0$, where $k = 1 \sim n$.

2.2. Discrete Wavelet Transform (DWT)

Due to the continuous change of the scale and time of the sample, the continuous wavelet transform (CWT) requires a lot of calculation. In order to reduce the amount of calculation without losing the information of the original signal, the scale and translation factors are appropriately discretized, which can reduce the calculation amount of wavelet transform. This is also the advantage of discrete wavelet transform. The ideas and views on the DWT and CWT are consistent. The CWT need to find a wavelet coefficient at each scale parameter, in order to reduce the amount of calculation, while the DWT use discrete scale and translation parameters to reduce the amount of calculation. DWT analysis was carried out by multi-resolution analysis, which had been applied in many practical structural health monitoring problems [28].

2.2.1. 1D Discrete Wavelet Transform

For the 1D discrete wavelet, if the scale and translation parameters were defined to $s = 2^j$ and $u = k2^j$ respectively, the definition of discrete wavelet transform can be described as :

$$C_{j,k} = 2^{-j} \int_{-\infty}^{+\infty} f(x) \psi^* (2^{-j} x - k) dx = \int_{-\infty}^{+\infty} f(x) \psi_{j,k}^*(x) dx \quad (3)$$

The information of high frequency signal is analyzed through the high frequency filter, and the information of low frequency signal is analyzed through the low frequency filter. A signal in the discrete wavelet transform is represented by approximations and details:

$$f(x) = A_j(x) + \sum_{j \leq J} D_j(x) \quad (4)$$

The detail at level j is defined:

$$D_j = \sum_{k \in \mathbb{Z}} s_{j,k} \psi_{j,k}(x) \tag{5}$$

where, k represents the set of positive integers.

The approximation at level j is defined:

$$A_j = \sum_{j>J} D_j \tag{6}$$

In the discrete wavelet transform, scaling function $\phi(x)$ must meet three conditions : (1) The integration of $\phi(x)$ is equal to 1 and $\int_{-\infty}^{+\infty} \phi(x) dx = 1$. (2) The unit energy value $\int_{-\infty}^{+\infty} |\phi(x)|^2 dx = 1$. (3) $\phi(x)$ and its positive integer translation are orthogonal and meet $\langle \phi(x), \phi(x-n) \rangle = \delta(n)$. The scaling function is also represented by:

$$\phi(n) = \sum_{j=0}^{N-1} C_j (2n - j) \tag{7}$$

$$\phi_{j,k}(x) = 2^{j/2} \sum_{j=0}^{N-1} \phi(2^j x - k) \tag{8}$$

It should be noted that not all of the wavelet function has its scaling function, and only orthogonal wavelet function has its scaling function.

2.2.2. 2D Discrete Wavelet Transform

2D DWT scaling function can be defined as $\phi(x, y)$ similar to 1D scaling function $\phi(x)$ which corresponding to 1D multi-resolution approximation $\{V_j\}_{j \in \mathbb{Z}}$. Similarly, 2D multi-resolution approximation is $\{W_j\}_{j \in \mathbb{Z}}$ which is defined by $W_j^2 = V_j \otimes V_j$. Where “ \otimes ” denotes the tensor product. If W_j^2 represents detail space, it will be the orthogonal element of V_j^2 in its lower resolution approximation space V_{j-1}^2 , V_{j-1}^2 can be expressed:

$$V_{j-1}^2 = V_j^2 \oplus W_j^2 \tag{9}$$

where, “ \oplus ” represent the sum of two orthogonal vector spaces. So, wavelet orthogonal basis in space $L^2(R^2)$ can be defined by using scaling function and wavelet function. Three wavelet function can be defined:

$$\psi^V(x, y) = \phi(x)\psi(y) \tag{10}$$

$$\psi^H(x, y) = \psi(x)\phi(y) \tag{11}$$

$$\psi^D(x, y) = \psi(x)\psi(y) \tag{12}$$

2D DWT is the sum of three types of wavelet coefficients component matrix (i.e. vertical, horizontal and diagonal wavelet coefficients). $\psi^V(x, y)$, $\psi^H(x, y)$ and $\psi^D(x, y)$ denote the signals of the directions of vertical, horizontal and diagonal, respectively. The basic decomposition steps have also been established. Symlet4 wavelet having 4 vanishing moments was selected as a tool to complete a 2D discrete wavelet transform in this paper. Symlet4 wavelet has the characteristics of bi-orthogonal, tight-support, symmetry and its support width is $2N-1$, which can perform discrete wavelet transform. The Symlet4 wavelet is chosen to implement related research. Through combined with the finite element model, the proposed method can be used to calculate the node displacement of the model and all the wavelet coefficients.

3. NUMERICAL EXAMPLE ANALYSES

The size of the rectangular thin plate model used in the calculation is 600 mm in length, 400mm in width and 4mm in thickness. Finite element modeling and mode analysis are performed through ANSYS software. The plate structure finite element type is Shell 181. The damage locations are described by the different positions of the single and multiple models which are shown in Fig. (1). Three types of damage are symmetrically set

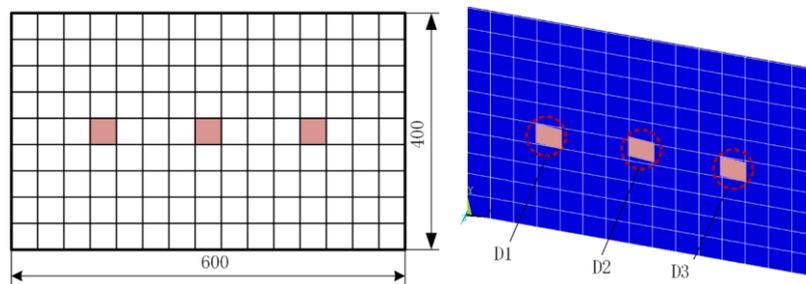


Figure 1: Damaged plate models.

in the center of the plate structure. The local damaged areas whose damaged degree is 50% are described by these damages, respectively. The main material parameters are shown in Table 1.

Table 1: Main Material Parameters

Main Material Parameters	Unit	Value
Elastic module	GPa	70
Poisson	--	0.3
Density	kg/m ³	2700

The mode nodes' displacements of the undamaged plate structure are shown in Fig. (2a). The 2D discrete wavelet transform method can be introduced to decompose the node displacement information and conducted identification. By using the 2D discrete wavelet transform, the four types of discrete signals can be calculated. cA is the wavelet coefficient of the profile component. cH is the horizontal wavelet coefficient, which is sensitive to the damage in the x-axis,

and the amplitude reflects the damage physical dimension. cV is the vertical wavelet coefficient, which is sensitive to the damage in the y-axis. cD is the diagonal wavelet coefficient, which is sensitive to the damage in the x or y-axis, and only being useful for the diagonal damage in the coordinate system.

The wavelet coefficients components (include cV, cH, cD) of the undamaged plate structure which are shown in Fig. (2b-d), respectively. It can be only found that these wavelet coefficients are slightly changed.

3.1. Different Damage Cases

3.1.1. Single Damage Case

This paper mainly considers two types of damage of thin rectangular plate structure: single damage and multiple damages, which are shown in Fig. (3). For the case of the single damage, the damage area is in the center of the plate, and the damage depth is 2mm and 50% damage. It is relatively difficult to observe the location, size and other information of the single damage only from the node displacement change which is seen

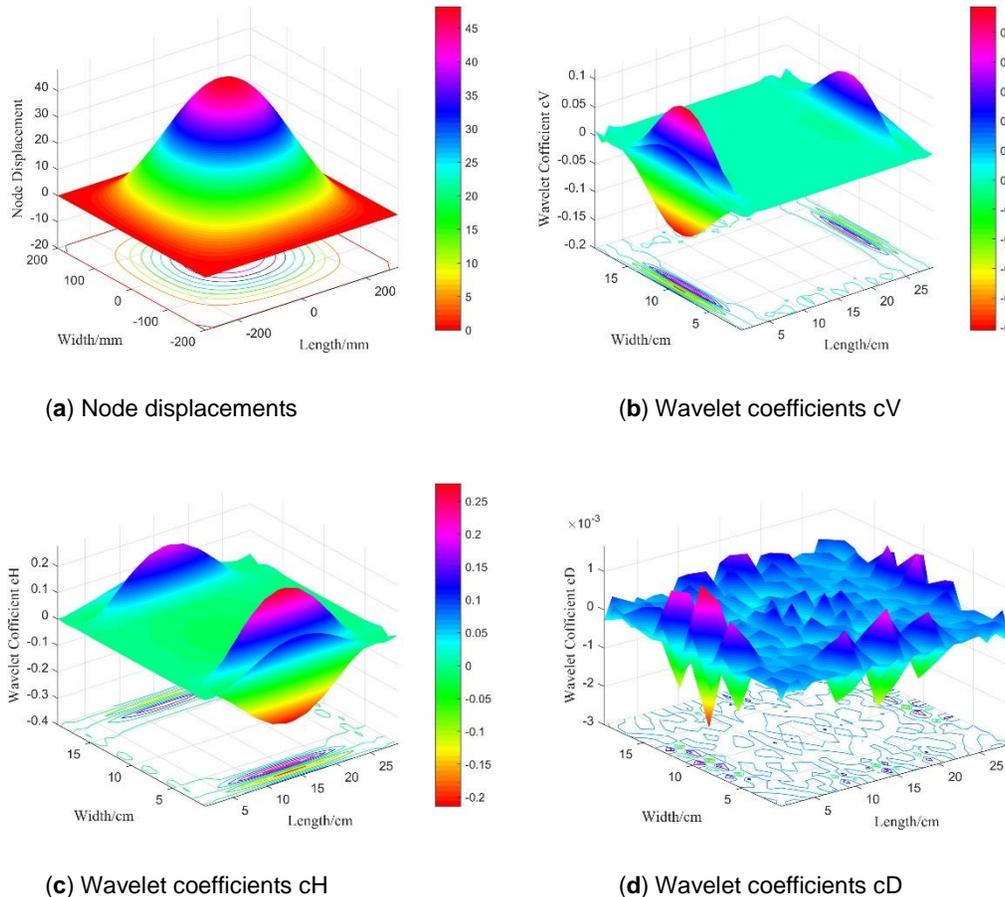


Figure 2: Undamaged plate node displacements and wavelet coefficients.

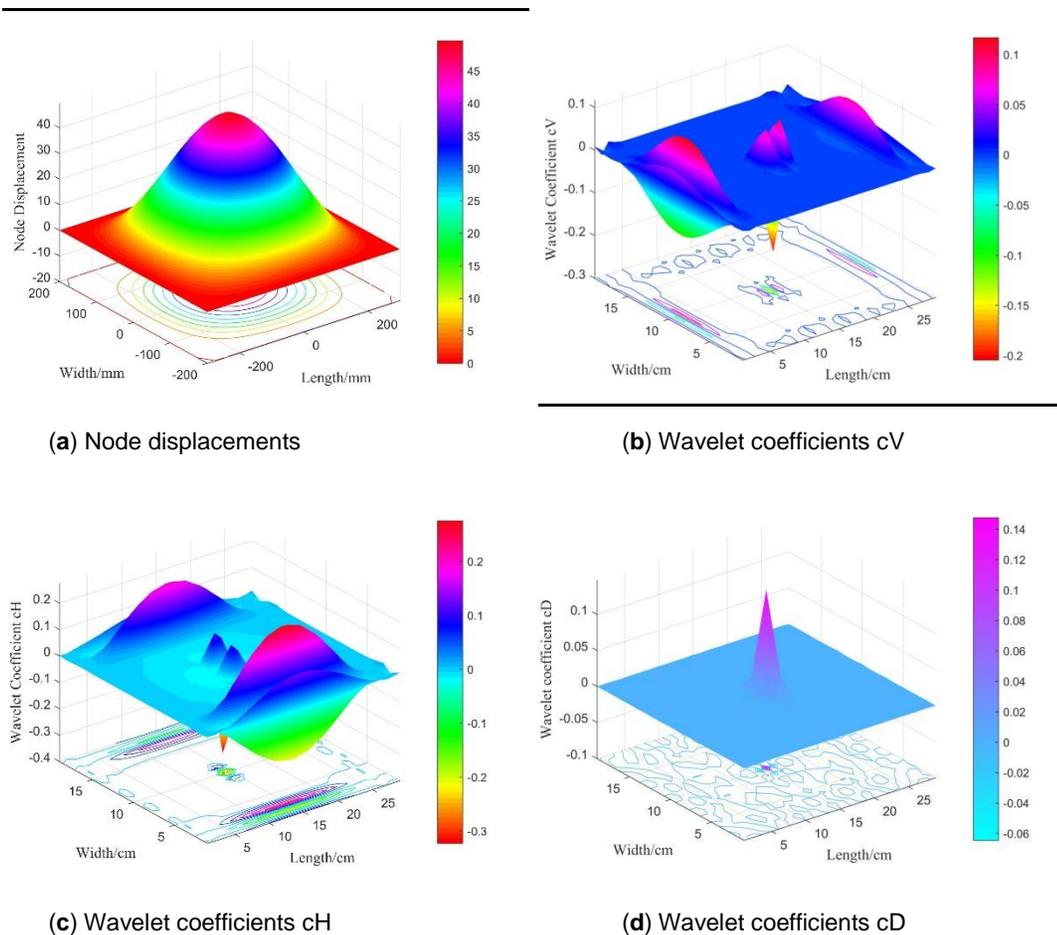


Figure 3: Single damaged plate node displacements and wavelet coefficients.

in Fig. (3a). But when a 2D discrete wavelet transform is performed to decompose the node displacement, obvious change in the damage area appeared, especially in the diagonal wavelet coefficient. So the proposed method is effective to identify single damage in the thin plate structure.

3.1.2. Multiple Damage Case

For the case of the multiple damages, damage areas are in the center of the plate and in the side of the 1/4 distance plate length, respectively, and the damage depth is 2mm and 50% damage. The node displacements and wavelet coefficients can be shown in Fig. (4).

Similarly, it can be seen that the location, size and other information of the multiple damages are relatively difficult only from the node displacement change which is shown in Fig. (4a). But when 2D discrete wavelet transform is performed to decompose the node displacement, obvious change in the damaged area appeared, especially in the diagonal wavelet coefficient and the center damage. So the proposed method is

effective to identify multiple damages in the thin plate structure.

From the above single damage and multiple damage identification, it can be concluded that the 2D discrete method can identify the plate structures' local damage effectively, including single damage and multiple damages. Although the location of the damage can be identified by a 2D discrete wavelet transform, the identification result of the diagonal wavelet coefficients cD is better, which has a certain guiding significance in future engineering.

3.2. Different Degree of Damage

The damage recognition result of the rectangular thin plate with a damage degree of 50% was discussed above, but the defect effect of the 2D discrete wavelet transform cannot be explained only by a single degree of damage. The thin rectangular plate structure with different damage levels (40%, 30%, 20%, 10%, 5% and 3%) are selected to discuss the recognition effect of 2D discrete wavelets. The calculation results are shown in Figs. (5-12).

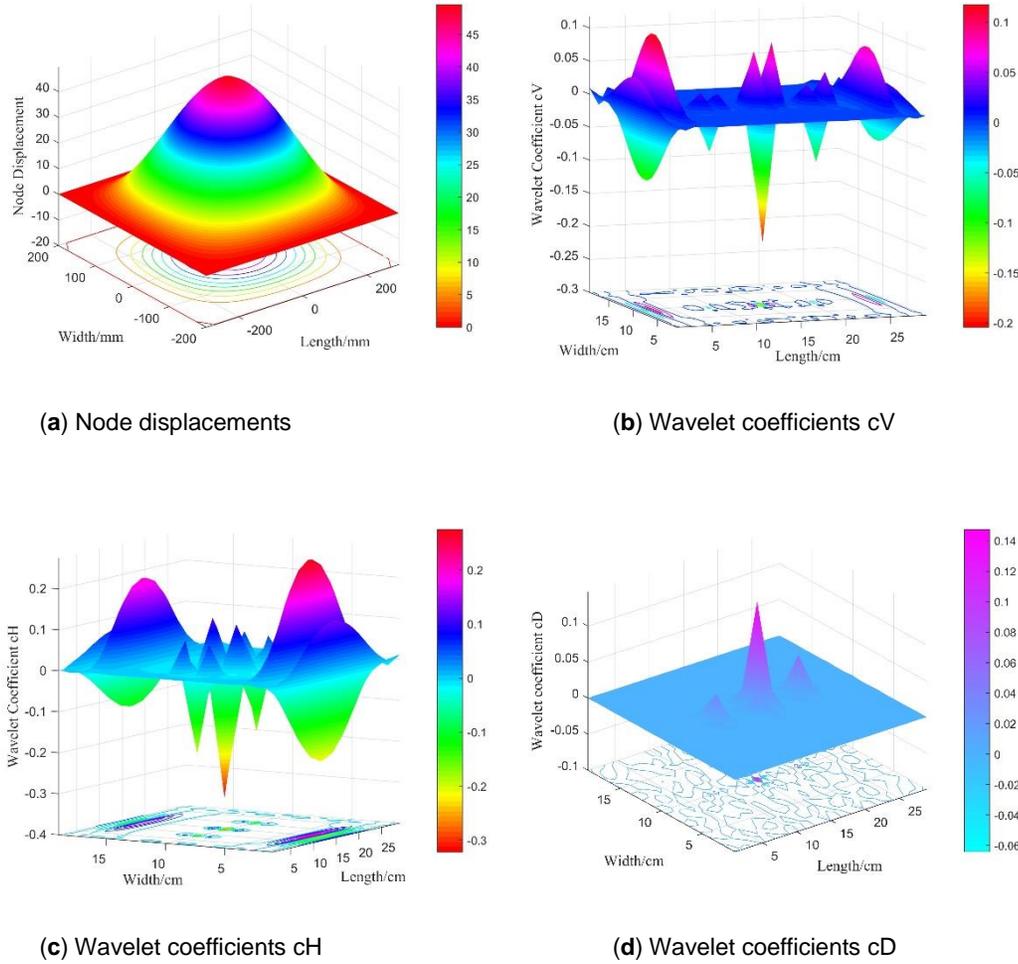


Figure 4: Multiple damaged plate node displacements and wavelet coefficients.

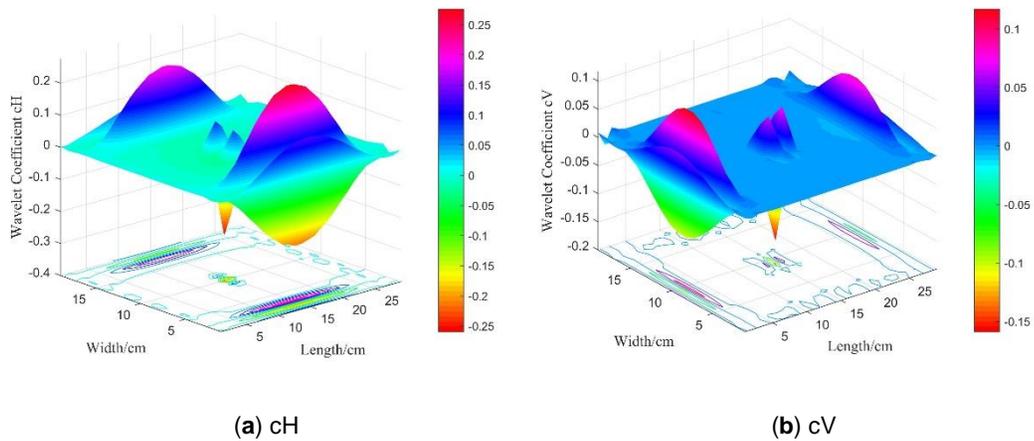


Figure (5): Wavelet coefficient cH and cV at 40% damage.

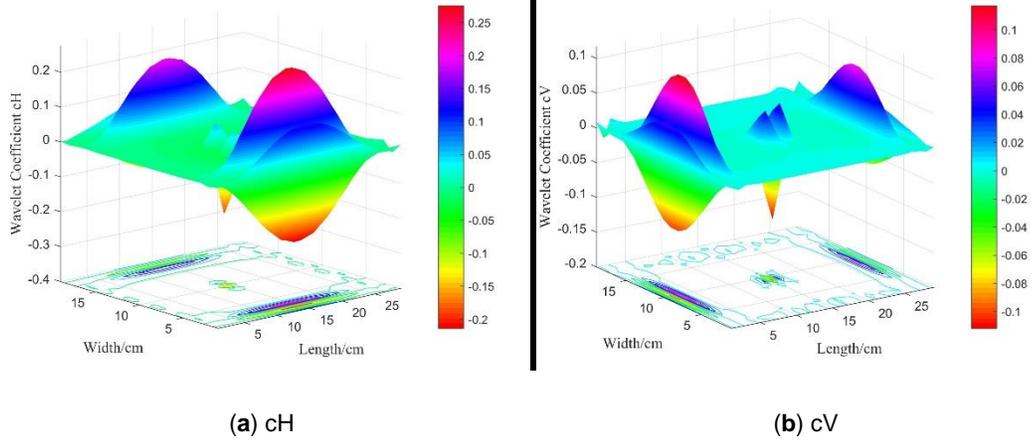


Figure (6): Wavelet coefficient cH and cV at 30% damage.

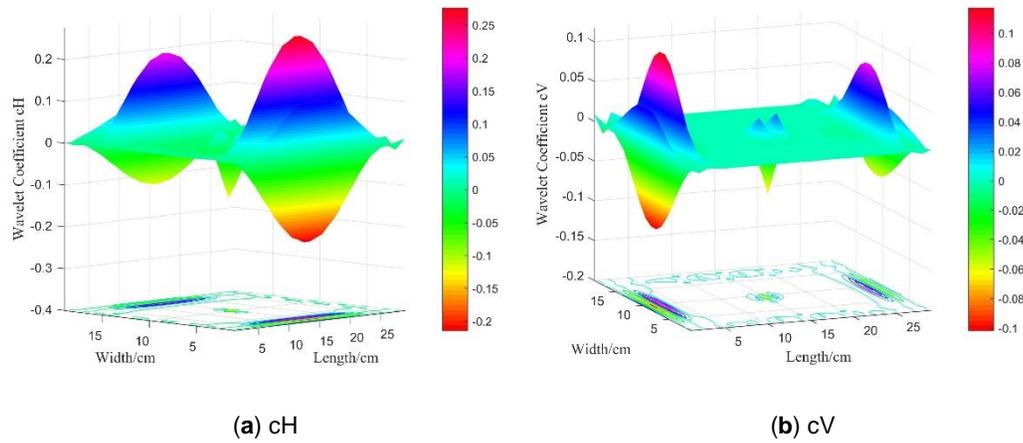


Figure (7): Wavelet coefficient cH and cV at 20% damage.

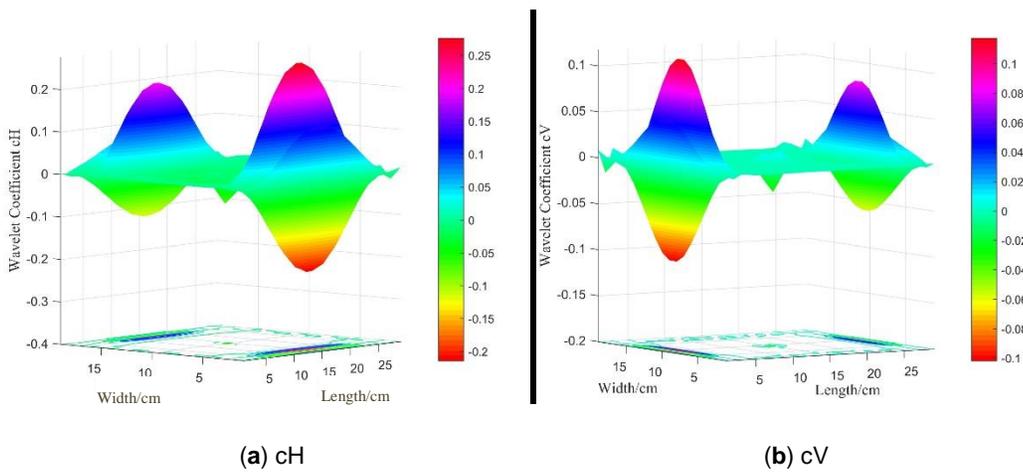


Figure (8): Wavelet coefficient cH and cV at 10% damage.

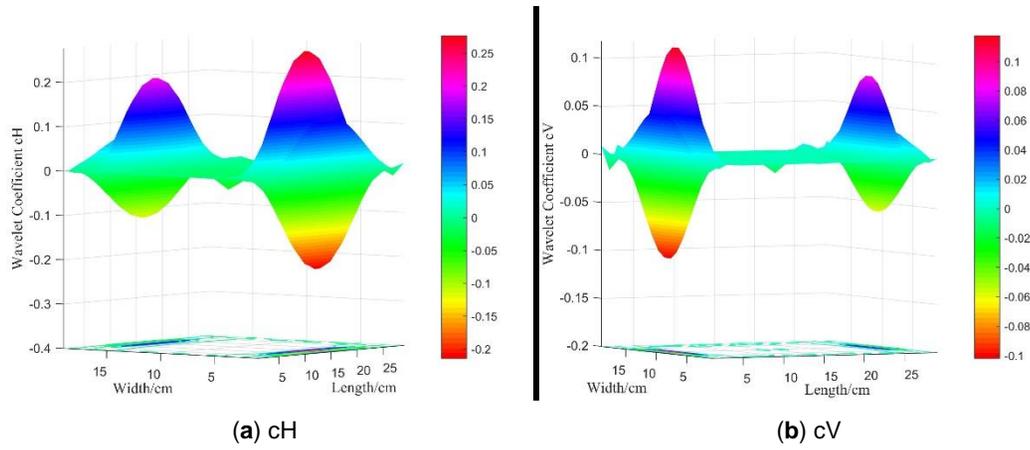


Figure (9): Wavelet coefficient cH and cV at 5% damage.

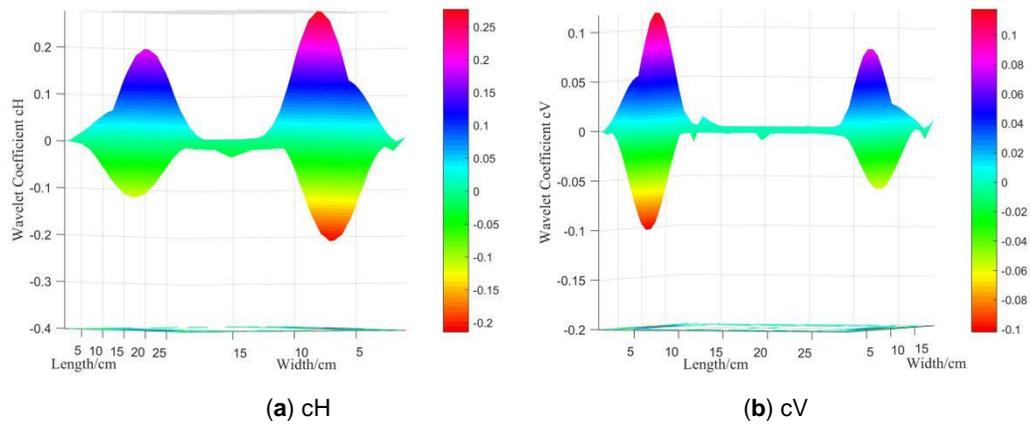


Figure (10): The Wavelet coefficient cH and cV at 3% damage.

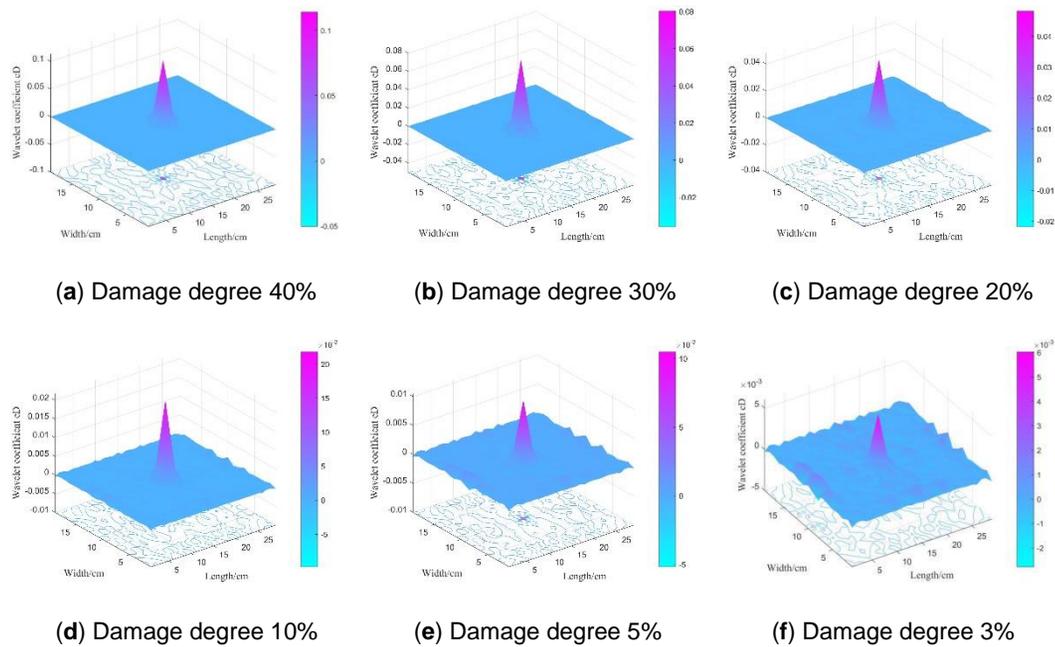


Figure (11): Recognition results of different damage levels by cD wavelet coefficient.

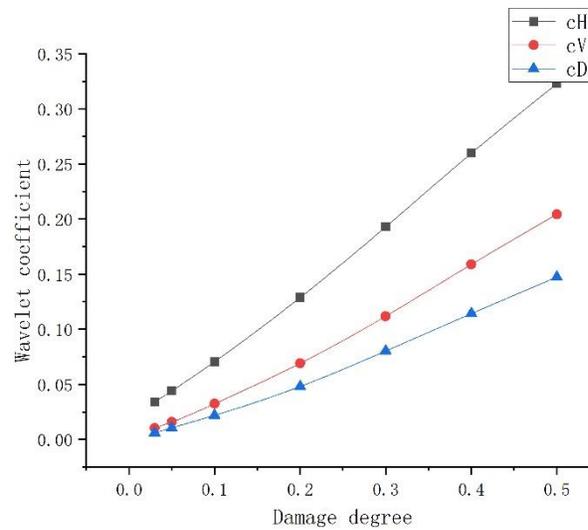


Figure (12): The relationship between different damage degree and wavelet coefficient.

By comparing Figs. (5 to 10), it can be found that the wavelet coefficient value decreases as the damage degree decreases. For small damage, the recognition effect of cH and cV is relatively poor, but the cD wavelet coefficient has a better recognition effect for different damage degrees, and the recognition effect is still very good even when the damage is 3% in Fig. (11).

By performing 2D discrete wavelet transform on rectangular plates with different degrees of damage, it can be found that as the degree of damage decreases, the value of wavelet coefficients is also decreasing, and the cD wavelet coefficient has the best effect on the identification of damage, especially small damage in Fig. (12).

3.3. The Effect of Noise

In actual measurement, it will inevitably be affected by noise. In order to verify the feasibility of 2D discrete wavelet transform in engineering applications, its noise immunity must be studied. In order to simulate the effect of noise, a series of noises need to be added to the modal data. In this study, a series of random noise is applied to the extracted node displacement. The expression of the new modal node displacement is:

$$z^* = z * (1 + (2 * rand - 1) * \rho) \quad (13)$$

where, $rand$ is a random number between 0-1, ρ is the noise ratio, z is the displacement of node, z^* is the displacement of node after adding noise.

Taking a single damage degree of 50% as an example, different proportions of noise such as 5%-50% are selected for noise immunity research. Since the recognition effect of cD wavelet coefficient is the best, only the recognition effect of cd wavelet is listed which are shown in Fig. (13), respectively.

From the analysis of the simulation results, the proposed damage recognition algorithm based on the two-dimensional discrete wavelet method is relatively accurate for the damage detection results of thin plates with different noise ratios, and has good noise resistance. These results also show that the diagonal component of 2D-Discrete wavelet method is larger and the detection accuracy is better. It has a certain reference value for the future engineering application of the plate structure recognition method.

4. CONCLUSIONS

In summary, in order to solve the problem of the detection accuracy and efficiency of the small damage of the damaged structure due to the changes of the modal displacement of the thin plate structure, a two-dimensional discrete wavelet damage identification method for the thin plate structure is proposed. The single and multiple damage detection effects of the plate structure under different noise levels are analyzed in detail. The innovation of this method is mainly to combine the discrete wavelet transform with the displacement mode parameters of the mode shape, and consider the identification effect of different wavelet coefficient components. The study found that different

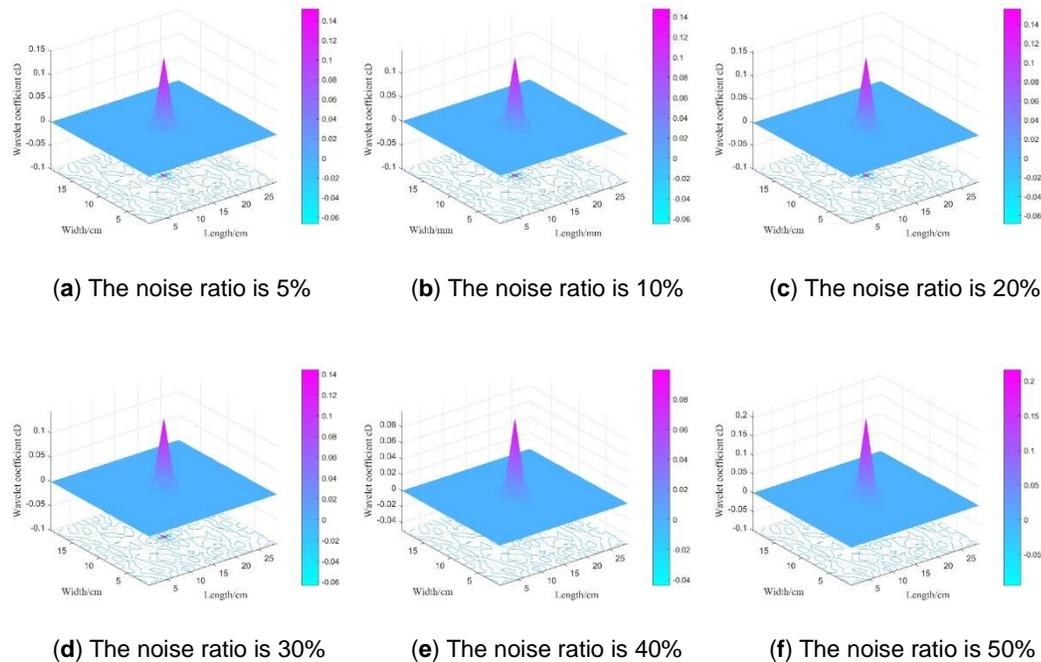


Figure 13: Recognition results of different noise ratio.

wavelet coefficient components can effectively identify the singular value change of the modal node displacement signal of the thin plate structure caused by the small damage feature in certain extent. Through comparison, it is also found that the use of diagonal component wavelet coefficients in the two-dimensional discrete wavelet transform can better identify the damage of the plate structure than other wavelet coefficient components. This is because the diagonal component wavelet coefficients can better avoid the influence of the boundary conditions than the vertical wavelet coefficients and the lateral wavelet coefficients. For the identification results of multiple damages of the structure, it is not particularly sensitive to the influence of the boundary conditions.

At the same time, this method can not only use the modal parameters to effectively identify the location and severity of the damage during the damage process of the thin plate structure, but also has better noise resistance. However, this study still has some shortcomings and needs to be improved in subsequent studies. For example, some wavelet coefficient components of this method are not ideal for identifying the types of multiple damages around the plate structure, which are more affected by boundary conditions. Obviously, the proposed algorithm needs further optimization and improvement. This method will combine neural network to improve the algorithm, and carry out

simulation and experimental verification on the wavelet damage identification algorithm of plate structure. In the future, this method will be combined with different vibration modes (such as modal curvature and modal strain energy, etc.) and optimization algorithms to explore better methods for thin plate structure damage identification.

ACKNOWLEDGEMENTS

This research was financially supported by the National Natural Science Foundation (51775456) and the Self-developed Research Project of the State Key Laboratory of Traction Power (2019TPL_T03).

REFERENCES

- [1] Das S, Saha P, Patro SK. Vibration-based damage detection techniques used for health monitoring of structures: a review [J]. *Journal of Civil Structural Health Monitoring*, 2016, 6(3): 477-507.
- [2] Rytter A, *Vibration Based Inspection of Civil Engineering Structures*. PhD Thesis, Aalborg University, Denmark. 1993.
- [3] Doebbling SW, Farrar CR, Prime MB, *et al.* Damage identification and health monitoring of structural and mechanical systems from change in their vibration characteristics: a literature review[J]. *The Shock and Vibration Digest*, 1996, 30(11): 2043-2049.
- [4] Bayissa W, Harutos N, Thelandersson S. Vibration-based structural damage identification using wavelet transform [J]. *Mechanical Systems and Signal Processing* 2008, 22(5): 1194-1215.
<https://doi.org/1-0.1016/j.ymsp.2007.11.001>.

- [5] Fan W, Qiao P. Vibration-based damage identification methods: a review and comparative study [J]. *Structural Health Monitoring*, 2011, 10(1): 83-111.
- [6] Jassim ZA, Ali NN, Mustapha F, *et al.* A review on the vibration analysis for a damage occurrence of a cantilever beam [J]. *Engineering Failure Analysis*, 2013, 31: 442-461.
- [7] Knitter-Piatkowska A, Garbowski T, Garstecki A. Damage detection through wavelet transform and inverse analysis[C]//VI International Conference on Adaptive Modelling and Simulation (ADMOS 2013). 2013: 389-400.
- [8] Miao BR, Whang MY, Yang SW, *et al.* An optimized damage identification method of beam using wavelet and neural network. *Engineering*, 2020, 12, 748-765.
- [9] Vafaei M, Alih SC, Rahman ABA, *et al.* A wavelet-based technique for damage quantification via mode shape decomposition [J]. *Structure and Infrastructure Engineering*, 2015, 11(7): 869-883. DOI:10.1080/15732479.2014.917114.
- [10] Palechor EUL, Bezerra LM, de Moraes MVG, *et al.* Damage detection in a reinforced concrete bridge applying wavelet transform in experimental and numerical data[J]. *Frattura ed Integrità Strutturale*, 2019, 13(48): 693-705.
- [11] Gholizad A, Safari H. Damage identification of structures using experimental modal analysis and continuous wavelet transform [J]. *Journal of Numerical Methods in Civil Engineering*, 2017, 2(1): 61-71.
- [12] Kim H, Melhem H. Damage detection of structures by wavelet analysis [J]. *Engineering Structures*, 2004, 26(3): 347-362.
<https://doi.org/10.1016/j.engstruct.2003.10.008>
- [13] Moavenian M, Khorrani H, Hasanzadeh S. An Investigation into the application of wavelet transform for crack detection in plates using Finite Element Method[C]// 16th Annual (International) Conference on Mechanical Engineering. Kerman: Shahid Bahonar University, 2008.
- [14] Bagheri A, Ghodrati AG, Khorasani M, *et al.* Structural damage identification of plates based on modal data using 2D discrete wavelet transform[J]. *Structural Engineering and Mechanics*, 2011, 40(1): 13-28.
<https://doi.org/10.12989/sem.2011.40.1.013>
- [15] Amiri GG, Bagheri A, Razzaghi SS, *et al.* Structural damage detection in plates using wavelet transform[M]// GHAFORI, N. Challenges, Opportunities and Solutions in Structural Engineering and Construction. Boca Raton Florida: CRC Press, 2009: 437-442.
- [16] Thuan LP, Trung NT, Hung NX, *et al.* Using 2D wavelet transform to identify crack in functionally graded material plates[C]// The 2012 International Conference on Green Technology and Sustainable Development(GTSD2012).
- [17] Masoumi M, Ashory MR. Damage identification in plate-type structures using 2-D spatial wavelet transform and flexibility-based methods [J]. *International Journal of Fracture*, 2013, 183(2): 259-266.
<https://doi.org/10.1007/s10704-013-9865-9>
- [18] Masoumi M, Ashory MR. Damage Identification from Uniform Load Surface Using Continuous and Stationary Wavelet Transforms [J]. *Latin American Journal of Solids and Structures*, 2014, 11(5): 738-754.
- [19] Reddy DM, Swarnamani S. Damage detection and identification in structures by spatial wavelet based approach [J]. *International Journal of Applied Science and Engineering*, 2012, 10(1): 69-87.
- [20] Zhao Y, Noori M, Altabay WA, *et al.* Mode shape-based damage identification for a reinforced concrete beam using wavelet coefficient differences and multiresolution analysis[J]. *Structural Control & Health Monitoring*, 2018, 25(1): e2041.
- [21] Sivasubramanian K, Umesha PK. Wavelet transform for damage identification in continuous beams [J]. *Int. Journal of Applied Sciences and Engineering Research*, 2013, 2(3): 294-306.
- [22] Mardasi AG, Wu N, Wu C. Experimental study on the crack detection with optimized spatial wavelet analysis and windowing [J]. *Mechanical Systems and Signal Processing*, 2018, 104: 619-630.
- [23] Janeliukstis R, Rucevskis S, Wesolowski M, *et al.* Damage Identification in Polymer Composite Beams Based on Spatial Continuous Wavelet Transform[C]// Iop Conference Series: Materials Science and Engineering. Baltic: 2016, 111(1): 012005.
- [24] Serra R, Lopez L. Damage detection methodology on beam-like structures based on combined modal wavelet transform strategy [J]. *Mechanics & Industry*, 2017, 18(8): 807.
- [25] Yang C, Oyadiji SO. Damage detection using modal frequency curve and squared residual wavelet coefficients-based damage indicator [J]. *Mechanical Systems and Signal Processing*, 2017, 83: 385-405.
- [26] Nikravesh SMY, Chegini SN. Crack identification in double-cracked plates using wavelet analysis [J]. *Meccanica*, 2013, 48(9): 2075-2098.
<https://doi.org/10.1007/s11012-013-9726-7>
- [27] Yang J-M, Yang Z-W, Tseng C-M. Damage detection in stiffened plates by wavelet transform [J]. *International Journal of Naval Architecture and Ocean Engineering*, 2011, 3(2): 126-135.
- [28] Yang J-M, Hwang C-N, Yang B-L. Crack identification in beams and plates by discrete wavelet transform method [J]. *Journal of Ship Mechanics*, 2008, 12(3): 464-472.

Received on 06-10-2020

Accepted on 28-11-2020

Published on 21-12-2020

DOI: <https://doi.org/10.31875/2409-9848.2020.07.5>© 2020 Miao *et al.*; Zeal Press.

This is an open access article licensed under the terms of the Creative Commons Attribution Non-Commercial License (<http://creativecommons.org/licenses/by-nc/3.0/>) which permits unrestricted, non-commercial use, distribution and reproduction in any medium, provided the work is properly cited.