# Small Sample Data-Driven Method for Interval Prediction on Structural Responses

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**Abstract:** This paper proposes an approach of intervals prediction on the structural responses under the limited samples by using Grey-BP neural networks (GBP) and genetic algorithm (GA). For the response prediction of complex structures, the presented method can ensure the necessary accuracy and can greatly reduce the cost of computation. Meanwhile, the method is different from the other traditional neural networks, which represents the superiority of dealing with the interval prediction under limited samples with help of the grey theory. In this paper, the Grey-BP neural networks are established by introducing the grey theory, which is used to achieve the mapping relationship between input and output of the system and accomplish the approximation of real mapping. The accuracy of built the mapping model can be tested by error analysis. Subsequently, the issue of interval prediction can be translated to optimal problem for extremum value. Although the explicit expression of the established mapping relationship is unknown, the fitness of every value includes the information of extremum. So, the best fitness and worst fitness can be searched in the given bound of uncertain variables based on genetic algorithm, and then achieve the upper bound and lower bound of the system response depending on the capability of global search. After proposed technologies are given in detail, one numerical example and one engineering example are presented and the results are discussed with common methods based on traditional BP neural network and Monte Carlo, which demonstrates the validity and reasonability of the developed methodology.

**Keywords:** Interval prediction, Grey-BP neural networks, Limited samples, Genetic algorithm, BP neural network.

## 1. INTRODUCTION

In many practical industry fields, the demand for accurate information prediction has been improved increasingly. Such as the financial field, it is the most significant that accurately predict the future trend of the financial data based on available data [1, 2]. Meanwhile, the prediction of vehicle flowrate and visitors' flowrate in the transportation industry is important for allocation of transportation resources [3-5]. The prediction of response of structures, including displacement, stress and strain in structural design industry, can provide the evidence for structure design [6-9]. Therefore, the technology of information prediction has become the highlighted focus.

Obviously, if the mapping relation between input data and output data is explicit, such as the solution for bending moment of cantilever beam, we can directly obtain the structural response under arbitrary conditions. However, the mapping relation is difficult to obtain the explicit expression in practical conditions, especially for the complex engineering issues. So, the other technologies must be provided, one of the common methods is response surface method (RSM). The response surface is constructed based on the available samples, and the samples can be generated by analysis means or the observation and statistics. The methodology fits the mapping relation by regression analysis and give simple polynomial expression, which avoids the complex repetitious analysis and reduce the computational cost for the large structures. Some studies are reported. The RSM is established to improve the accuracy of approximate model base on proposed algorithm using orthogonal polynomials is proposed to determine the necessary polynomial orders [10-13]. Different improved RSM are proposed to account properly for the second order effects in the response surface with acceptable computational effort [14-17]. RSM is also applied to the optimization of structure to reduce the computational cost for the large complex structure. The RSM is used as the means of approximating the mapping relation [18-21].

However, the RSM also presents some deficiencies in the process of application. For the simpler mapping relation, the accuracy of RSM is acceptable, but for the complex mapping relation, the accuracy is difficult to satisfy the requirements. Therefore, another method artificial neural network (NN) is provided, which is also an approximate model by simulating the model of brain learning. The method appears the superiority of approximating the complex mapping relation, especially

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for the strongly nonlinear problem[22-24]. Some significant works about the neural network are reported. Chokshi [25] proposed a novel artificial neural network (ANN) based phase distribution prediction model for tailored hot stamping and the predicted results shown the agreement with the experimentally generated data. Biswas [26] dealt with the problem of predicting the energy utilization with help of the neural network approach. In his work, the complex nonlinear model of energy consumption was established based on the neural network and achieve better results. Ping [27] predicted the flow stress for Ti-15-3 alloy based on the neural network model under hot compression experiments. The neural network better approximated the nonlinear relation between the temperature and strain rate and obtained better results. The relevant researches have been carried out by Fu [28], Escrivá [29] and Mazloumi [30].

It is emphasized that the above works only achieve the prediction of point to point, namely the input value and output value are the determined variables. In fact, the input variables are not determined, which exists the range of fluctuation, such as the materials property and external load, which are described by the mean and variance. In other words, the uncertainty widely exists and must be considered. Therefore, intervals prediction analysis based on NN model has been a hot topic. For examples, Weerdt [31] introduced the polynomial set method in the neural network to predict the bound of interval, which improved the computational efficiency and accuracy for the high-nonlinear problem. Pierce [32] predicted the fatigue life of the glass fiber sandwich materials, using the interval set theory and neural network regression model. Meanwhile, the robustness of the model was investigated and demonstrated the validity of the method. Khosravi [33] propose a new, fast, yet reliable method for the construction of prediction intervals for neural network predictions. The proposed lower upper bound estimation (LUBE) method constructs a neural network for estimating the prediction interval bounds.

Although some studies have achieved some significant developments by using the NN models, the number of samples for the training is still the factor of limited application in complex structures. In the practical engineering, especially for the aeronautics and astronautics, the trouble that we have to deal with is high test cost and limited test samples. To solve the problem, a new model grey neural network (GNN) is proposed and some exploratory studies have been conducted. In the literature [34], the grey model and artificial neural network were employed to predict the suspended solids and chemical oxygen demand in the effluent from sequence batch reactors of a hospital wastewater treatment plant. The results presented better predicted accuracy under the limited samples. Liu [35] propose an improved grey neural network model to predict transportation disruptions. This improved model of grey neural networks exceeds the conventional grey model GM(1,1) with respect to the fact that the raw data tend to show exponential growth, demonstrating the good attribute of nonlinear approximation in terms of neural networks. The literature [36] predicted the energy consumption in Spanish economic sectors, using the grey neural network and input-output combined forecasting model. The results of energy consumption forecast are used to validate the effectiveness of the proposed model. The other exploratory works are conducted by Abdulshahed [37], Yang [38] and Ming [39]. Unfortunately, the researches about interval prediction under the limited number of samples are seldomly reported. But the uncertainty of the input parameters definitely exists. Therefore, it is necessary to propose one interval prediction method under the limited number of samples.

In view of above discussion, this paper proposes an approach of intervals prediction method under the limited samples by using Grey-BP neural networks (GBP) and genetic algorithm (GA). Where, the uncertainty of variables is considered, and utilize the grey theory to establish the grey neural networks, which reduces the dependence on the number of samples to a certain extent. Meanwhile, for the response prediction of complex structures, the presented method can ensure the necessary accuracy and can greatly reduce the cost of computation. In section 2, the grey neural networks are described in detail, combining the grey theory with artificial neural networks. In section 3, the method of intervals analysis based on constructed grey neural networks, with help of improved genetic algorithm (IGA), is proposed and the details are illustrated. In addition, in section 4, one numerical example and one engineering example are provided to elaborate the whole proposed procedure, followed by some conclusions in section 5.

## 2. CONSTRUCTION OF THE GREY NEURAL NETWORK MODEL BASED ON GREY THEORY

### 2.1. The Grey Mathematical Theory

The grey mathematical theory is proposed by Deng [40] to solve the statistic problem with the small samples. The grey mathematical theory is different from the probabilistic statistic, the information is partly known and partly unknown due to the limitation of the samples. In addition, the small amount of data usually shows the greater degree of dispersion. Obviously, the characteristics of the whole system is difficult to obtain depending on the limited samples. The grey theory claims that the regularity is always existed even though the complex representation and scattered data. Therefore, the grey mathematical theory is proposed to solve the problem. In order to excavate the essential regularity of the scatter data, the method of accumulated generating operation (AGO) is proposed to deal with the original data. Furthermore, the original data is changed as the increasing sequence and the randomicity of the original data is weaken, which is conducive to the searching the regularity of the data.

The original data sequence is defined as the

$$\boldsymbol{X}_i^{(0)} = \left\{ \boldsymbol{X}_1^{(0)}, \boldsymbol{X}_2^{(0)}, \cdots \boldsymbol{X}_M^{(0)} \right\} \quad (i = 1, 2, \cdots, M) \tag{1}$$

where, the superscript 0 denotes the original data, the $\boldsymbol{X}_i^{(0)}$ is the original data of the $i-th$ variable, $\boldsymbol{X}_i^{(0)} = \{X_i^{(0)}(1), X_i^{(0)}(2), \cdots X_i^{(0)}(u)\}$ and the $\boldsymbol{X}_i^{(1)}$ is defined as the 1-GAO sequence, the $\boldsymbol{X}_i^{(1)}$ is written as

$$X_i^{(1)}(t) = \sum_{t=1}^{u} X_i^{(0)}(t) = X_i^{(1)}(t-1) + X_i^{(0)}(t) \tag{2}$$

where, the superscript 1 denotes the 1-AGO, the $X_i^{(1)}(t)$ is the $t-th$ data of the $i-th$ variable in the 1-AGO sequence. Then, the model of $GM(0,N)$ is defined as

$$\boldsymbol{X}_1^{(1)} = b_2 \boldsymbol{X}_2^{(1)} + b_3 \boldsymbol{X}_3^{(1)} + \cdots + b_i \boldsymbol{X}_i^{(1)} + a \tag{3}$$

We defined the two matrixes $\boldsymbol{Y}$ and $\boldsymbol{B}$ based on the Eq.(3)

$$\boldsymbol{Y} = \begin{bmatrix} X_1^{(1)}(1) \\ X_1^{(1)}(2) \\ \vdots \\ X_1^{(1)}(u) \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} X_2^{(1)}(1) & X_3^{(1)}(1) & \cdots & X_i^{(1)}(1) & 1 \\ X_2^{(1)}(2) & X_3^{(1)}(2) & \cdots & X_i^{(1)}(2) & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ X_2^{(1)}(u) & X_3^{(1)}(u) & \cdots & X_i^{(1)}(u) & 1 \end{bmatrix} \tag{4}$$

The parameters in Eq.(3) is

$$b = [b_2, b_3, \cdots b_q, a] \tag{5}$$

The parameters are solved through the least square method, the estimation of the parameters is written as

$$\hat{\boldsymbol{b}} = (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T \boldsymbol{Y} \tag{6}$$

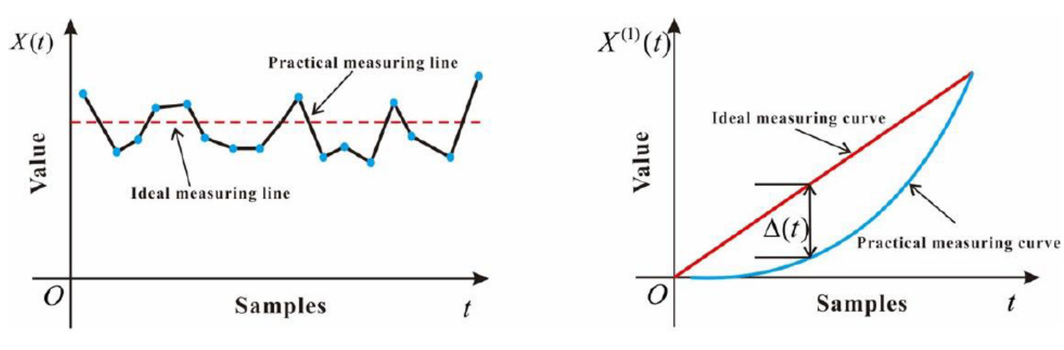
Substitute the $\hat{\boldsymbol{b}}$ into the Eq.(3), we can obtain the predicted value based on the $GM(0,N)$. It is necessary to point that the predicted value is obtained based on the new sequence with help of the AGO method. Therefore, we need to inverse the predicted value to acquire the real data. The inverse accumulated generating operation (IAGO) is given as follows

$$X_i^{(0)}(t) = X_i^{(1)}(t) - X_i^{(1)}(t-1) \tag{7}$$

where, all the notations are the same meaning as above equations. The Figure **1** shows the original data sequence and the 1-AGO data sequence.

### 2.2. The Model of Grey-BP Neural Network

The classical BP neural network has been reported and applied widely. The advantages of BP neural network represent the powerful nonlinear fitting ability and learning ability. In addition, the grey mathematical theory is applicable for small samples, but the




(a) Original measuring data sequence (b) Accumulated measuring data sequence

**Figure 1:** The sketch of the original sequence and the cumulative sequence.

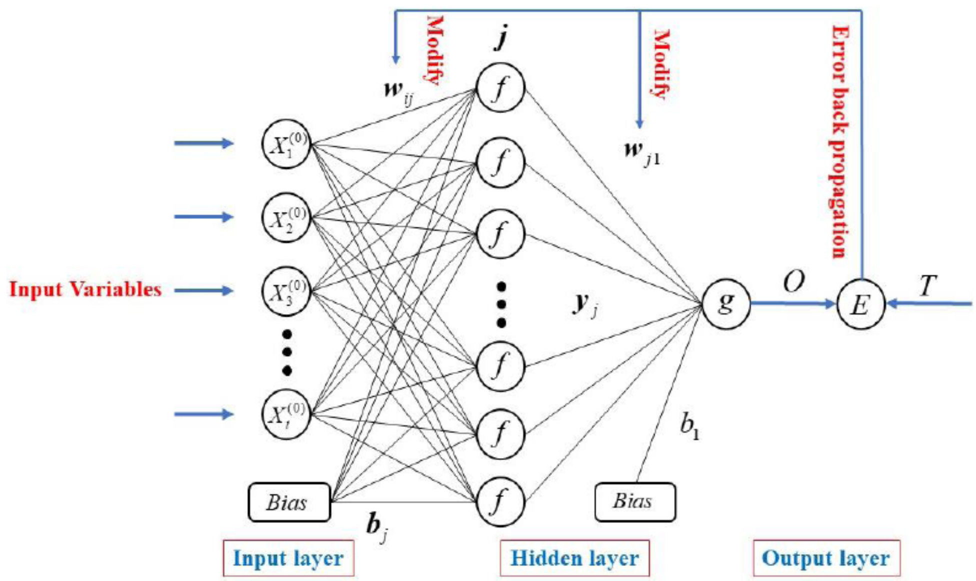



**Figure 2:** The three-layer structure of the classical BP neural network.

capability of dealing with the nonlinear problem is insufficient. Therefore, the Grey-BP neural network can be established to solve the prediction of nonlinear problem with the small samples. The structure of classical BP neural network includes input layer, hidden layer and output layer. Each layer has the nodes (Neurons) correspondingly, which are determined by the number of input variable and output variable. The information is transferred by the weight among the nodes of each layer. The three-layer structure of classical BP is shown in Figure **2**.

The BP algorithm includes two parts: forward propagation and backward propagation. In the process of forward propagation, input information is handled by the node of hidden layer to the output layer. If the desired output can't be obtained in the output layer, the error signal will be back-propagated to minimize the error signal by modifying the connection weights between neurons in each layer. In the process of network learning, the sigmoid function and pure linear function were employed as the transfer function of hidden and output layer ( $f(x)=1/(1+e^{-x})$, $g(x)=x$ ), respectively. The error is defined as fitness function, and the function is given as follows

$$E=\frac{1}{2N}\sum_{j=1}^{N}(T_j-O_j)^2 \tag{8}$$

where, $N$ is the number of all input samples, the $O$ is the target output of the experimental data and the $T$ is the predicted output of the neural network. The $O$ is obtained by the followed equation.

$$O=g\left(\sum_{j=1}^{N}w_{j1}y_i+b_1\right)=g\left(\sum_{j=1}^{N}w_{j1}f(\sum_{i=1}^{M}\sum_{j=1}^{N}w_{ij}X_i+b_j)+b_1\right) \tag{9}$$

where, the $w_{j1}$ is the weight value that connects the hidden layer and output layer. In this paper, the output layer has only one output value. The $w_{ij}$ is the weight value that connects the input layer and hidden layer and the $b$ is the neuron bias. $X_i$ is the input data of the net and the $y_j$ is the output of the hidden layer.

Combining with the grey theory, the new data sequence obtained by the AGO is used as the input data instead of the original data. The advantage is that the limited samples can be presented increasing format, which can excavate the regulation of the data. Therefore, the grey layer is added before the input layer and the whitening layer is added before the output layer. The whitening layer plays the role of obtaining the real data sequence by the IAGO method. Furthermore, the Grey-BP neural network is established, and the structure is shown in Figure **3**.

In order to acquire the better prediction of the neural network, the IGA algorithm is introduced to optimize the initial weight value and threshold of the BP neural network. We can calculate the weight value and threshold of the BP neural network based on the input samples, and then the best individual is selected by the calculating the fitness with help of the IGA algorithm. The fitness is obtained based on the fitness function (Eq.6). Subsequently, the weight value and threshold

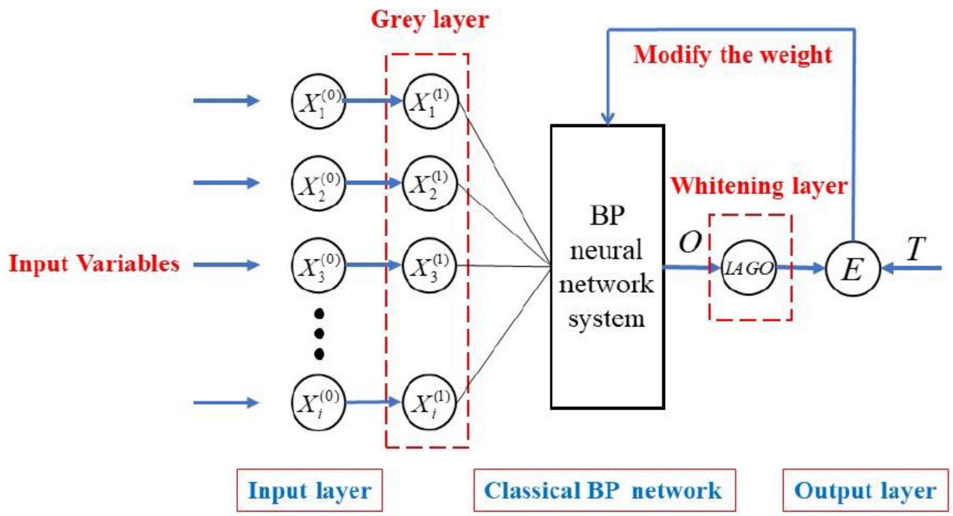



**Figure 3:** The architecture of the Grey-BP neural network.

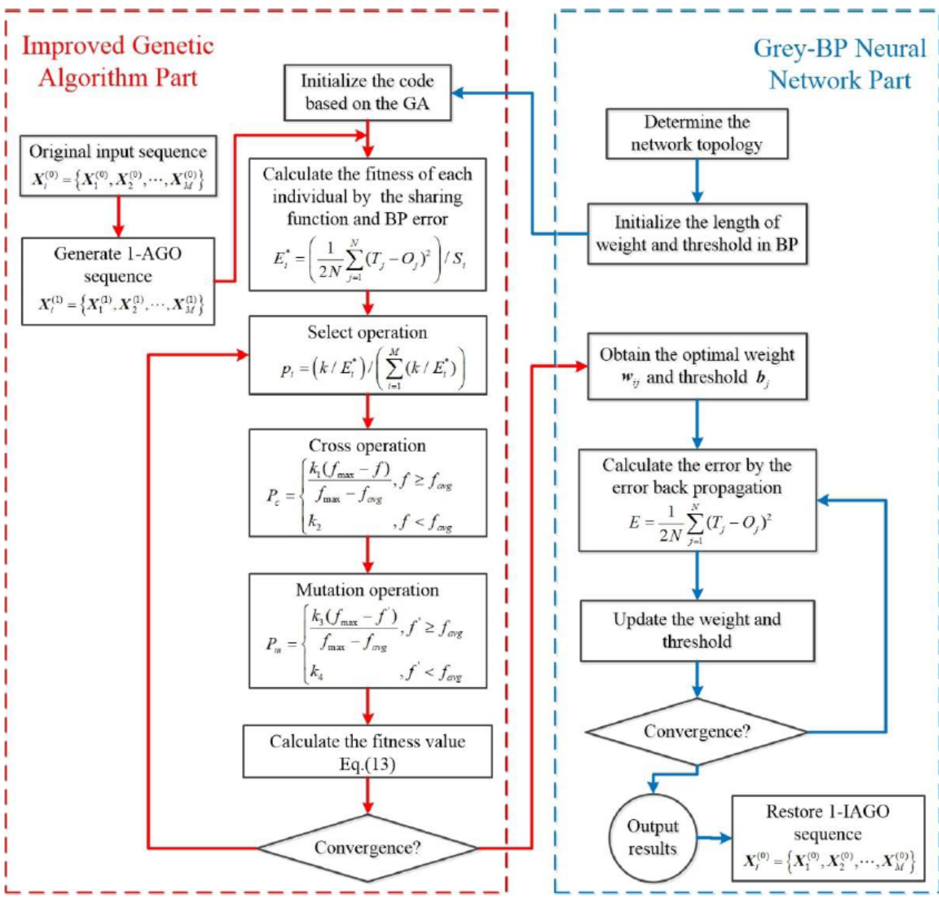



**Figure 4:** The optimal process of the Grey-BP based on the IGA method.

based on the best individual can be acquired and used to the training of the BP network. The optimal process of the BP based on the IGA method is shown in Figure **4**.

## 3. INTERVAL PREDICTION USING THE GREY-BP NEURAL NETWORK MODEL

### 3.1. Optimization Problem Formulation

The essence of the intervals analysis is the optimal problem. Therefore, the interval analysis is conducted by using the GA algorithm when obtaining the grey neural network model. The upper and lower of the interval, namely maximum value and minimum value are the target. The $X_i(j)\ (i=1,2,\cdots M, j=1,2,\cdots N)$ is defined as the $j-th$ samples of the $i-th$ variable. Generally, the variable varies in the certain scope when considering the uncertainty of variables. the variables can be rewritten as the interval format.

$$\boldsymbol{X}=\left[\boldsymbol{X}^L,\boldsymbol{X}^U\right]=\begin{bmatrix}[X_1^L(j),X_1^U(j)]\\ [X_2^L(j),X_2^U(j)]\\ \vdots\\ [X_i^L(j),X_i^U(j)]\end{bmatrix}=\begin{bmatrix}X_1(1) & X_1(2) & \cdots & X_1(j)\\ X_2(1) & X_2(2) & \cdots & X_2(j)\\ \vdots & \vdots & \vdots & \vdots\\ X_i(1) & X_i(2) & \cdots & X_i(j)\end{bmatrix} \tag{10}$$

$$\begin{aligned}\boldsymbol{X}^L &= \boldsymbol{X}^C - Cv\times\boldsymbol{X}^C\\ \boldsymbol{X}^U &= \boldsymbol{X}^C + Cv\times\boldsymbol{X}^C\end{aligned} \tag{11}$$

where $\boldsymbol{X}$ is the interval variable, $\boldsymbol{X}^L$ is the lower bound of the variable, the $\boldsymbol{X}^U$ is the upper bound of the variable, $\boldsymbol{X}^C$ is the mean value of the variable, the $Cv$ is the variational coefficient, and the $X_i(j)$ is the existing sample.

Since the input variables are interval format, and then the output variables are also the interval format. In order to acquire the range of interval, we introduce the optimal algorithm to solve the maximum and minimum value. For the unknown expression of function, we established the grey BP neural network based on the existing sample, namely the surrogate model. Furthermore, utilize the artificial optimal algorithm to search the extremum to determine the range of response. The classical optimal formulation is shown as follows

$$\begin{cases}\max f(\boldsymbol{X})\ \ or\ \ \min f(\boldsymbol{X})\\ s.t.\ \boldsymbol{X}\in[\boldsymbol{X}^L,\boldsymbol{X}^U]\end{cases} \tag{12}$$

where, $f(\boldsymbol{X})$ is the objective function, which is established depending on the specific problem. In this paper, the objective function is fitted based on the grey neural network. The $\boldsymbol{X}$ are the optimal variables and limited in the interval. Furthermore, the interval analysis is transferred to a solving extremum problem.

### 3.2. The Improved GA Optimal Policy

GA algorithm is one global optimization method, which is proposed by the Holland. The method simulates the biological genetic mechanism to search the optimum solution. The design variables are coded based on the evolutional theory, and then the screen the individuals that satisfy with the fitness requirements through select cross and variation. The best individuals are stored and the others are eliminated. However, the GA presents the high sensitivity to the fitness, crossover probability and mutation probability. Although the diversity of solution can be guaranteed in the pre-stage of optimization, the large number of individuals are concentrated on one extremum during the post-stage of optimization.

In this paper, we select the improved GA algorithm as the optimal algorithm for the optimal problem. The difference between adaptive GA algorithm and classical GA algorithm is reflected in two aspects:

(1) The sharing function is introduced to adjust the fitness of individuals, which guarantees the diversity of the colony. The sharing function reflects the degree of correlation between two individuals, which is defined as $S(x_{ij},x_{i(j+1)})$. The two individuals are the similar, the value of share function is big, otherwise, the value is small. The share function is calculated as follows

$$S(x_{ij},x_{i(j+1)})=\begin{cases}1-\dfrac{d_1(x_{ij},x_{i(j+1)})}{\sigma_1} & d_1(x_{ij},x_{i(j+1)})<\sigma_1, d_2(x_{ij},x_{i(j+1)})\ge\sigma_2\\ 1-\dfrac{d_2(x_{ij},x_{i(j+1)})}{\sigma_2} & d_1(x_{ij},x_{i(j+1)})\ge\sigma_1, d_2(x_{ij},x_{i(j+1)})<\sigma_2\\ 1-\dfrac{d_1(x_{ij},x_{i(j+1)})d_2(x_{ij},x_{i(j+1)})}{\sigma_1\sigma_2} & d_1(x_{ij},x_{i(j+1)})<\sigma_1, d_2(x_{ij},x_{i(j+1)})<\sigma_2\\ 0 & others\end{cases} \tag{13}$$

where, the $d_1(x_{ij},x_{i(j+1)})$ and $d_2(x_{ij},x_{i(j+1)})$ are the Hamming distance and fitness distance between two individuals, respectively. The $\sigma_1$ and $\sigma_2$ are the niche radius. Subsequently, the sharing degree is defined as $S_i=\sum_{j=1}^{N}S(d_{ij})\ (i=1,2,\cdots M)$ and then we can adjust the fitness degree according with the sharing degree. The fitness can be rewritten as

$$E_i^*=E/S_i\ \ (i=1,2,\cdots M) \tag{14}$$

(2) The other improvement is about estimation of crossover probability and mutation probability, which can vary with the individual fitness varying. The crossover probability $P_c$ and mutation probability $P_m$ are defined as follows

$$P_c=\begin{cases}\dfrac{k_1(f_{\max}-f)}{f_{\max}-f_{avg}}, & f\ge f_{avg}\\ k_2, & f<f_{avg}\end{cases},\quad P_m=\begin{cases}\dfrac{k_3(f_{\max}-f^{'})}{f_{\max}-f_{avg}}, & f^{'}\ge f_{avg}\\ k_4, & f^{'}<f_{avg}\end{cases} \tag{15}$$

where the $f_{\max}$ is the maximum fitness of populations, $f_{avg}$ is the average fitness, $f$ is the bigger value between two cross individuals, $f^{'}$ is the fitness of mutable individual, $k_1, k_2, k_3, k_4$ are the constants of experiments.

### 3.3. Interval Prediction of IGA Optimization Strategy for Grey BP Neural Network

In this section, the whole procedure of interval analysis for the grey BP neural network is summarized in detail.

Step 1: Initialize the uncertain parameters, generating the input samples $X_i$ and the output samples $y_i \ (i = 1,2,\cdots M)$ by the experiment or simulation analysis. In the paper, we consider the condition of small samples, the number of samples is less than 20.

Step 2: Generate the grey sequence $\boldsymbol{X}^{(1)}$, dealing with the original data based on the grey mathematical theory. In this paper, we utilizing the 1-AGO method to process the original data, and then whitening the data by 1-IAGO method.

Step 3: Construct the Grey-BP neural network. The new sequence of input is used as the training samples to complete the construction of the Grey-BP neural network, and then the output is whitened by the 1-IAGO method. Meanwhile, the GA algorithm is introduced to optimize the weight and threshold to improve the solution.

Step 4: Interval prediction based on the Grey-BP neural network and IGA iterative policy. After the Grey-BP neural network obtained in step 3, the interval analysis can be transferred to an optimal problem. Therefore, we can complete the searching of extremum with help of the IGA method in section 3.2. Furthermore, the interval bound of the response is obtained under the limited number of samples.

## 4. NUMERICAL EXAMPLES

### 4.1. A Simple Mathematical Problem

As the first example, we consider one mathematical problem. The nonlinear function is expressed as follows:

$$y = \sin \pi x_1 + \cos \pi x_2 + x_1 x_2 \tag{16}$$

where, the $x_1$ and $x_2$ are used as the input variables and the $y$ is output variable. The expression is a high-nonlinear and multimodal problem, the Figure **5** shows the features of the function.

In order to illustrate the advantage of Grey-BP neural network under the limited samples, we select the different number of samples to compare with the classical BP neural network. The number of training samples is 5, 10, 15, 20, 25, 30, respectively, and 10 true samples are compared with the predicted results. In this example, the IGA method is introduced to optimize the weight and threshold in Grey-BP neural network. The size of population is 100 and the

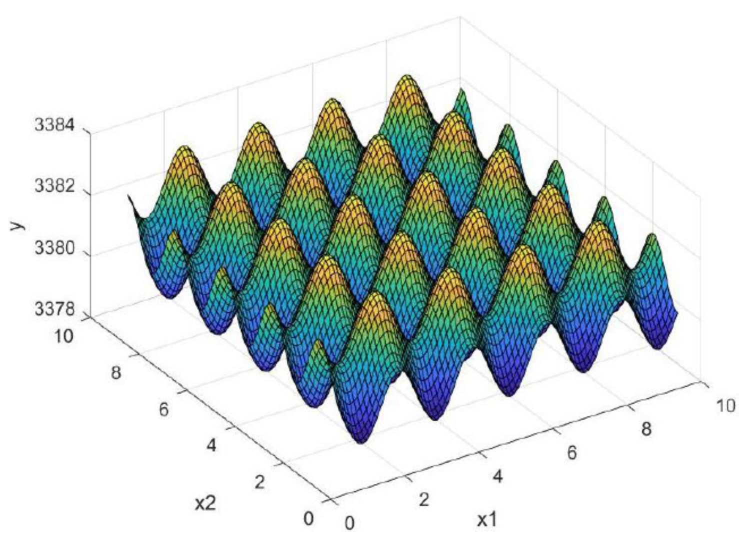
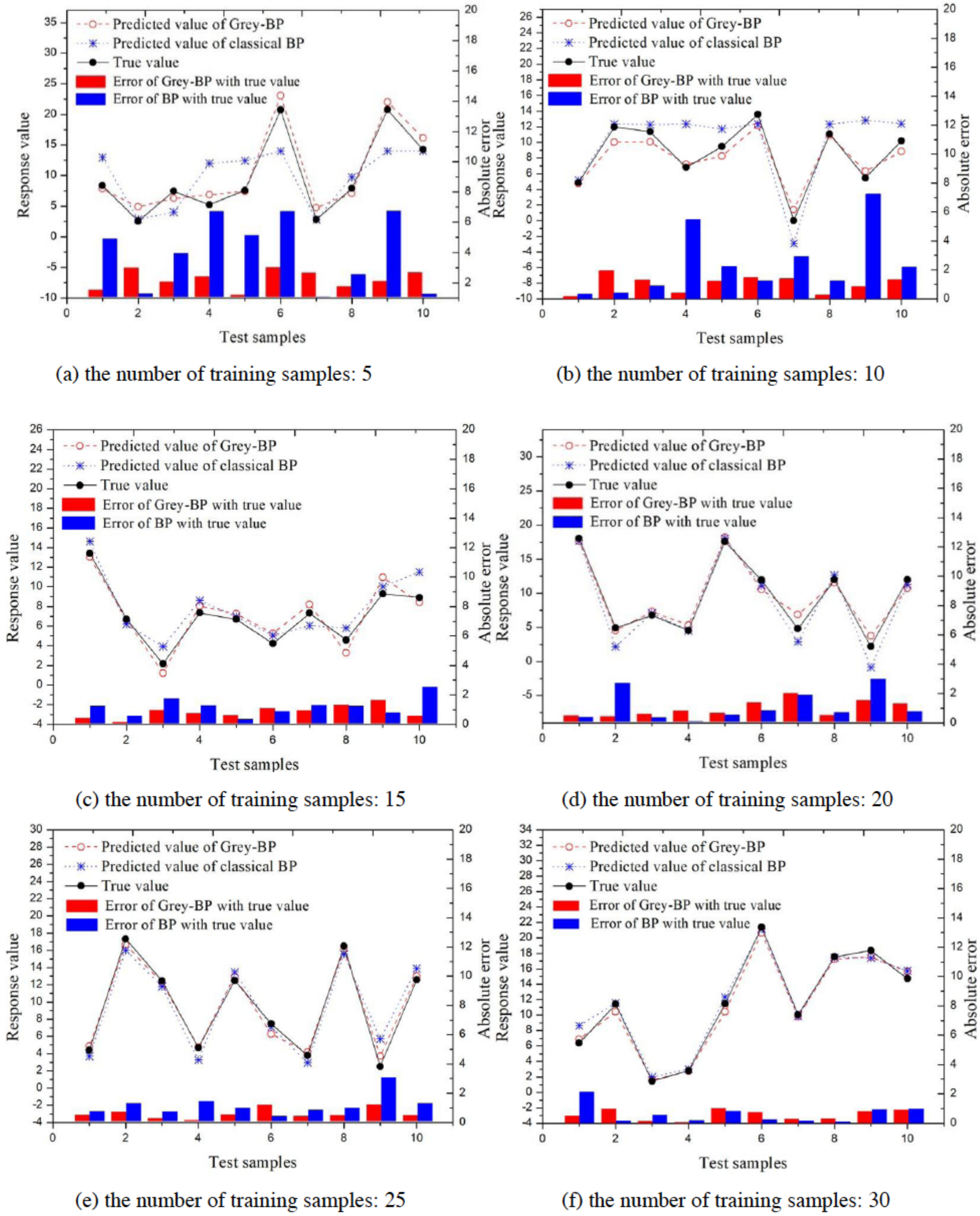



**Figure 5:** The sketch of the nonlinear function.




(a) the number of training samples: 5

(b) the number of training samples: 10

(c) the number of training samples: 15

(d) the number of training samples: 20

(e) the number of training samples: 25

(f) the number of training samples: 30

**Figure 6:** The predicted value of different method and the absolute error.

evolutionary generation is 10. The results of different conditions are shown in Figure **6**.

The accuracy of the proposed Grey-BP neural network is compared with classical BP network, and the results are listed in Table **1**.

Subsequently, we introduce the IGA method to solve the extremum value of the Grey-BP neural network. Furthermore, the interval prediction is transferred to the optimal problem. In the example, the $x_1^C = 3$, $x_2^C = 4$. We predict the intervals under different variational coefficient of the input variables $Cv = 0.05,\ 0.1,\ 0.15,\ 0.2$ and compare with the classical BP network and the Monte Carlo simulation. Meanwhile, we consider the effect of the number of training samples, the 25 training samples, 15 training samples and 5 training samples are selected to discuss. The interval prediction is shown in Figure **7** and the results are listed in Table **2**.

**Table 1: Results of Both Predicted Models under Different Training Samples**

| The Number of Samples | Grey-BP Model | | Classical BP Model | |
|---|---|---|---|---|
| | Absolute Error | Relative Error | Absolute Error | Relative Error |
| 30 | 0.5477 | 5.8% | 0.6405 | 6.8% |
| 25 | 0.5811 | 6.15% | 1.1108 | 11.7% |
| 20 | 0.8013 | 11.4% | 1.1699 | 16.8% |
| 15 | 0.9435 | 11.67% | 1.1777 | 21.3% |
| 10 | 1.0187 | 13.43% | 2.4300 | 32.0% |
| 5 | 1.4027 | 15.47% | 3.5463 | 39.1% |

**Table 2: Interval Prediction of the Both Predicted Models**

| Variation Coefficient | The Number of Samples | GBP Model | | BP Model | |
|---|---|---|---|---|---|
| | | Predicted Value | Interval Error | Predicted Value | Interval Error |
| 0.05 | 25 | [12.139,13.638] | 0.77% | [12.191,13.657] | 1.34% |
| | 15 | [12.139,13.638] | 0.77% | [11.891,13.657] | 2.2% |
| | 5 | [11.487,13.241] | 7.54% | [9.868,10.987] | 37.53% |
| 0.1 | 25 | [10.679,14.463] | 0.77% | [10.454,14.939] | 2.2% |
| | 15 | [10.679,14.363] | 3.94% | [10.454,14.639] | 8% |
| | 5 | [10.265,13.222] | 11% | [9.142,11.803] | 31.48% |
| 0.15 | 25 | [9.392,15.104] | 4.08% | [9.465,15.519] | 7.71% |
| | 15 | [9.392,15.104] | 4.08% | [8.865,15.519] | 11.71% |
| | 5 | [8.779,13.587] | 12.90% | [8.289,12.195] | 27.68% |
| 0.2 | 25 | [7.968,15.780] | 3.39% | [8.135,15.967] | 6.74% |
| | 15 | [7.968,15.780] | 3.39% | [7.135,16.317] | 14.07% |
| | 5 | [7.467,14.687] | 10.04% | [6.884,13.581] | 24.6% |

From the results in Figure **6**, Figure **7**, Table **1** and Table **2**, the following points can be summarized:

- The results show the predicted accuracy of the Grey-BP neural network under different number of training samples, and compared with the classical BP neural network. The predicted accuracy of both models is gradually improved with the increase of the training samples (Figure **7**). The number of samples is 30, the error of both models is the nearly same. The absolute error is calculated based on $\sum_{j}^{N}\left|y_{predict}^{j}-y_{true}^{j}\right|/N$, and the relative error is computed by $\sum_{j}^{N}(\left|y_{predict}^{j}-y_{true}^{j}\right|/y_{true}^{j})$. It is also demonstrated the number of samples determines the accuracy of the neural network.
- The predicted accuracy of the Grey-BP neural network represents the advantage when the number of samples is deduced. Although the accuracy shows the tendency of decline with the reduction of samples, the accuracy can be guaranteed compared with the classical BP neural network. In the case of the minimal samples, the training samples has only 5, the predicted error of the Grey-BP is 1.4027, but the error of classical BP is 3.5463. Therefore, the proposed Grey-BP neural network is more suitable for prediction of the small samples.
- Through the comparison of the interval prediction by different methods, the Grey-BP neural network has higher accuracy than the classical BP model. In the Figure **7**, both models have acceptable accuracy in the case of the 25 training samples. But The predicted error

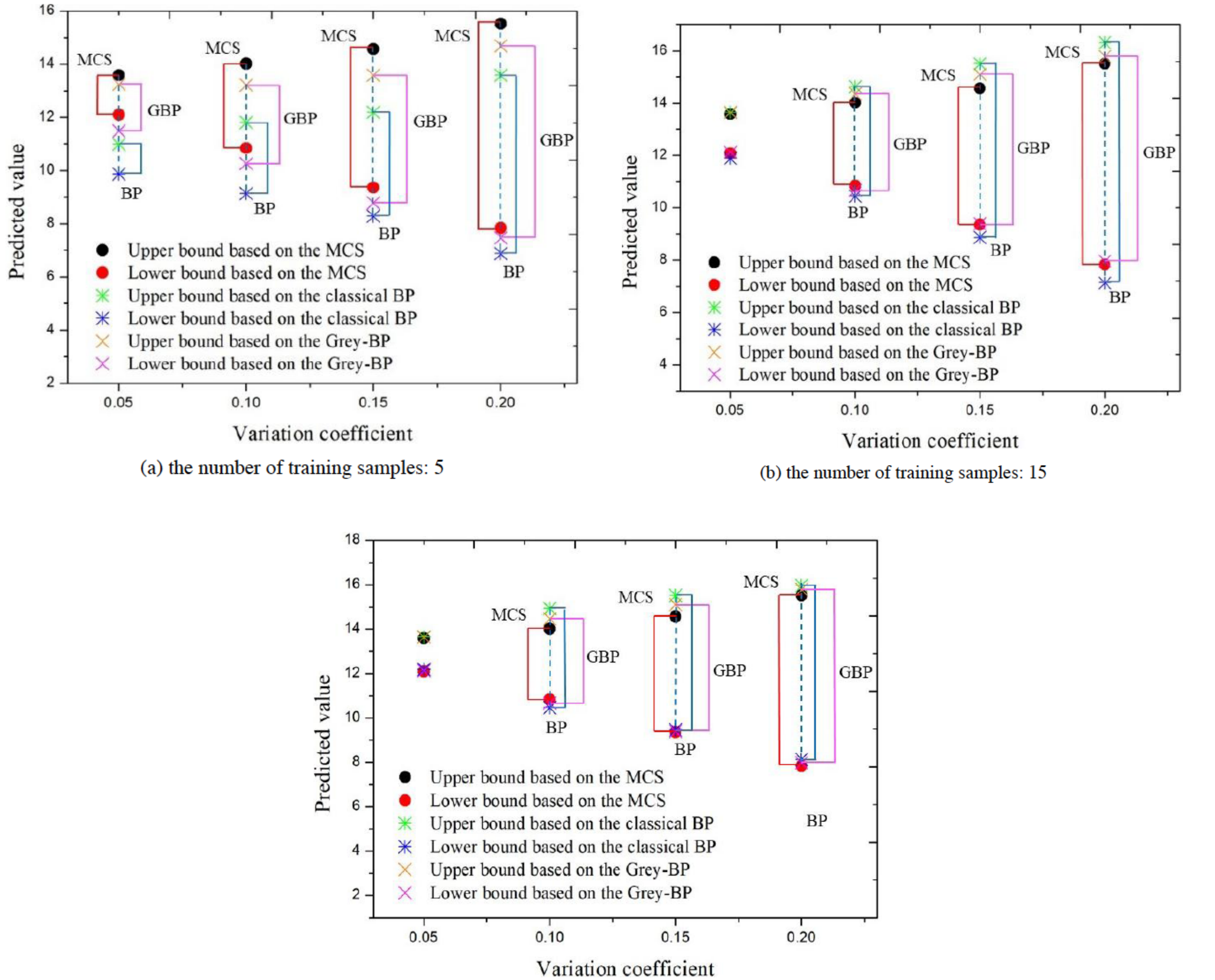



**Figure 7:** The comparison of interval prediction under different number of training samples.

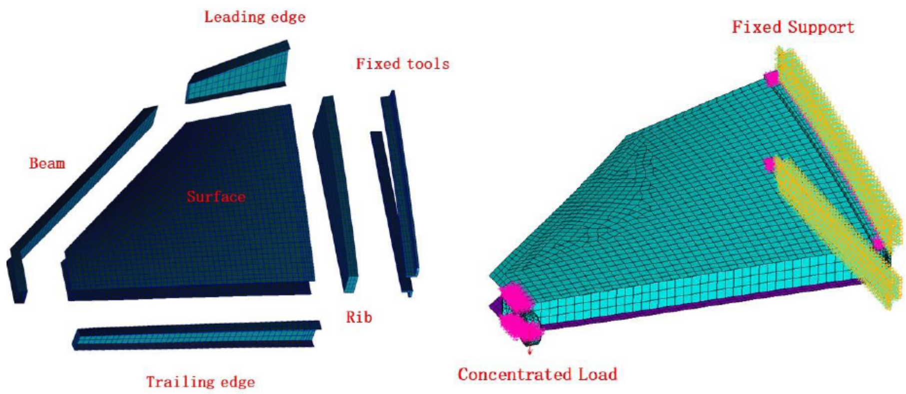
becomes higher with the sample sizes reducing, especially for the 5 training samples, predicted error of the both models reach the maximum. However, in the case of the 5 training samples, the interval error of classical BP model is 37.53%, the error of Grey-BP model is only 12.90%. Therefore, the Grey-BP model is suitable for interval prediction under the limited of samples. the Interval error is defined as

$$\left(\left|y^U_{predict} - y^U_{true}\right| / y^U_{true} + \left|y^L_{predict} - y^L_{true}\right| / y^L_{true}\right) \times 100\% \tag{17}$$

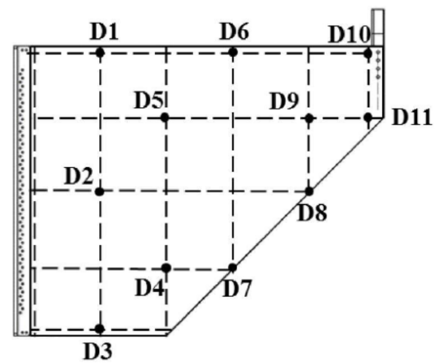
where, the $y^U_{predict}$ and $y^L_{predict}$ are upper bound and lower bound of predicted interval, respectively. The $y^U_{true}$ and $y^L_{true}$ are upper bound and lower bound of true interval, which are obtained by the MCS method.

### 4.2. A Composite Wingtip Structure of Reusable Aircraft

In this example, we consider one engineering structure, composite wingtip of the reusable aircraft (in Figure **8**), to predict the displacement interval of measuring points ($D_1 \sim D_{11}$) under external load. Similarly, both classical BP neural network model and Grey-BP neural network model are employed to predict the interval and discuss the results. The example is different from above mathematical problem, the expression of this example is implicit, the input variables are more than 2 and the output variables are 11. We select 5, 15 group samples as the training samples and 2 group samples are used as the test samples to demonstrate the applicability and rationality of the Grey-BP model. Subsequently, we complete the interval prediction with help of the IGA method and the results obtained by both models are compared to be discussed.




(a) Finite element information and boundary conditions




(b) Distribution of displacement measuring points

**Figure 8:** The model of composite wingtip of reusable aircraft.

**Table 3: The Interval Variables of Materials $T300$ and Aluminum Alloy**

| Label | $E_{11}$ | $E_{22}$ | $G_{12}$ | $\mu_{12}$ | $E_{Al}$ | $\mu_{Al}$ |
|---|---|---|---|---|---|---|
| Median | 125000 MPa | 9250 MPa | 4450 MPa | 0.35 | 210000 MPa | 0.27 |
| Radius | 12500 MPa | 925 MPa | 445 MPa | 0.035 | 21000 MPa | 0.027 |

Considering the dispersity of materials, the Poisson ratio $\mu_{12}$, $\mu_{Al}$ and elastic modulus $E_{11}, E_{22}, G_{12}$, $E_{Al}$ are treated as the interval variables. The wingtip is made of the carbon fiber reinforced materials ($T300$), the fixed tools is made of the aluminum alloy and the material properties are listed in Table **3**. Meanwhile, the different region has the different layer information, which is given in the Table **4**. The wing root is imposed by the fixture tools and the external load ($F_y = 14.5\text{kN}$) is applied on the special position, which is shown in Figure **8**.

**Table 4: The Stacking Information of the Composite Wingtip**

| Components | Stacking Sequence |
|---|---|
| Beam | $[0/\pm45/90/\pm45]_s$ |
| Trailing edge | $[0/\pm45/90/0/90/\pm45/0]_s$ |
| Rib | $[0/\pm45/90]_s$ |
| Leading edge | $[0_2/\pm45/90_2/\pm45]_s$ |
| Surface | $[0/\pm45/90/0/\pm45/90]_s$ |

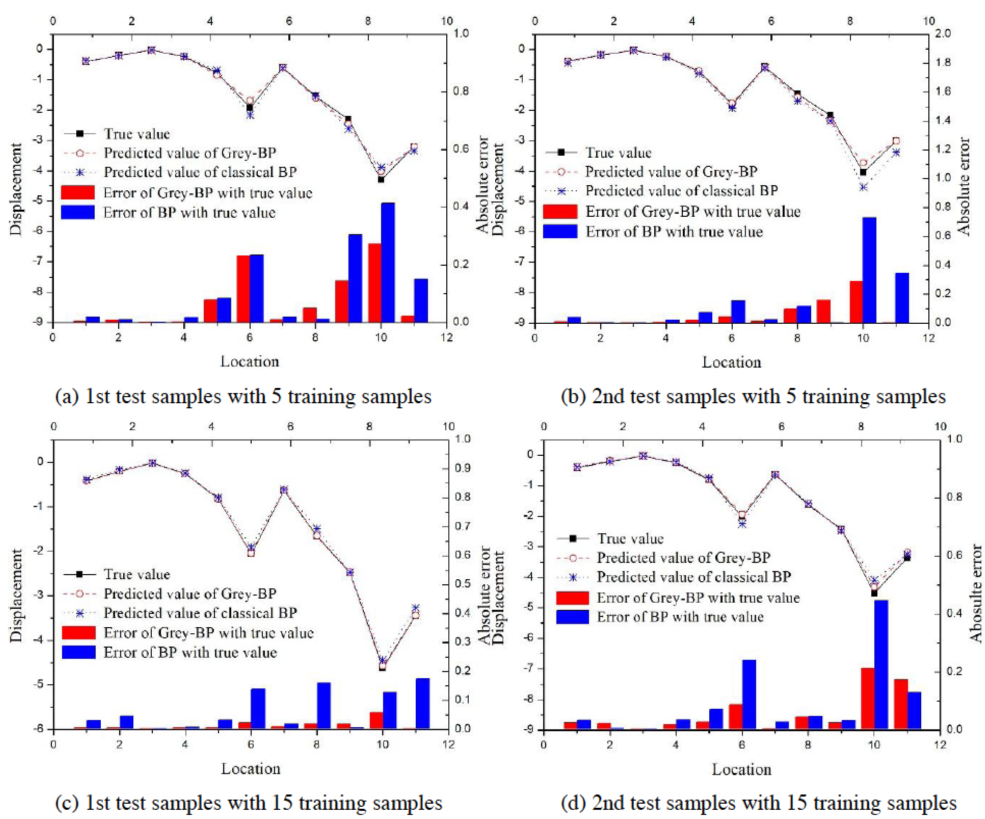
In the example, the number of input variables has 6, the output variables have 11, the hidden node is estimated by empirical formula $l = \sqrt{(m+n)} + a$. The $m$ is the number of input nodes, $n$ is the number of output nodes and $a$ is the constant varying in range $[0,10]$. In this example, $a = 4$, and then the number of hidden nodes $l = 8$. We can generate 22 groups of samples by utilizing the finite element analysis, in which we select 5 groups and 15 groups as the training samples, and the 2 groups are used as the test samples. The results obtained by the Grey-BP model (GBP) and classical

**Table 5: Error Comparison of Two Predicted Models under Different Training Samples**

| Location | 5 Training Samples (Error) | | | | 15 Training Samples (Error) | | | |
|---|---|---|---|---|---|---|---|---|
| | 1st Test Sample | | 2nd Test Sample | | 1st Test Sample | | 2nd Test Sample | |
| | GBP | BP | GBP | BP | GBP | BP | GBP | BP |
| 1 | 0.0032 | 0.0189 | 0.0099 | 0.0427 | 7.77e-4 | 0.0271 | 0.0211 | 0.0286 |
| 2 | 0.006 | 0.0086 | 0.0029 | 0.0018 | 8.78e-4 | 0.0432 | 0.0179 | 0.002 |
| 3 | 7.81e-6 | 1.13e-4 | 1.43e-4 | 2.60e-4 | 2.60e-4 | 6.08e-4 | 5.32e-5 | 1.18E-04 |
| 4 | 0.0012 | 0.0151 | 0.0059 | 0.0205 | 0.0016 | 0.0049 | 0.0128 | 0.0308 |
| 5 | 0.0785 | 0.0842 | 0.0178 | 0.0771 | 0.0015 | 0.0291 | 0.0223 | 0.066 |
| 6 | 0.2334 | 0.2359 | 0.0483 | 0.1666 | 0.0188 | 0.1392 | 0.0808 | 0.2324 |
| 7 | 0.0068 | 0.0189 | 0.0117 | 0.0216 | 0.0057 | 0.0158 | 6.02e-4 | 0.0234 |
| 8 | 0.0475 | 0.0099 | 0.1058 | 0.1238 | 0.0157 | 0.1615 | 0.0394 | 0.0424 |
| 9 | 0.1461 | 0.3070 | 0.1719 | 1.72e-5 | 0.0138 | 0.0019 | 0.0206 | 0.0277 |
| 10 | 0.2760 | 0.4186 | 0.3125 | 0.7985 | 0.0567 | 0.1271 | 0.2048 | 0.4343 |
| 11 | 0.0202 | 0.1509 | 9.53e-4 | 0.3779 | 5.97e-4 | 0.1752 | 0.1669 | 0.1233 |
| Total error | 0.8189 | 1.2681 | 0.6878 | 1.6308 | 0.1160 | 0.7256 | 0.5873 | 1.011 |

BP model (BP) are listed in Table **5** and shown in Figure **9**.

In addition, the interval can be predicted based on the different models with help of IGA, the neural networks built based on the 5 group training samples is selected as the objective function and the results of different models are listed in Table **6** and shown in Figure **10**.




(a) 1st test samples with 5 training samples (b) 2nd test samples with 5 training samples

(c) 1st test samples with 15 training samples (d) 2nd test samples with 15 training samples

**Figure 9:** The predicted results and error of different models under different training samples.

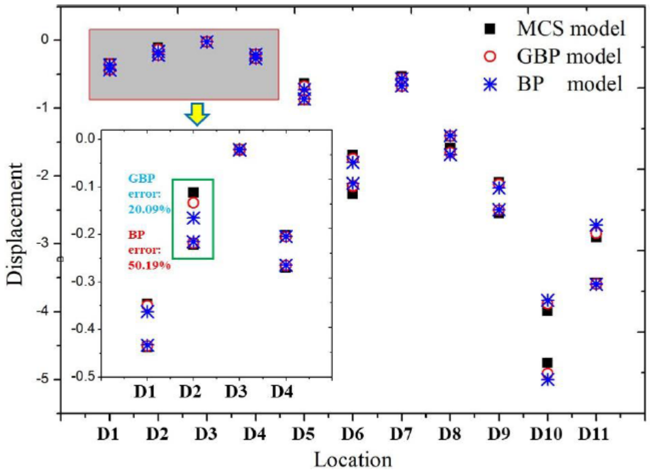



**Figure 10:** Interval prediction of different models under 5 training samples.

In this example, we complete the interval prediction based on the proposed GBP model for the engineering structure. Meanwhile, the results are compared with classical BP model under the 5 group training samples, which shows that the accuracy of the GBP model is more suitable for the limited number of samples (Figure **9**, Figure **10**, Table **6**).

## 5. CONCLUSIONS

At present, the engineering system becomes more and more complex, and the experiment cost bring the new challenge for design of structures. Therefore, a large number of experimental samples are difficult to obtain. However, uncertain factors widely exist in the structure design. We must consider the effects of uncertainties and predict the range of response as accurately as possible to provide the reference for structure design.

In view of the above discussion, this paper proposed a new method of interval prediction under limited number of samples for complex structure. The method was proposed based on the grey mathematical theory and BP neural networks. Combining with the advantages of the two methods, we built Grey-BP neural networks and completed the interval prediction with help of the GA method. Results and discussion of this study can be summarized as follows:

(1) The classical BP neural networks can guarantee the prediction accuracy with enough training samples. In Figure **6**, the number of training samples is 30, the accuracy can be satisfied with the requirement and the accuracy will be higher with the training samples increasing. However, the serious error is presented with the training samples decreasing, which is not satisfied with the requirement and also demonstrates unsuitable for the limited samples.

(2) The Grey-BP neural networks represent the superiority, compared with the classical BP neural networks. The prediction accuracy of different models under different number of training samples also illustrates the advantages of the GBP model for dealing with issue of the limited samples. Obviously, the accuracy of that is higher than classical BP and more stable. For the engineering structure, the method may

**Table 6: Comparison of Interval Prediction under 5 Training Samples**

| Location | MCS Model | GBP Model | | BP Model | |
|---|---|---|---|---|---|
| | | Predicted Value | Interval Error | Predicted Value | Interval Error |
| 1 | [-0.4368,-0.3466] | [-0.4346,-0.3500] | 1.50% | [-0.4325,-0.3627] | 5.65% |
| 2 | [-0.2215,-0.1122] | [-0.2186,-0.1333] | 20.09% | [-0.2151,-0.1653] | 50.19% |
| 3 | [-0.0230,-0.0215] | [-0.0224,-0.0214] | 2.77% | [-0.0223,-0.0214] | 3.48% |
| 4 | [-0.2704,-0.2019] | [-0.2651,-0.2050] | 3.50% | [-0.2643,-0.2040] | 3.30% |
| 5 | [-0.8861,-0.6355] | [-0.8721,-0.6813] | 8.79% | [-0.8630,-0.7223] | 16.27% |
| 6 | [-2.2673,-1.6903] | [-2.1472,-1.7463] | 8.61% | [-2.1108,-1.8010] | 13.45% |
| 7 | [-0.6585,-0.5382] | [-0.6743,-0.5576] | 6.00% | [-0.6716,-0.5642] | 6.82% |
| 8 | [-1.5893,-1.4173] | [-1.6554,-1.4099] | 4.68% | [-1.6845,-1.4046] | 6.89% |
| 9 | [-2.547,-2.0963] | [-2.5034,-2.1222] | 2.95% | [-2.4974,-2.1759] | 5.74% |
| 10 | [-4.7565,-3.992] | [-4.9132,-3.8875] | 5.91% | [-4.9980,-3.8331] | 9.06% |
| 11 | [-3.5795,-2.9079] | [-3.5901,-2.8421] | 2.56% | [-3.5934,-2.7231] | 6.74% |

provide reliable theoretical support for an engineering application.

(3) Consider the uncertainties existing in engineering, the interval prediction is completed with help of IGA method. Similarly, the interval results obtained by different models are compared, which shows the applicability of GBP model for limited samples. In Table **6**, the maximum interval error of GBP is 20.09%, the interval error of BP model is 50.19%.

Apparently, the response prediction of complex structure is a complicated issue, especially for the limited samples due to the high experimental cost. However, the accurate prediction is most important for structure design. Therefore, in this paper, we proposed one relatively reasonable strategy of interval prediction for the above problem and continue to modify in the follow-up work.

In the future, the credibility of intelligent learning models and methods under small sample conditions is a problem that needs to be addressed. Based on the credible quantification of uncertain parameters under small sample conditions, we will study the credibility of the predicted results of structural response interval.

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