All-World Solar Collector Optimum Slope Determination Basing on Site Latitude and Day Number

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Abstract: A rule of thumb says that a solar collector should be orientated towards Equator; is it valid all over the world? A rule of thumb says that solar collector should have a latitude tilt value; is it valid all over the world and all over the year? The present work focuses on presenting an algorithm for determining the optimum tilt angle all over the world and for any collector azimuth angle. The Earth surface, located between latitudes 66.45°S and 66.45°N, is divided into 3 characteristic zones. The first zone is the tropical between latitudes 23.45°S and 23.45°N. The second zone is the mid-latitude zone between 23.45°N and 43.45°N and between 23.45°S and 43.45°S. The third zone is the high-latitude zone between 43.45°N and 66.45°N and between 43.45°S and 66.45°S. For each of these zones an adequate method is proposed for calculating the solar collector optimum tilt. Moreover, four simple equations are proposed for predicting daily optimum tilt angle and optimum tilt angle for any number of consecutive days. The yearly possible energy gain in relation that received by a horizontal surface and latitude tilted surface is also calculated on the basis of daily optimum tilts. It is found that the above mentioned rules of thumb are not applicable for a number of consecutive days in the year.

Keywords: Optimum tilt, general formulae, pole facing, equator facing, energy gain.

1. INTRODUCTION

The performance of a solar collector is highly influenced by its orientation (with respect to the Equator) and its angle of tilt with respect to the local horizontal plane. This is due to the fact that both the orientation and tilt angle change the solar radiation reaching the surface of the collector. Taking into consideration that designing an installation to yield maximum annual energy helps to minimize the necessary installed capacity and reduce the cost of equipment. To achieve this, a solar collector must be mounted at right angles to the Sun's rays. The best way to collect maximum daily energy is to use tracking systems. A tracker is a mechanical device that follows the direction of the Sun on its daily course across the sky. The most effective tracking could be achieved by mounting the collector on a two-axis tracker that continuously tracks the Sun.

There are number of studies showing that tracking systems enable significant amount of solar energy compared to fixed systems. Abdallah [1] found that tracking systems increase total daily energy collection of about 43.87% as compared with a fixed system. A very detailed review of energy gain of different trackers is done in Mousazadeh *et al*. [2]. In that paper the authors report a boost of collected solar energy by means of a tracking system in the range of 10-100% depending on different time periods and geographical conditions. Tomson [3] reported increasing of seasonal energy yield by 10-20% if using the two-positional tracking system that positions collectors in the morning and in the afternoon. Chang [4] found substantial gains of 51.4%, 28.5% and 18.7% from the extraterrestrial, predicted and observed radiations, respectively, by using a single-axis tracking system. However, sun-tracking systems are quite expensive and energy intensive. Therefore, sun tracking is quite cumbersome and inconvenient practically and sun trackers are not recommended for use with small solar panels ([2, 3]). Thus, the majority of installations are with fixed mountings. Therefore, it is often practicable to orient the solar collector at an optimum tilt angle, B_{opt} and to correct the tilt from time to time. So, optimally orienting the collector maximizes the solar energy collected. For this purpose, one should be able to determine the optimum slope of the collector at any latitude, for any surface azimuth angle, and on any day or any period of the year. Various schemes have been proposed for optimizing the tilt angle and orientation of solar collectors designed for different geographical latitudes or possible utilization periods. However, the tilt angle optimization has been extensively addressed in many articles and several attempts were made to determine, or at least to estimate, the optimum tilt angle B_{opt} theoretically and experimentally.

There are number of studies that were carried out in order to find the optimum tilt angle of solar collectors

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around the world (Carbondale, Illinois [5], Izmir, Turkey [6], Sanliurfa, Turkey [7], Dhaka [8], 30 cities in China [9], Madinah, Saudi Arabia [10], Jordan [11], Helwan, Egypt [12], Brunei Darussalam [13], Syria [14], Cyprus [15], Burgos, Spain [16], Brisbane, Australia [17], Malaysia [18], Philippine [19], [20], Athens basin area [21], Mediterranean region [22], Zaria, Nigeria [23], WA, Ghana [24], Kano [25], Zomba district, Malawi [26] and many more).

Soulayman [27] proposed a general algorithm for calculating daily $B_{opt,d}$, monthly $B_{opt,m}$, seasonally $B_{opt,s}$, and yearly $B_{opt,y}$, for a south facing collector at any latitude from 0°N to 60°N. Soulayman and Sabbagh [28] proposed an algorithm for determining $B_{opt,\mathrm{d}}$, $B_{opt,\mathrm{m}}$, $B_{opt,\mathrm{s}}$, and $B_{opt,\mathrm{y}}$ at any latitude, L, and for any direction (surface azimuth angle, G). Stanciu, and Stanciu [29] proposed a simple formula for determining the optimum tilt of a south facing collector at latitudes from 0°N to 80°N. Nijegorodov *et al*. [30] presented 12 equations (one for each month), for determining optimum tilt angle for any location that lies between latitude 60°S to 60°N. Mujahid [31] computed the optimum slope angle for latitude of 10°N to 50°N and concluded that if the collector is adjusted by the seasonally optimum angles, 10% more energy is received compared with the zero slope angle. Calabrò [32] proposed an algorithm to calculate the optimum tilt angle of solar panels by means of global horizontal solar radiation data, provided from Earth-based meteorological stations. Some comments in regard to the applicability of the formula $B_{opt,d} = L\text{-}\delta$ proposed in [29] is given in [33]. The effects of latitude, solar reflectivity, and clearness index were considered by Elsayed [34] in determining the optimal tilt angle analytically. Tang and Wu [9] used the monthly horizontal radiation to develop a simple mathematical procedure allowing the determination of the optimal tilt angle. For the cold seasons, Chiou *et al*. [35] developed a method for calculating the optimal tilt angle for south orientating collectors. Darhmaoui and Lahjouji [22] proposed to determine the yearly optimum tilt $B_{opt,y}$ as a function of latitude L as follows $B_{opt,y}$=1.25351L-0.00728944L^2. So, in the previous studies, no definite value or relation has been found by the researchers for the optimum tilt angle. Moreover, it can be noticed from the literature, that the optimal tilt angle is considered in many studies as location-dependent.

In this paper, a general algorithm is proposed for treating B_{opt} all over the world. The theoretical aspects that determine the optimal tilt angle, regarding to maximum solar energy collection, are examined. A computer program using the mathematical equation of the proposed algorithm is implemented. The computer program is used in determining the optimal tilt angle of any site at the Earth's surface between 66.45°S and 66.45°N. A regression analysis using site's latitude, solar declination angle and its corresponding optimal tilt angle is conducted to develop a mathematical model that allows the determination of the optimal tilt angle at which maximum solar radiation is collected using only the site's latitude and the day number of the year. A comparison with available experimental and theoretical results from other researchers is provided.

2. METHODOLOGY / ALGORITHM

Radiation data are the best source of information for estimating average incident radiation. Lacking such data, it is possible to use empirical relationships to estimate radiation from hours of sunshine or cloudiness, relative humidity and ambient temperature, which are widely available from many hundreds of stations in most countries. The main part of empirical relationships is restricted to sunshine duration or cloudiness. However, these relationships could be written as:

$$H = f\left(\frac{n}{N}, C, RH, T\right) H0 \qquad (1)$$

where H = monthly average daily radiation on a horizontal surface, H0 = the monthly average daily extraterrestrial solar radiation on a horizontal surface, n = monthly average daily hours of bright sunshine, N = monthly average of the maximum possible daily hours of bright sunshine (i.e., the day length of the average day of the month), C = monthly average daily cloud cover, RH = monthly average relative humidity and T = monthly average ambient temperature. Supposing Eq. (1) is applicable for daily values and differentiating it in relation to surface tilt angle B, a nonlinear algebraic equation is obtained. By equating the left part of the derived equation to zero, the daily optimum tilt angle $B_{opt,d}$ could be obtained. So, the daily optimum tilt angle, $B_{opt,d}$ is the solution of the following nonlinear algebraic equation:

$$C(N)\left\{\begin{array}{l} \left(\frac{\partial A2}{\partial B}\right)\left[\sin\left(W_{SS}\right)-\sin\left(W_{Sr}\right)\right] \\ +A2\left[\cos\left(W_{SS}\right)\left(\frac{\partial W_{SS}}{\partial B}\right)-\cos\left(W_{Sr}\right)\left(\frac{\partial W_{Sr}}{\partial B}\right)\right]+\left(\frac{\partial A1}{\partial B}\right)\left(W_{SS}-W_{Sr}\right) \\ +A1\left(\frac{\partial W_{SS}}{\partial B}-\frac{\partial W_{Sr}}{\partial B}\right)-\left(\frac{\partial A3}{\partial B}\right)\left[\cos\left(W_{SS}\right)-\cos\left(W_{Sr}\right)\right] \\ +A3\left[\sin\left(W_{SS}\right)\left(\frac{\partial W_{SS}}{\partial B}\right)-s\,\mathrm{in}\left(W_{SS}\right)\left(\frac{\partial W_{Sr}}{\partial B}\right)\right] \end{array}\right\}=0 \qquad (2)$$

in relation to the surface tilt B, where C(N) is the N^{th} day correction factor for the Sun-Earth average distance:

$$C(N) = 1 + 0.034\cos\left(\frac{2\pi N}{365}\right) \quad (3)$$

δ^o is the solar declination angle which could be calculated using the equation of Cooper [36]:

$$\delta = 23.45\sin,\left[\frac{2\pi(N+284)}{365}\right] \quad (4)$$

Wss (rad) is the sunset hour angle on a tilted surface:

$$Wss = \min\left\{\begin{array}{l}-\arccos\left[-\tan(\delta)\tan(L)\right], \\ -\arccos\left(-\frac{A1}{A4}\right)+\arcsin\left(\frac{A3}{A4}\right)\end{array}\right\} \quad (5)$$

Wsr (rad) is the sunrise hour angle on a tilted surface:

$$Wsr = \max\left\{\begin{array}{l}-\arccos\left[-\tan(\delta)\tan(L)\right] \\ ,-\arccos\left(-\frac{A1}{A4}\right)+\arcsin\left(\frac{A3}{A4}\right)\end{array}\right\} \quad (6)$$

A1, A2, A3 and A4 are functions of solar and collector angles:

$$A1 = \sin(\delta)\left[\sin(L)\cos(B) - \sin(B)\cos(L)\cos(G)\right] \quad (7)$$

$$A2 = \cos(\delta)\left[\cos(L)\cos(B) - \sin(B)\sin(L)\cos(G)\right] \quad (8)$$

$$A3 = \cos(\delta)\sin(B)\sin(G);\ A4 = \left(A2^2 + A3^2\right)^{1/2} \quad (9)$$

The analytical solution of Eq. (2), in the case of Wss tilt angle independence, is:

$$Bopt,d = L - \arctan\left[Ws\tan(\delta)/\sin(Ws)\right] \quad (10)$$

where Ws (rad) is the sunset hour angle on a horizontal surface:

$$Ws = arccos\left[-\tan(\delta)\tan(L)\right] \quad (11)$$

As for Equator facing (EF) and Pole facing (PF) surfaces in both Northern Hemisphere (NH) and Southern Hemisphere (SH) Wss = -Wsr and sin(G) =0. So, A3 =0. Wss is independent of tilt angle on PF surfaces at equator and for EF and PF surfaces for other latitudes in NH and SH for the period starting on 22/9 and ending on 21/3 in NH and for the period starting on 22/3 and ending on 21/9 in SH. For other periods Newton's iteration scheme could be applied for searching $B_{opt,d}$.

Here, it should be noted that before calculating the $B_{opt,d}$ it is reasonable to search about the interval of the possible change of this angle. The following condition should be satisfied:

$$sign\left(B_{opt,d}\right) = sign(L-\delta)\ elsewhere B_{opt,d} = 0^o \quad (12)$$

This means that, ịfimagining an axis of solar noon incident angle on a horizontal surface, θ_{noon} = L-δ changes as *θnoon* is positive when solar rays are incident from south direction and *θnoon* is negative when solar rays are incident from north direction; then the sign of *θnoon* determines the period where the solar collector should be oriented to Equator or to opposite direction. This may be done by studying the dependence of *θ noon* as function of N.

The optimum tilt, $B_{opt,p}$ over a period of several consecutive days, is the solution of the following nonlinear algebraic equation:

$$\sum C(N)\left\{\begin{array}{l}\left(\frac{\partial A2}{\partial B}\right)\left[\sin(W_{SS})-\sin(W_{Sr})\right] \\ +A2\left[\cos(W_{SS})\left(\frac{\partial W_{SS}}{\partial B}\right)-\cos(W_{Sr})\left(\frac{\partial W_{Sr}}{\partial B}\right)\right]+\left(\frac{\partial A1}{\partial B}\right)(W_{SS}-W_{Sr}) \\ +A1\left(\frac{\partial W_{SS}}{\partial B}-\frac{\partial W_{Sr}}{\partial B}\right)-\left(\frac{\partial A3}{\partial B}\right)\left[\cos(W_{SS})-\cos(W_{Sr})\right] \\ +A3\left[\sin(W_{SS})\left(\frac{\partial W_{SS}}{\partial B}\right)-\sin(W_{Sr})\left(\frac{\partial W_{Sr}}{\partial B}\right)\right]\end{array}\right\} = 0 \quad (13)$$

Where the summation covers the period in consideration. In the case of Wss tilt angle independence,

$$Bopt,p = L - \arctan\left[\sum C(N)Ws\sin(\delta)/\sum C(N)\cos(\delta)\sin(Ws)\right] \quad (14)$$

is the analytical solution of Eq. (13) which should be solved in accordance with condition (11).

This case takes place on Equator all over the year and in NH, for the period starting on 22/9 and ending on 21/3, and in SH, for the period starting on 22/3 and ending on 21/9. For other periods Newton's iteration scheme could be applied for searching $B_{opt,p}$.

The solution of Eqs (2) and (13) will be provided for Earth's surface, located between 66.45°S and 66.45°N after dividing it into 3 characteristic zones. The first zone is the tropical that is located between 23.45°S and 23.45°N. The second zone is the mid-latitude zone that is located between 23.45°N and 43.45°N and between 23.45°S and 43.45°S. The third zone is the high-latitude zone that is located between 43.45°N and 66.45° and between 43.45°S and 66.45°S.

3. TROPICAL ZONE

3.1. General Formulae for Predicting Optimum Tilts

When applying the above mentioned algorithm on the latitudes of the tropical region (23.5°S<L<23.5°N) and analyzing the obtained results as a function of δ and L it is found that Eq. (10) could be applied with a high accuracy for determining $B_{opt,d}$ all over the year. The absolute difference between the results of Eq. (10) and precise results, obtained from Eq. (2), does not exceed 0.2°. Figure **1** shows the results of applying Eq. (10) and Eq. (2) in determining $B_{opt,d}$ for three different latitudes. It is seen from Figure **1** that the differences between the results of these two equations are negligible. Moreover, when calculating the optimum tilt angle over a period of several consecutive days, it was found that Eq. (14) could be applied with a high accuracy for determining $B_{opt,d}$ all over the year. The absolute difference between the results of Eq. (14) and precise results, obtained using Eq. (13), does not exceed 0.2°. Figure **2** shows the results of applying Eq. (14) and Eq. (13) in determining $B_{opt,d}$ for three different latitudes. It is seen from Figure **2** that the differences between the results of these two equations are negligible.

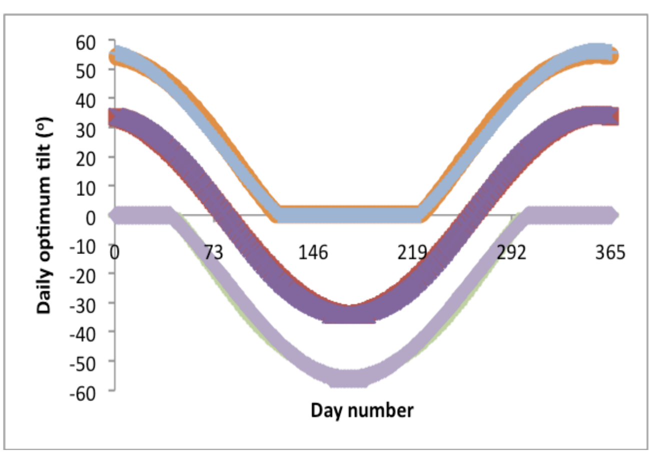

Figure 1: Daily optimum tilt angle for latitudes 23.5°S (- Eq. (2), ♦ Eq. (10)), 0° (x Eq. (2), ■ Eq. (10)) and 23.5°N (+ Eq. (2), ● Eq. (10)).

3.2 Comparison with Available Methods

Bari [18] proposed a procedure for calculating daily optimum tilt angle, $B_{opt,d}$, and optimum tilt angle, $B_{opt,p}$, for any number of days for different latitudes (1°N, 3°N, 5°N, 7°N) in Malaysia. Later on, he applied his procedure for the same purpose in Philippine (odd latitudes from L=5°N to L=19°N) [20] and Thailand (even latitudes from L=4°N to L=20°N) [37]. Stanciu and Stanciu [29] proposed a simple formula for determining the optimum tilt of a solar collector at latitudes from 0° to 80°N. Soulayman and Sabbagh [38] proposed an algorithm for determining daily $B_{opt,d}$ and for any number of consecutive days $B_{opt,p}$ in the tropical region and introduced a new idea regarding Equator facing, North Pole facing and South Pole facing conditions. Nigegorodov *et al*. [30] presented 12 equations (one for each month) for determining optimum tilt angle for any location that lies between 60°S to 60°N. Oko and Nnamchi [39] studied theoretically the optimum tilt angles for the territory of Nigeria (L=4.86- 13.02°N) and provided expressions for monthly different optimum tilt angles with respect to low latitudes.

Here it should be mentioned that [29] formula is just to calculate solar noon incidence angle on a horizontal plane. So, it is not suitable for calculating $B_{opt,d}$. The comparison with other methods proposed for calculating $B_{opt,m}$ and $B_{opt,p}$ is provided in [38]. So the comparison with [18] is required. Figure **2** shows the calculated results using Bari [18] method and the method of this work for latitudes 1°N and 7°N. It is seen that the Bari equation overestimates $B_{opt,d}$ a little bit during summer and underestimates $B_{opt,d}$ a little bit

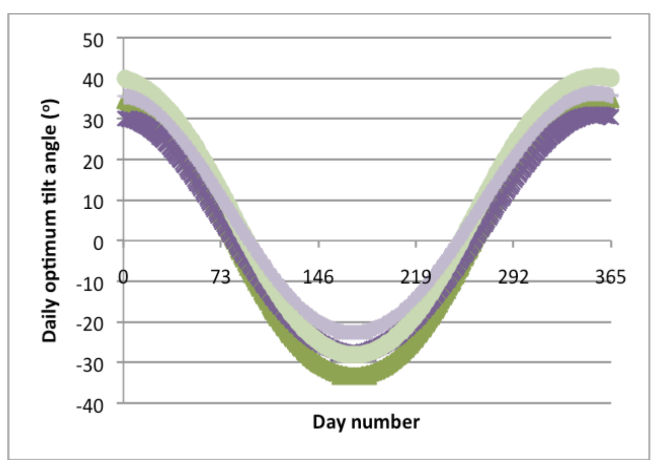

Figure 2: Daily optimum tilt angle for latitudes 1° and 7° calculated using [18] (violet for 7° and grey for 1°) and present work (green for 7° and light green for 1°).

during winter. Moreover, a reasonable agreement is achieved between these applied methods especially for latitudes L<10°N. In addition to the complexity of the Bari method, this does not pay attention to the condition (12) which means that Sun should be seen from the surface. So, with increasing L the number of days for which $B_{opt,d}$ =0° increases and the agreement between the results of Eq. (10) and Bari [18] method becomes more and more worthly.

4. MID-LATITUDE ZONE

4.1. General Formulae for Predicting Optimum Tilts

When applying the above mentioned algorithm on the mid-latitudes zone (23.45°S<L<43.45°S and 23.45°N<L<43.45°N) and analyzing the obtained results as a function of δ and L it was found that Eq. (10) could be applied with a reasonable accuracy for determining $B_{\text{opt,d}}$ all over the year. The absolute difference between the results of Eq. (10) and precise results, obtained from Eq. (2), increases with L but it does not exceed 2.6°. Therefore, the precise results calculated for mid- and high-latitude zones in NH and SH were fitted in a polynomial form:

$$Bopt,d = a\boldsymbol{\delta}^2 + b\boldsymbol{\delta} + c \quad (15)$$

where "a", "b" and "c" are functions of L. These functions are of the following form:

$$a = 0.0000027(-1)^i L^2 + 0.0001L + 0.01247(-1)^i \quad (16)$$

$$b = 0.00007328L^2 + 0.0036486(-1)^i L - 1.48819 \quad (17)$$

$$c = 0.0000026783(-1)^i L^2 + 1.01971L + 0.0104399(-1)i \quad (18)$$

where i=0, 1 for SH and NH, respectively. The absolute difference between the results of Eq. (15) and precise results, obtained from Eq. (2) does not exceed 1.5°.

The results of applying Eq. (15), Eq. (10) and the present algorithm are shown in Figure **3** for NH and Figure **4** for SH on the example of 30°N, 43.45°N, 30°S and 43.45°S. It is seen from these Figures that all three approaches are applicable in the mid-latitude zone.

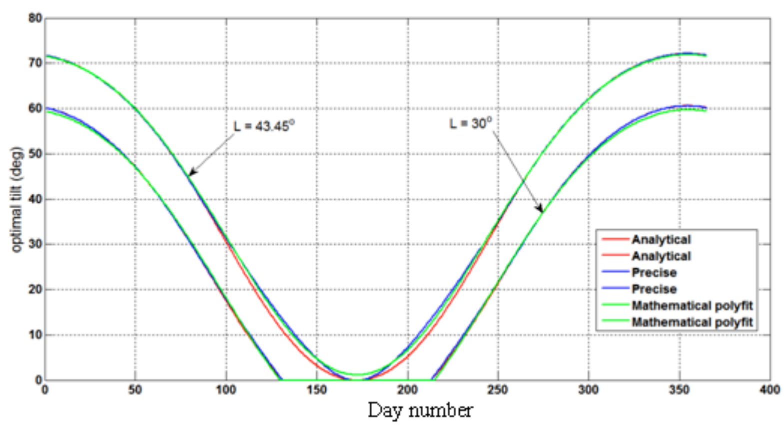

Figure 3: Calculated $B_{opt,d}$ for latitudes 30°N and 43.45°N using different approaches.

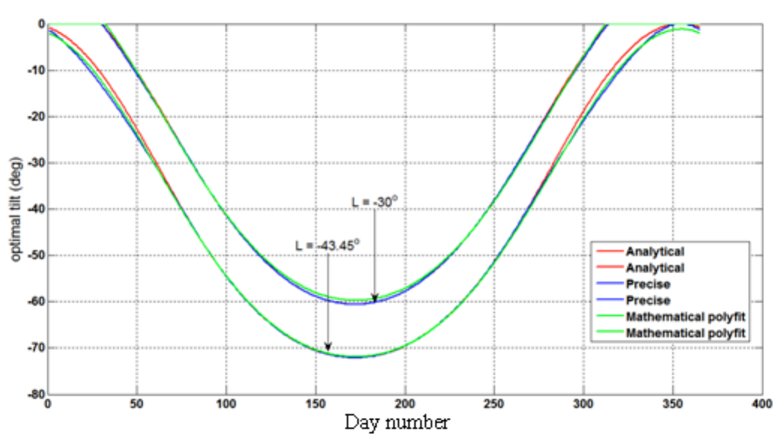

Figure 4: Calculated $B_{opt,d}$ for latitudes 30°S and 43.45°S using different approaches.

4.2. Monthly Optimum Tilt Angle

When integrating Eq. (15) for obtaining optimum tilt angle $B_{opt,p}$ at any period of time one obtains the following formula:

$$Bopt,p = \left[\frac{N2-N1}{N2-N1+1}\right] \left\{ c + 549.9a \left[\begin{array}{l} 0.5 - \left(\frac{365}{4\pi}\right) \\ \cos\left[\left(\frac{2\pi}{365}\right)(N2-N1+568)\right] \\ *\sin\left[\frac{2\pi}{365}\right](N2-N1) \end{array} \right] \right\} + \left[\frac{23.45\left(\frac{365}{\pi}\right)b}{N2-N1+1}\right] \sin\left[\left(\frac{\pi}{365}\right)(N2-N1)\right] * \sin\left[(\pi/365)(N2+N1+568)\right] \quad (19)$$

where N_1 and N_2 are the day numbers of the period beginning and ending respectively.

When applying Eqs (13), (14) and (19) in determining $B_{opt,p}$ for the latitudes of the mid-latitude zone it was found that, the agreement between their results is of the same order as that for optimum daily tilt angle $B_{opt,d}$ in NH and SH. The absolute difference between the results of Eq. (19) and precise results, obtained from Eq. (14), does not exceed 2.6^{o}. Figures **5** and **6** show the differences between the precise results of Eq. (2) and those resulted by applying the analytical Eq. (10) and the polynomial Eq. (15) in determining $B_{opt,d}$ for 2 different latitudes, respectively. It is seen from these Figures that the mentioned differences do not exceed 2.6^{o}.

4.3. Comparison with Available Methods

El-Kassaby [40] proposed a procedure for calculating daily optimum tilt angle, $B_{opt,d}$, for any latitude in NH using Eq. (10). The results of the present work

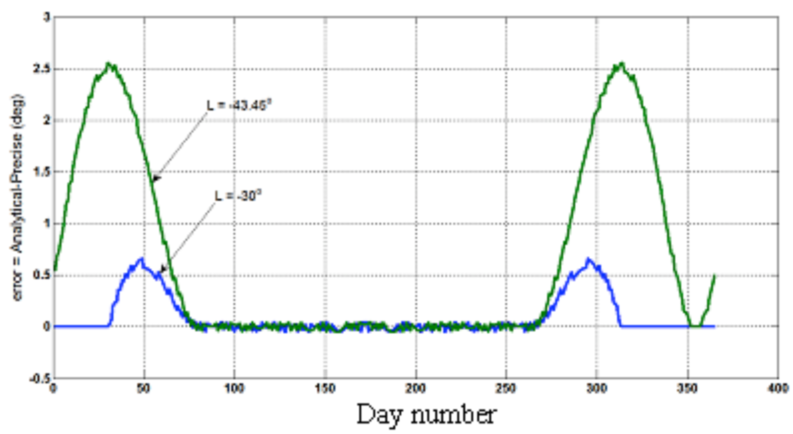

Figure 5: The differences between the calculated results of $B_{opt,d}$ using the precise approach and the analytical formula for latitudes 30°S and 43.45°S.

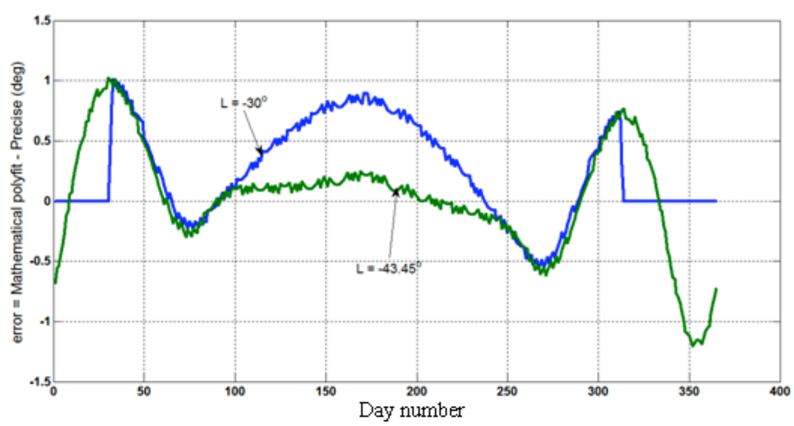

Figure 6: The differences between the calculated results of $B_{opt,d}$ using the precise approach and the polynomial fit for latitudes 30°S and 43.45°S.

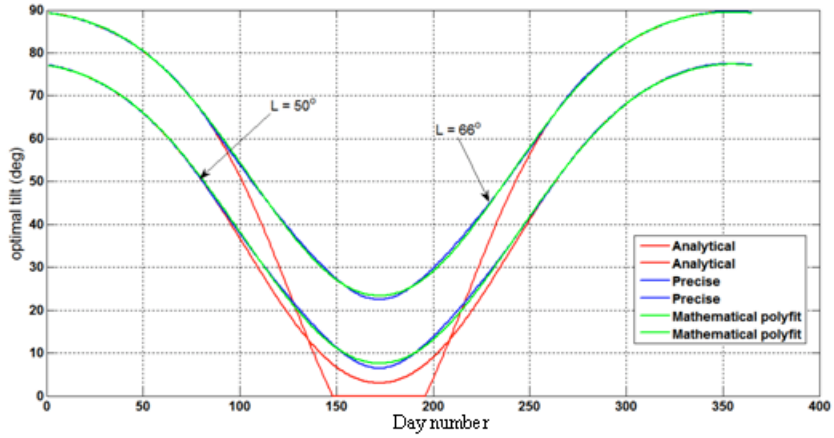

Figure 7: Calculated $B_{opt,d}$ for latitudes 50°N and 66°N using different approaches.

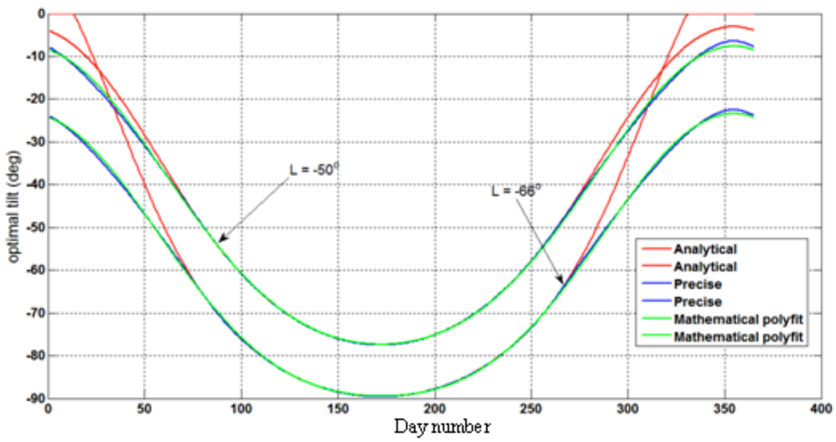

Figure 8: Calculated $B_{opt,d}$ for latitudes 50°S and 66°S using different approaches.

demonstrate clearly that El-Kassaby proposal should be:

- Conjoined with the condition (12).
- Restricted to the latitudes lower than 40°.

The same comments could be addressed to the work of Skeiker [14]. Soulayman and Sabbagh [28] and [41] discussed in [14] in detail.

5. HIGH-LATITUDES ZONE

5.1. General Formulae for Predicting Optimum Tilts

When applying the above mentioned algorithm to the high-latitude zone (43.5°S <L<66.45°S and 43.5°N<L<66.45°N) and analyzing the obtained results as a function of δ and L it is found that Eq. (10) could not be applied with an acceptable accuracy for determining $B_{opt,d}$ all over the year. Eq. (10) is a precise solution of Eq. (2) over a half-year (from 22/9 to 21/3 in NH and from 22/3 to 21/9 in SH) while the polynomial fit [Eq. (15)] describes the $B_{opt,d}$ dependence on latitude and day number very well. Figures **7** and **8** show the results of applying these three approaches in NH and SH, respectively. It is seen from Figures **7** and **8** that the analytical approach [Eq. (10)] is restricted to a period the length of which decreases with increasing L, while the polynomial fit [Eq. (15)] is a very good approximation (see Figures **9** and **10**).

It is seen from Figure **9** that the absolute difference between the results of Eq. (10) and the precise results, obtained using Eq. (2), increases with L increase and it reaches 28°, while this difference does exceed 1.2° in the case of the polynomial fit [Eq. (15)].

Therefore, the polynomial fit could be applied for predicting $B_{opt,d}$ all over the year in the studied third zone (high-latitude one). This means that the optimum tilt angle over any period of consecutives days could be calculated using Eq. (19) with a very good accuracy. So, Eq. (19) could be applied for calculating the weekly, fortnightly, monthly, seasonally, half-yearly and yearly optimum tilt angles.

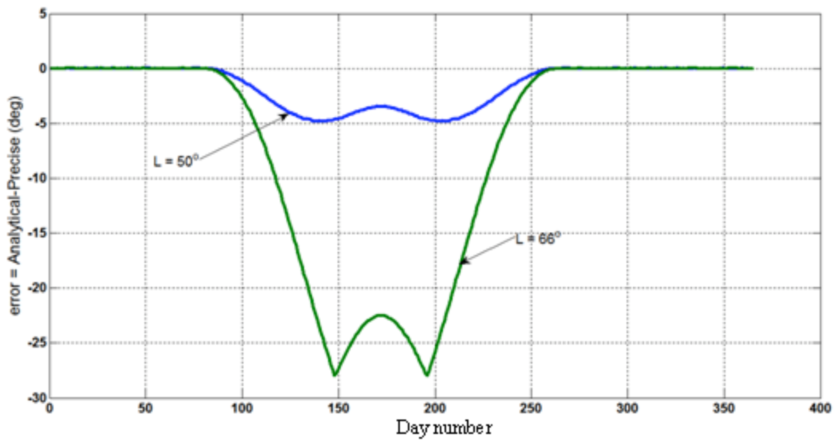

Figure 9: The differences between the calculated results of $B_{opt,d}$ using the precise approach and the analytical formula for latitudes 50°N and 66°N.

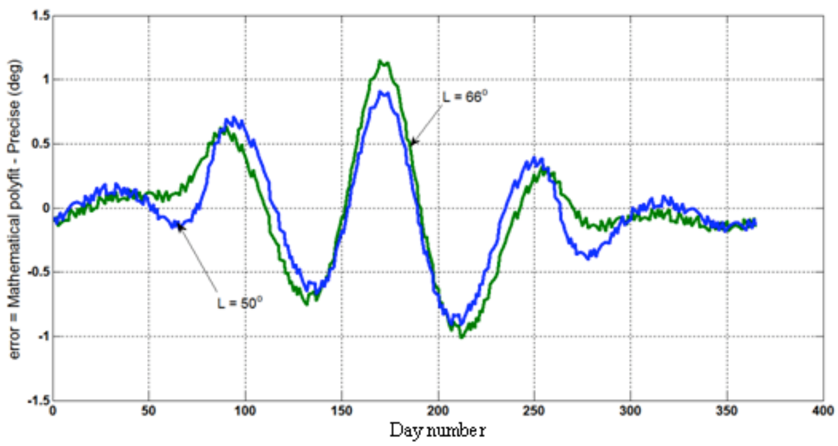

Figure 10: The differences between the calculated results of $B_{opt,d}$ using the precise approach and the polynomial fit for latitudes 50°N and 66°N.

5.2. Comparison with Available Methods

El-Kassaby [40] proposed a formula for determining $B_{opt,d}$ at latitudes up to 60° in NH by verifying the applicability of his formula during the period from 22/9 to 21/3 but his formula suffered from uncertainty during the other period of the year (22/3 to 21/9) (see [27] for more details). Skeiker [14] presented a study aiming to develop an analytical procedure to obtain a formula for determining $B_{opt,d}$ for any chosen day at any latitude in NH but he repeated the same formula of El-Kassaby [40] (see [28] for more details). So, no need to compare the results of the present work with those of [40] and [14]. Figure **11** shows the results of applying Eqs (2) and (15) for determining $B_{opt,d}$ for 60°S. Figure **11** shows also the results of applying Eq. (2), the algorithm of [28] and the results of [27] for 60°N. It is seen from Figure **11** that the results of the present work agree very well with those of [27] and the algorithm of [28]. Moreover, Eq. (15) gives an excellent fit of the calculated results of the algorithm in this work.

When calculating the monthly optimum tilt angle at 60°S using Eq. (19) and comparing the obtained results with those of set of equations in [30] and set of equations in [42] with taking into consideration the latitudes interval of the applicability of the mentioned references it is observed that in the SH the results of the present work are in a very good agreement with those of [30], and [42] (see Table **1**). However, the equations of [30] give a little bit higher values of optimum angle for the months of April, May, and August as compared with those of [42] and present work while the agreement for other months is very good.

It could be noted that the methodologies used in [30], [42] and present work are based on the optimization using mathematical techniques without taking into account the localized patterns of solar radiation falling over a particular location (region). So, one or more assumptions of [30] are responsible for this little disagreement. Nijegorodov *et al*. [30] used

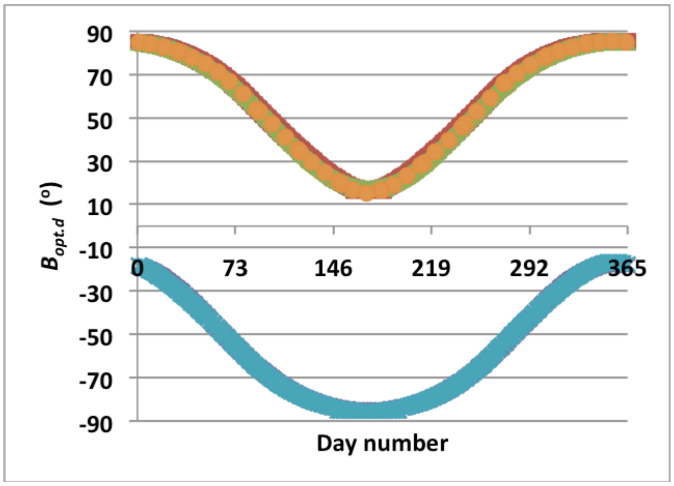

Figure 11: Daily optimum tilt angle for latitudes 60° S in SH (lower curve) and 60°N in NH (upper curve).

mathematical models for calculating the hourly total radiation and then integrated them to obtain the total daily solar radiation. The main assumption of [30] is the clearness index is constant and equal to 0.7. As the clearness index is not generally constant and varies for each area between two values the slight difference could be due to this assumption.

Table 1: Monthly Optimum Tilt Angles at 60° S

Month	L=60°S		
	Eq. (19)	[30]	[42]
1	-22.3	-24.4	-23.7
2	-36.6	-41.2	-38.5
3	-54.6	-56	-56.3
4	-69.9	-70	-72.6
5	-78.9	-79.8	-81.9
6	-82.1	-86.2	-85.6
7	-80.7	-83.4	-83.9
8	-74.0	-75.2	-76.6
9	-60.8	-62	-62.6
10	-42.8	-48	-44.0
11	-25.9	-30.8	-27.4
12	-17.6	-18.2	-17.7
MBE		2.41766	1.425014
RMSE		2.98281	2.234652
t		4.58982	2.745675
R^2		0.994	0.999

6. ENERGY GAIN

In order to evaluate the possible solar energy gain using tilt angle adjustment the total yearly solar radiation at $B = 0^o$, $B = B_{opt}$, $B = L$ should be calculated on a daily, monthly, seasonally as well as on a half – yearly and yearly basis. It was found that the solar radiation incident on a surface with optimum tilt angle contains maximum values all over the year. When comparing solar radiation received on surface of optimum tilt, H_1, with that received on surface of latitude angle tilt, H_2, it was found that H_1 is greater than H_2 remarkably over the period from 22/9 to 21/3 while these values are near to each other during the remaining half year (from 22/3 to 21/9).

By taking the ratio between the values related to the surface with optimum tilt to those on a horizontal one and to those tilted at the latitude angle for the same period, the corresponding tilt factors, $R_b = H_1/H_0$ and $R_{b1} = H_1/H_2$, could be calculated on a daily, monthly, seasonally, half-yearly and yearly basis. These results are given in Figures **12-14**. It is seen from Figure **12**

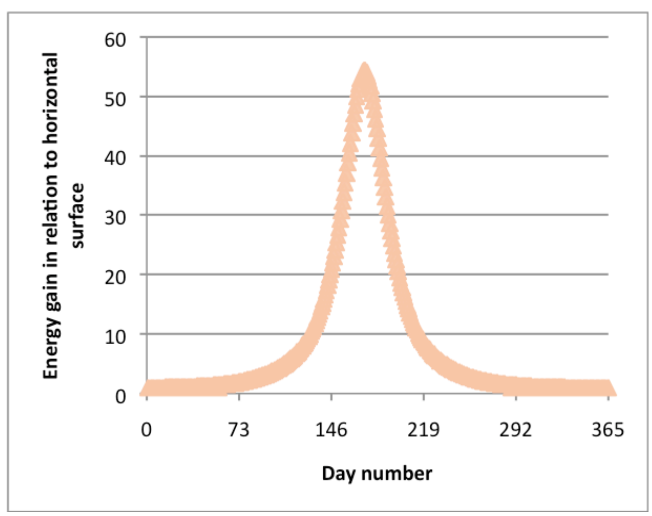

Figure 12: Daily solar energy gain in relation to horizontal surface for latitude 65°S.

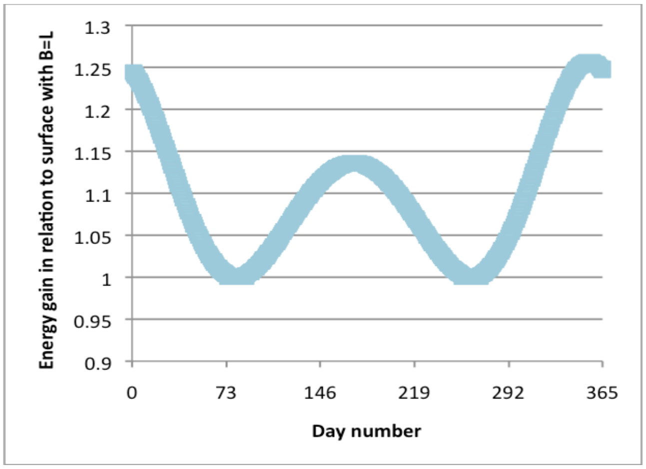

Figure 13: Daily solar energy gain of $B_{opt,d}$ in relation to that tilted surface with *B=L* at 65°S.

that for the latitude 65°S the daily solar energy gain can reach about 55 times of the solar radiation on a horizontal plane while this gain is less than 2 times that of a solar radiation on horizontal plane during the period from 22/9 to 21/3.

Figure **13** shows that the daily solar energy gain can arrive 1.27 times that of the solar radiation received on a plane tilted at the latitude during the period from 22/9 to 21/3 while this gain is less than 1.15 times that of the solar radiation on latitude tilted plane during the period from 22/3 to 21/9. Finally, Figure **14** shows that the monthly adjustment is sufficient in solar applications.

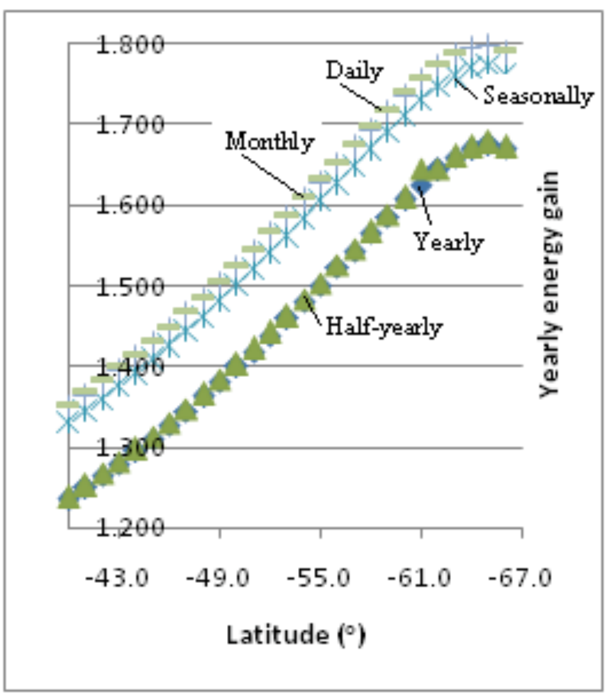

Figure 14: Solar energy gain in relation to a horizontal surface for latitudes [40°S - 66°S].

CONCLUSIONS

A mathematical model was applied for determining the optimum tilt angle of a solar collector at any latitude of the interval [66.45°S, 66.45°N] on the Earth's surface. The optimum tilt angle was computed by searching for the values for which the radiation on the collector surface is maximum for a particular day or a specific period. It should be notified that:

- The rule of thumb that says that a solar collector should be orientated towards the Equator is true for mid- and high-latitudes zones only while is partially true for the tropical zone.
- A second rule of thumb that says that a solar collector should be tilted at an angle equal to the latitude of the location is not valid at all in the tropical zone while it could be applied with a reasonable accuracy during half a year in mid- and high-latitude zones.
- A general algorithm was proposed for determining the optimum daily tilt angle and optimum tilt angle for any number of days.
- General formulae were proposed for determining the optimum daily tilt angle and optimum tilt angle for any number of days with a very good accuracy.
- The applicability of the proposed formulae was verified.
- It is sufficient to adjust the solar collector's tilt angle weekly (once a week) as this adjustment leads to the daily gain approximately.
- It is sufficient to adjust the solar collector's tilt angle 12 times (once a month) as this adjustment leads to the daily gain approximately (see Figure **14**).
- It is practically sufficient to adjust the solar collector's tilt angle 4 times (once a season) as the losses in the energy gain are not important (see Figure **14**).

It is practically sufficient to orientate the solar collectors at a tilt angle equal to the latitudes of the location during half a year as the losses in the energy gain are less than 0.2.

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NOMENCLATURE

- B Tilt angle (°) which is positive for EF case and negative for PF cases
- $B_{opt,d}$ Daily optimum tilt angle (°)
- $B_{opt,w}$ Weekly optimum tilt angle (°)
- $B_{opt,m}$ Monthly optimum tilt angle (°)
- $B_{opt,p}$ Optimum tilt angle over a number of days (°)
- δ Solar declination angle (°)
- EF Equator facing
- EFP Equator facing period
- G Collector azimuth angle (°)
- H0 Monthly average daily extraterrestrial solar radiation on a horizontal surface (MJ/m^2)
- H Monthly average daily solar radiation on a horizontal surface (MJ/m^2)
- L Latitude (°) which is positive at NH and negative at SH
- N Day number in the year starting from January 1st
- NH Northern Hemisphere
- NPF North Pole facing
- N_1 Day number of the beginning of the period
- N_2 Day number of the end of the period
- PF Pole facing
- PFP Pole facing period
- R_b Geometric tilt factor
- SH Southern Hemisphere
- SPF South Pole facing

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